HFOFO study

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2/21/2025

Discussion item

- RF cavity design in cooling channels
- Next goal

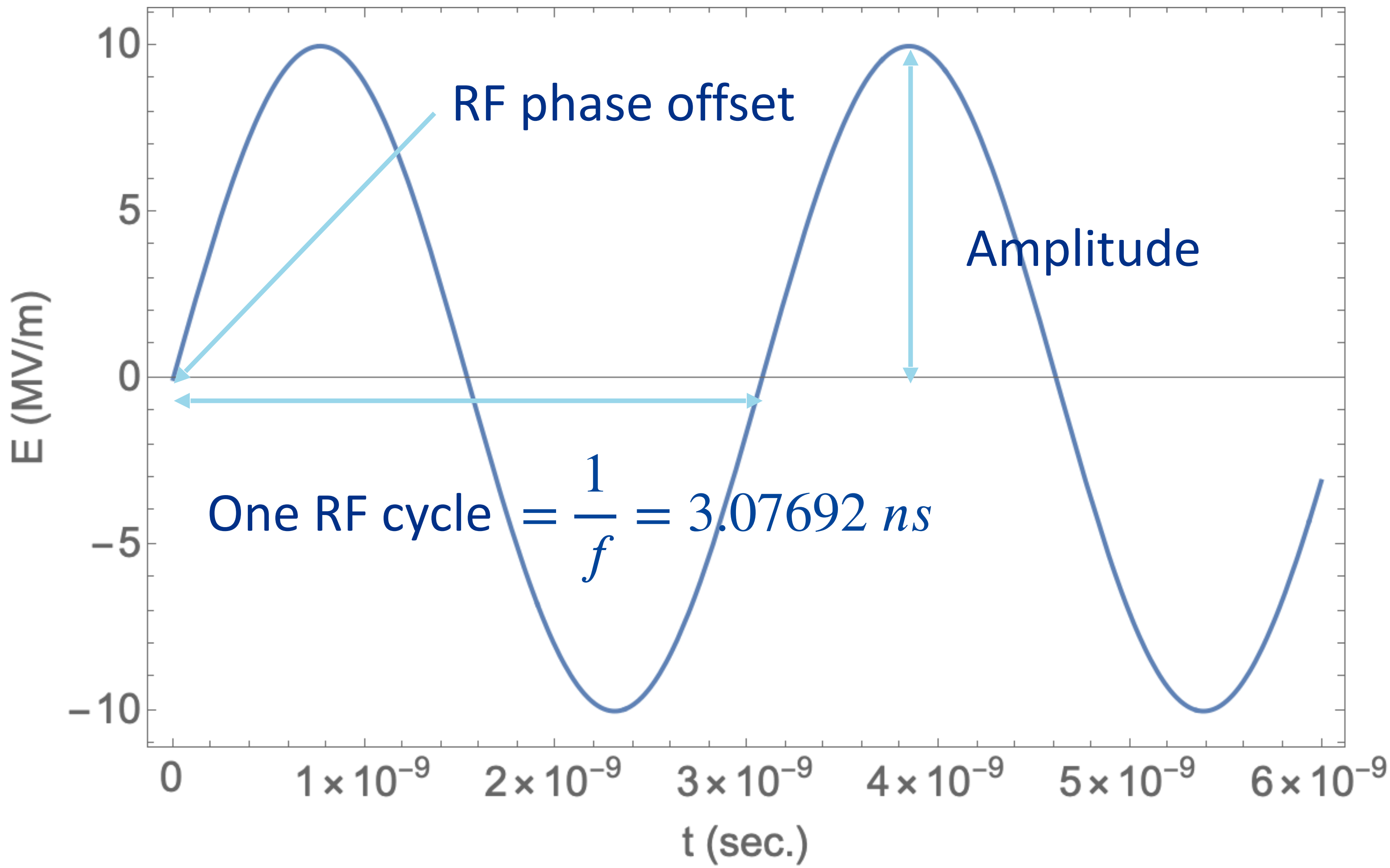
RF cavity in g4beamline (General concept)

- Pillbox
  - TEM010 (fundamental mode)
- Function
  - Gain/restore kinetic energy of muons through RF electric fields
- Fundamental RF parameter in g4beamline
  - Peak RF gradient (MV/m): A
  - Resonant frequency (MHz): f
  - Phase offset (degrees): ϕ_0, or time offset (ns): t_0
  - $E(t) = A \bullet sin\left(2\pi f \bullet t - \phi_0\right) = A \bullet sin\left(2\pi f \bullet \left(t - t_0\right)\right) \rightarrow \phi_0 = 2\pi f \bullet t_0$
- Distribute RF cavity along the beam path

