Fermilab

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Characterize Solenoid-based channel

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HFOFO meeting

U.S. DEPARTMENT of ENERGY
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