RF electric field (General concept)

$f = 325\ MHz$

$A = 10\ MV/m$



RF phase offset

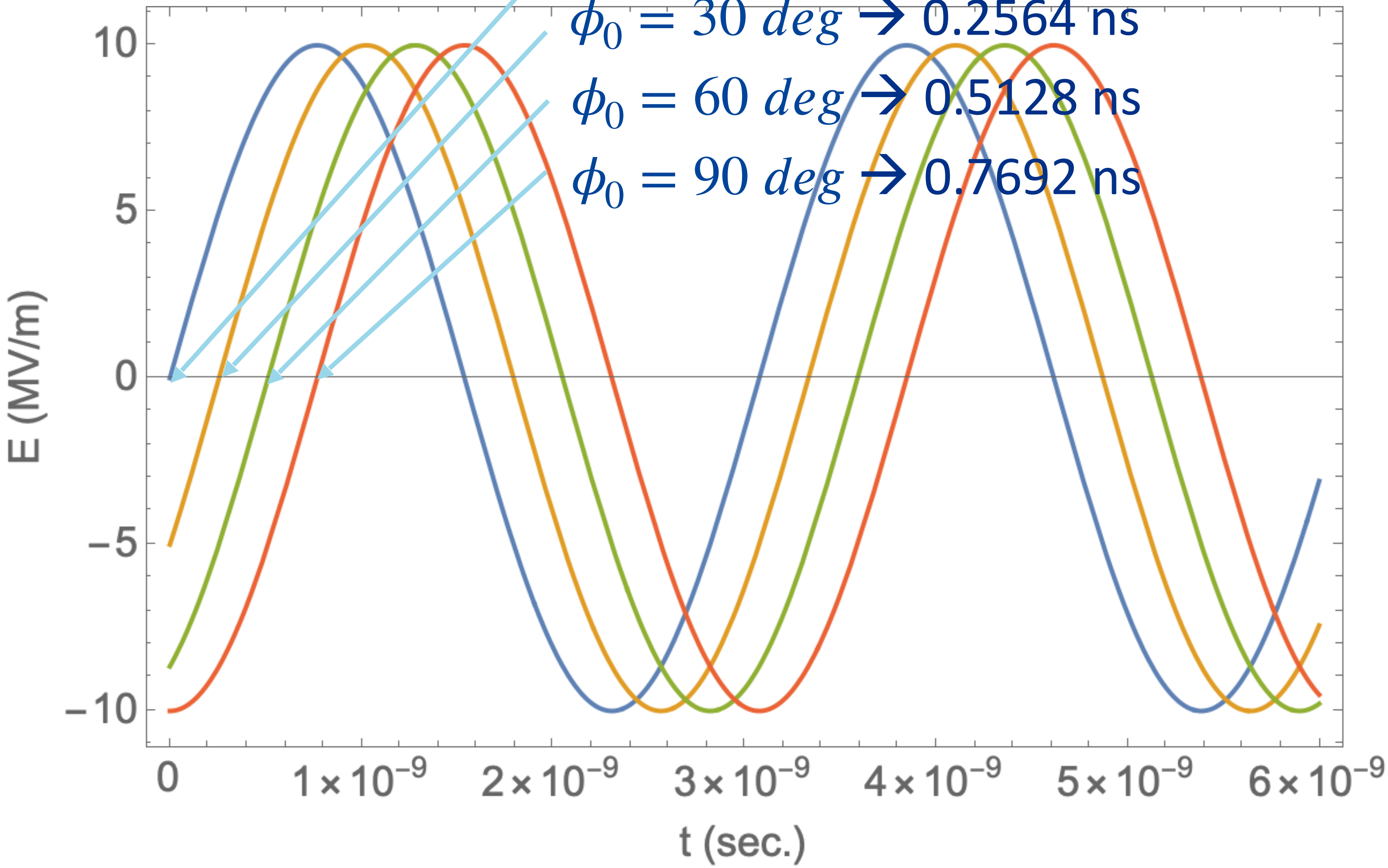
$$t_0 = \frac{\phi_0}{2\pi f}$$

$\phi_0 = 0$

$\phi_0 = 30\ deg$ → 0.2564 ns

$\phi_0 = 60\ deg$ → 0.5128 ns

$\phi_0 = 90\ deg$ → 0.7692 ns



Time of flight (HFOFO specific)

param beamstart=-700 # from the rotator solenoid exit at z=0
param beamtime=-0.671 # average time of mu+ in initial.dat modulo RF period TRF

param toffs0=$beamtime-$beamstart/275.89-.07 # time needed to travel from beamZ to 0

Let us get this number

place RFC0 z=-425. timeOffset=$toffs0-0.77123842
place RFC0 z=-175. timeOffset=$toffs0+0.13491993
place RFC0 z=75. timeOffset=$toffs0+1.0410783
place RFC0 z=325. timeOffset=$toffs0+1.9472366
place RFC0 z=575. timeOffset=$toffs0+2.853395

$$\beta = \frac{p}{E},\ \gamma = \frac{E}{m}$$

If p=248 MeV/c, m =105.7 MeV

$$v = \beta c = \frac{p}{E}c = \frac{pc}{\sqrt{p^2 - m^2}} = 2.7579 \bullet 10^8 \text{ m/s}$$

=0.27579 m/ns = 275.79 mm/ns

Close! My ref p may be a little bit small.

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Let us convert timeoffset to RF phase offset

$$\phi_0 = 2\pi f t_0$$

-0.77123842 ns → -90.2349 deg
+0.13491993 ns → +15.7856 deg
+1.0410783 ns → +121.806 deg
+1.9472366 ns → +227.827 deg
+2.8853395 ns → +337.585 deg

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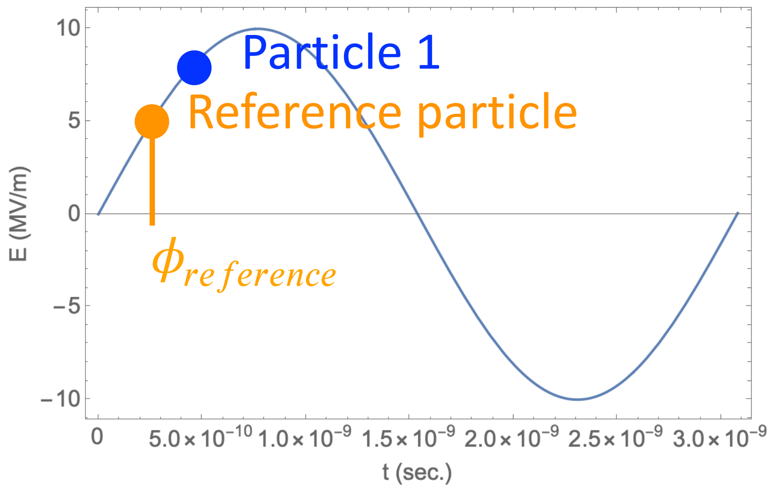
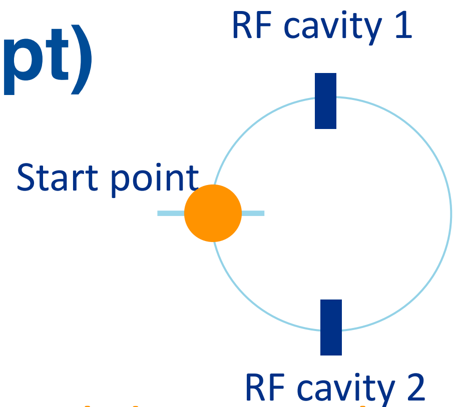
RF acceleration (I) (General concept)



RF cavity 1

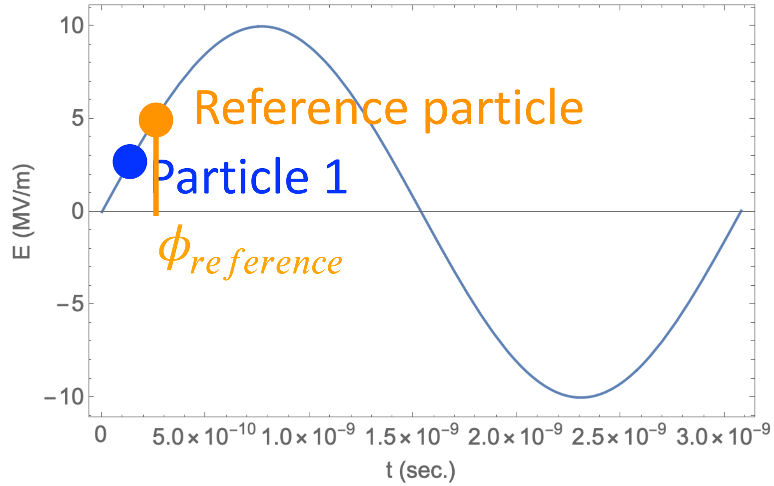
s = s1



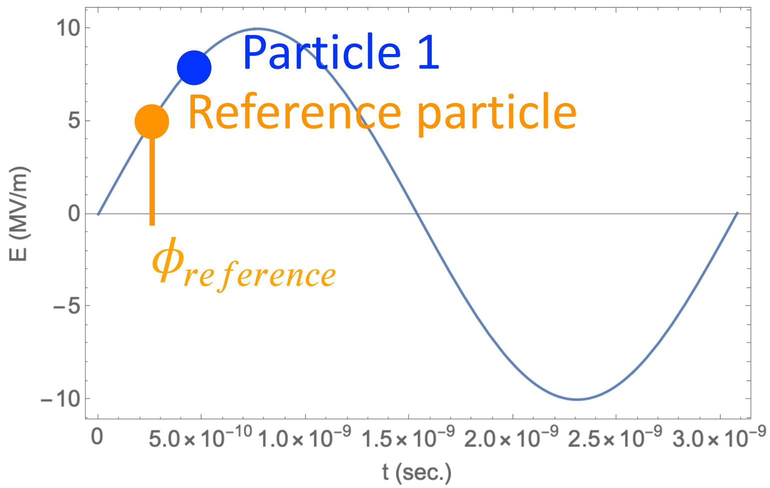
Most channel designs, the RF phase of reference particle is the same for each RF cavity.

RF cavity 2

s = s1+s2



RF acceleration (II) (General concept)



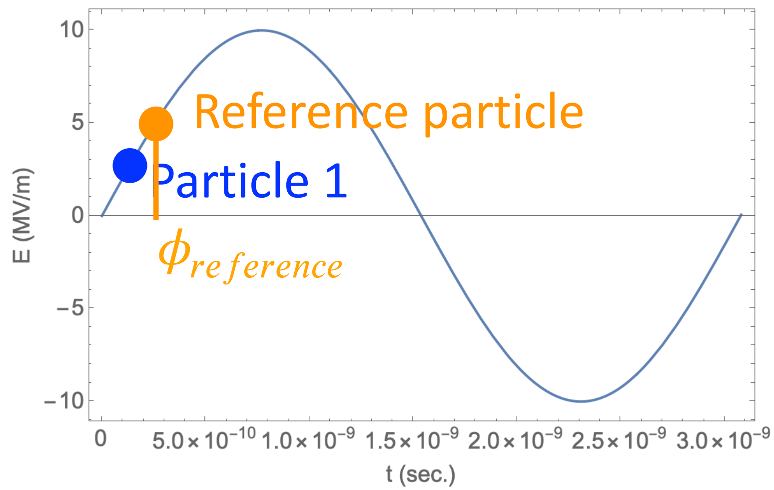
RF cavity 1

s = s1

While particles have a different RF phase at each RF cavity.

RF cavity 2

s = s1+s2



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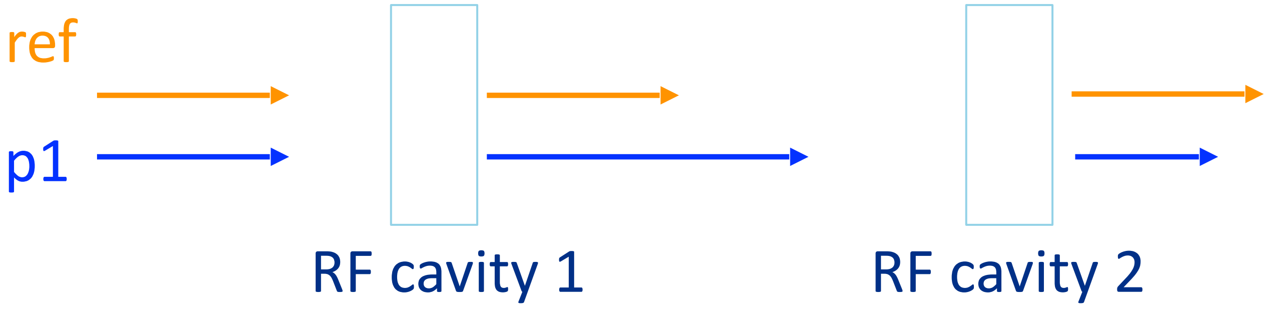
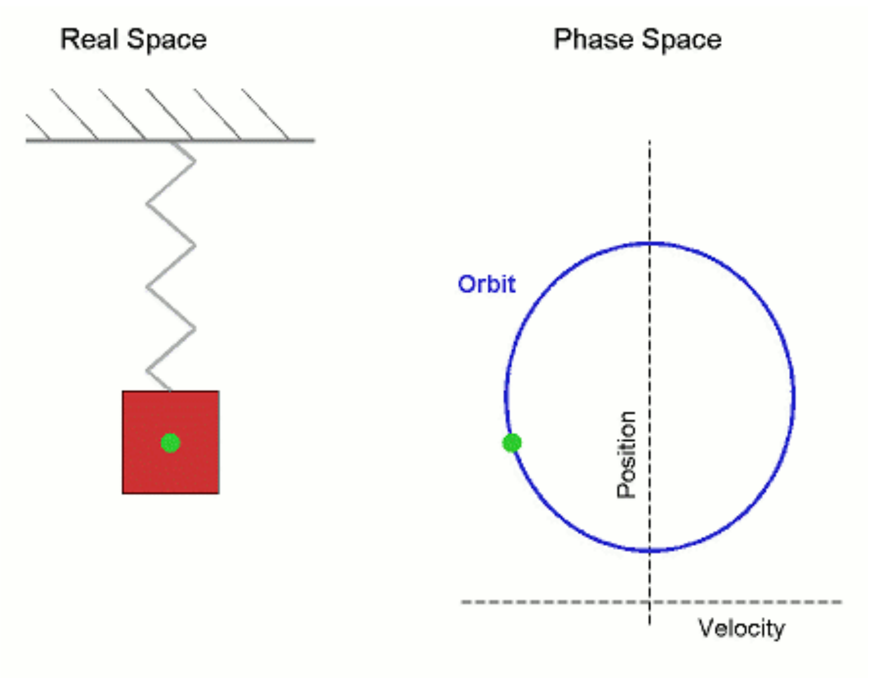
Synchrotron motion (General concept)

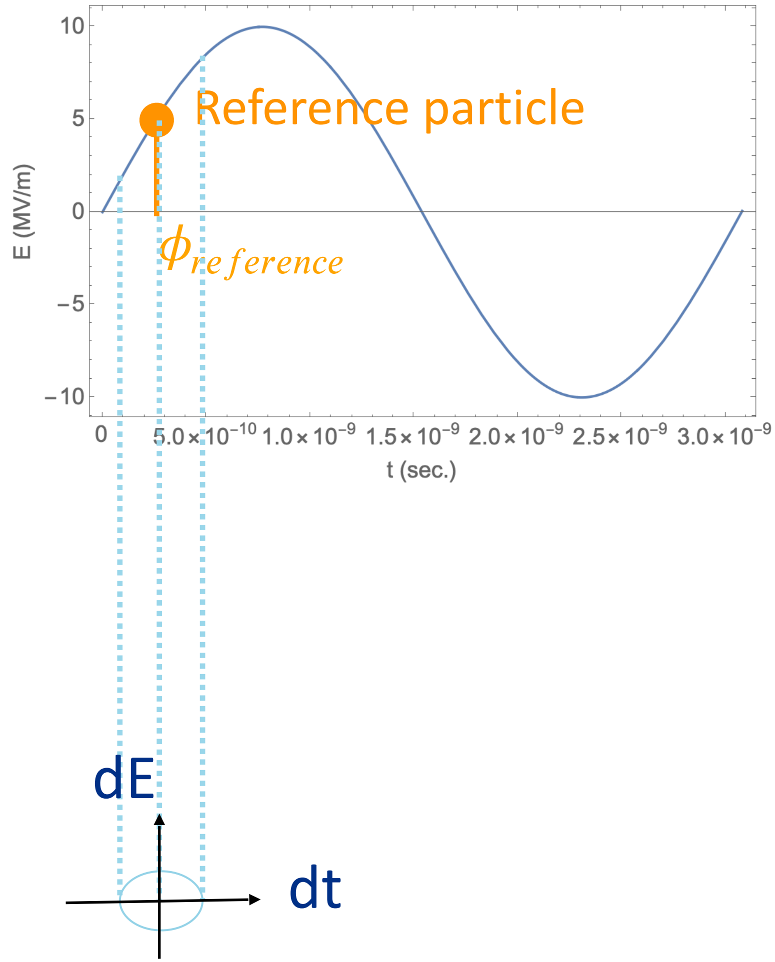
- Particle 1 gains higher energy than the reference particle at RF cavity 1 since the RF gradient of Particle 1 is higher than the reference particle
- At RF cavity 2, Particle 1 arrives earlier than the reference particle since Particle 1 has more kinetic energy at RF cavity 1, then Particle 1 gain less energy than the reference particle
- As a result, Particle 1 is oscillated around the reference particle



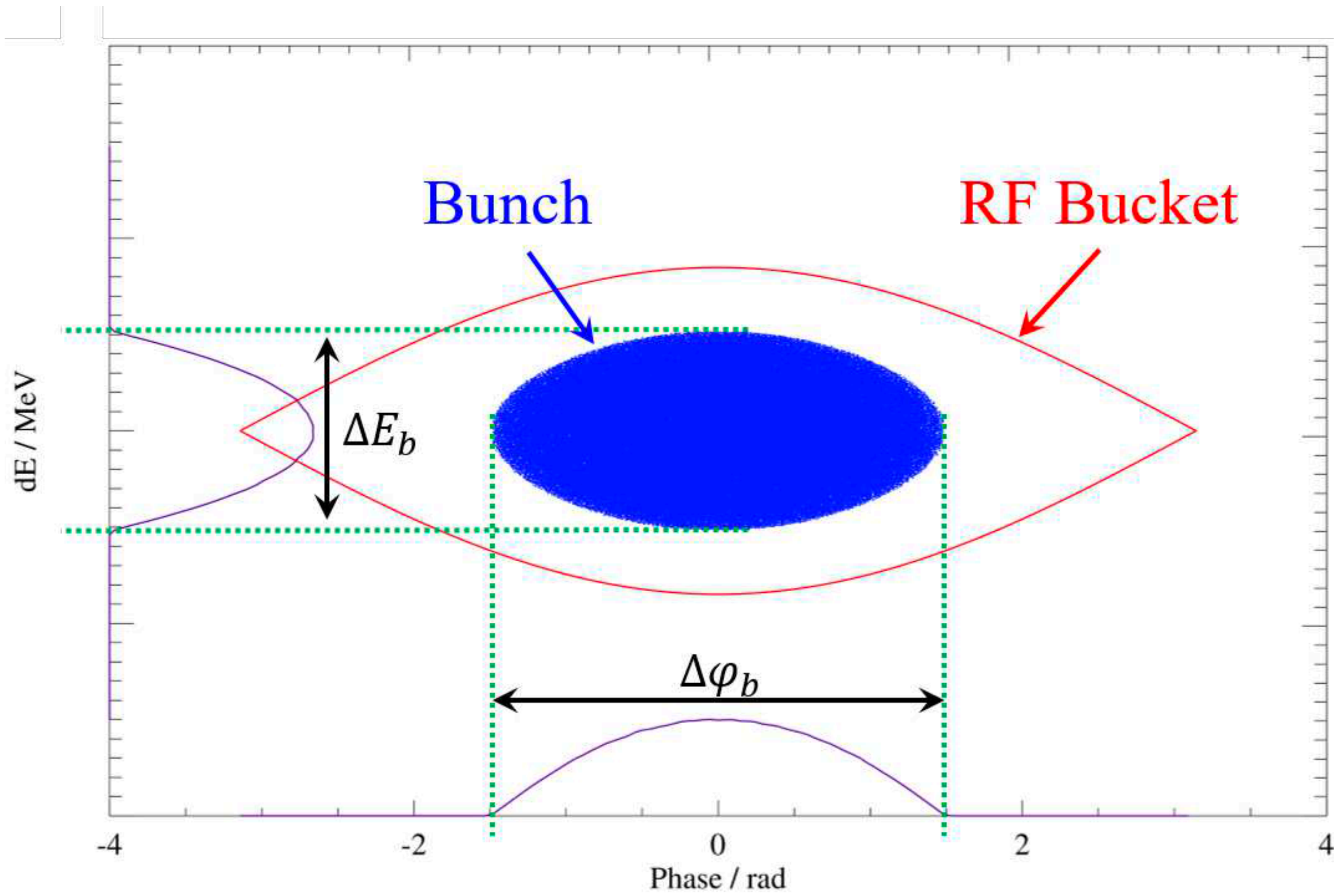
Analogy of Pedram (General concept)

From wikipedia

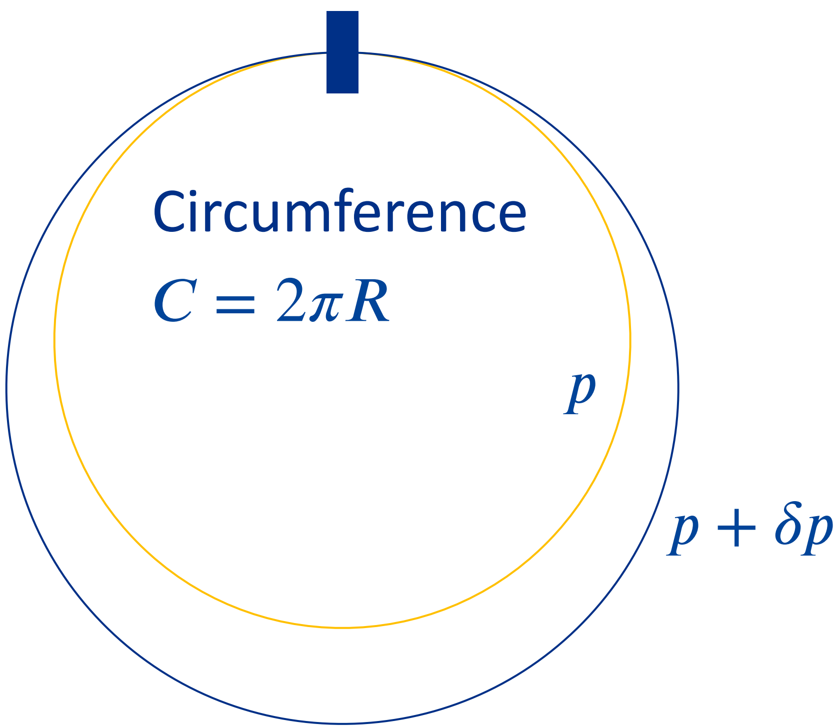




Separatrix (General Concept)



Traveling path length adjustable by momentum (General concept)



Momentum compaction factor

(path length variation by p)

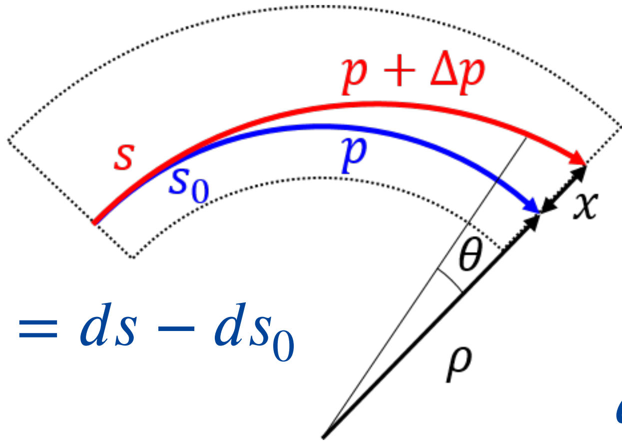
$$\alpha_c = \frac{p}{2\pi R}\frac{2\pi dR}{dp} = \frac{p}{R}\frac{dR}{dp} = \frac{p}{C}\frac{dC}{dp}$$

Phase slip factor

(particle revolution variation by p)

$$\eta = \frac{p}{f_r}\frac{df_r}{dp}$$

Dispersion, Momentum compaction and Slip factor (General concept)



$$dl = ds - ds_0$$

For individual dipole, a particle position is varied by its momentum

$$ds_0 = \rho d\theta \rightarrow ds = (\rho + x)d\theta$$

Dispersion is given

$$\frac{ds - ds_0}{ds_0} = \frac{x}{\rho} = \frac{D}{\rho}\frac{dp}{p} \rightarrow D = x\frac{p}{dp}$$

In circular periodic motion,

$$\Delta C = \oint dl = \oint x \bullet d\theta = \oint x \bullet \frac{ds_0}{\rho} = \oint x \bullet \frac{p}{dp} \bullet \frac{dp}{p}\frac{ds_0}{\rho} = \oint D \bullet \frac{dp}{p}\frac{ds_0}{\rho}$$

$$\alpha_c = \frac{p}{C}\frac{dC}{dp} = \frac{1}{C}\oint D \bullet \frac{ds_0}{p} \rightarrow \frac{1}{C}\sum_i \bar{D}_i \bullet \theta_i$$

$\bar{D}_i$: Average dispersion per beam element

Dispersion, Momentum compaction and Slip factor (General concept)

$$f_r = \frac{v}{2\pi R} = \frac{\beta\gamma}{2\pi R} \rightarrow \frac{df_r}{f_r} = \frac{d\beta}{\beta} - \frac{dR}{R} = \frac{d\beta}{\beta} - \alpha_c \frac{dp}{p}$$

$$p = \beta\gamma mc \rightarrow \frac{dp}{p} = \frac{d\beta}{\beta} + \frac{d\gamma}{\gamma} = \gamma^2 \frac{d\beta}{\beta}$$

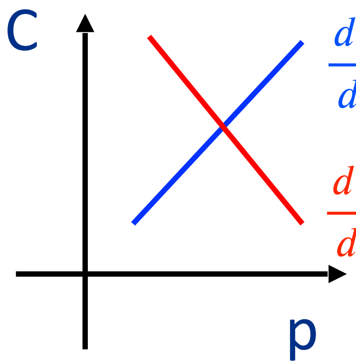
$$\frac{df_r}{f_r} = \frac{d\beta}{\beta} - \alpha_c \frac{dp}{p} = \frac{1}{\gamma^2}\frac{dp}{p} - \alpha_c \frac{dp}{p} = \left(\frac{1}{\gamma^2} - \alpha_c\right)\frac{dp}{p}$$

(Phase) Slip factor

$$\eta = \left(\frac{1}{\gamma^2} - \alpha_c\right)$$

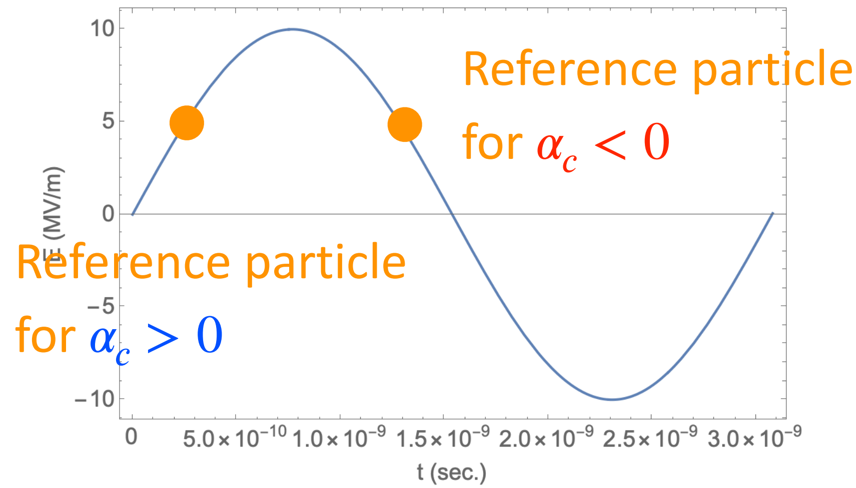
Interpret momentum compaction and slip factor (General concept)

$$\alpha_c = \frac{p}{C}\frac{dC}{dp}$$

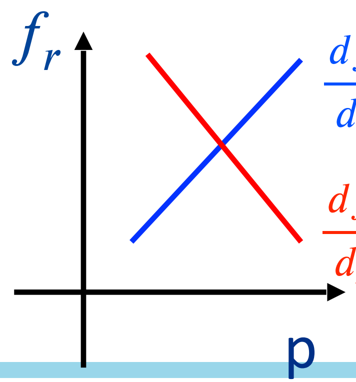


$$\frac{dC}{dp} > 0 \rightarrow \alpha_c > 0$$

$$\frac{dC}{dp} < 0 \rightarrow \alpha_c < 0$$



$$\frac{df_r}{f_r} = \left(\frac{1}{\gamma^2} - \left|\alpha_c\right|\right)\frac{dp}{p} = \eta\frac{dp}{p}$$



$$\frac{df_r}{dp} > 0 \rightarrow \eta > 0$$

$$\frac{df_r}{dp} < 0 \rightarrow \eta < 0$$

- $\eta = 0$: Transition
- $\eta > 0$: Below transition, higher momentum particle revolute faster
- $\eta < 0$: Above transition, higher momentum particle revolve slower

Tuning RF timing in G4beamline (HFOFO specific)

- There are two ways to adjust the RF timing for each cavity
- One: Set timing offset
  - This is what Yuri sets
  - However, this is not intuitive
- Two: Set RF phase offset
  - This is what I usually do
  - I guess that this method is more intuitive than the time offset
    - For example, if we know the dE/dx of reference particle, we can easily set the RF phase offset,
    - $\frac{dE}{dx} = A \bullet sin(\phi_0)$
    - Of course, $\frac{dE}{dx} = A \bullet sin(2\pi f t_0)$ works as well

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Next step

- I propose a new cooling optimization study
- Optimize HFOFO with a constant reference momentum
  - Easy to optimize cooling performance
  - Understand more on HFOFO and its critical parameters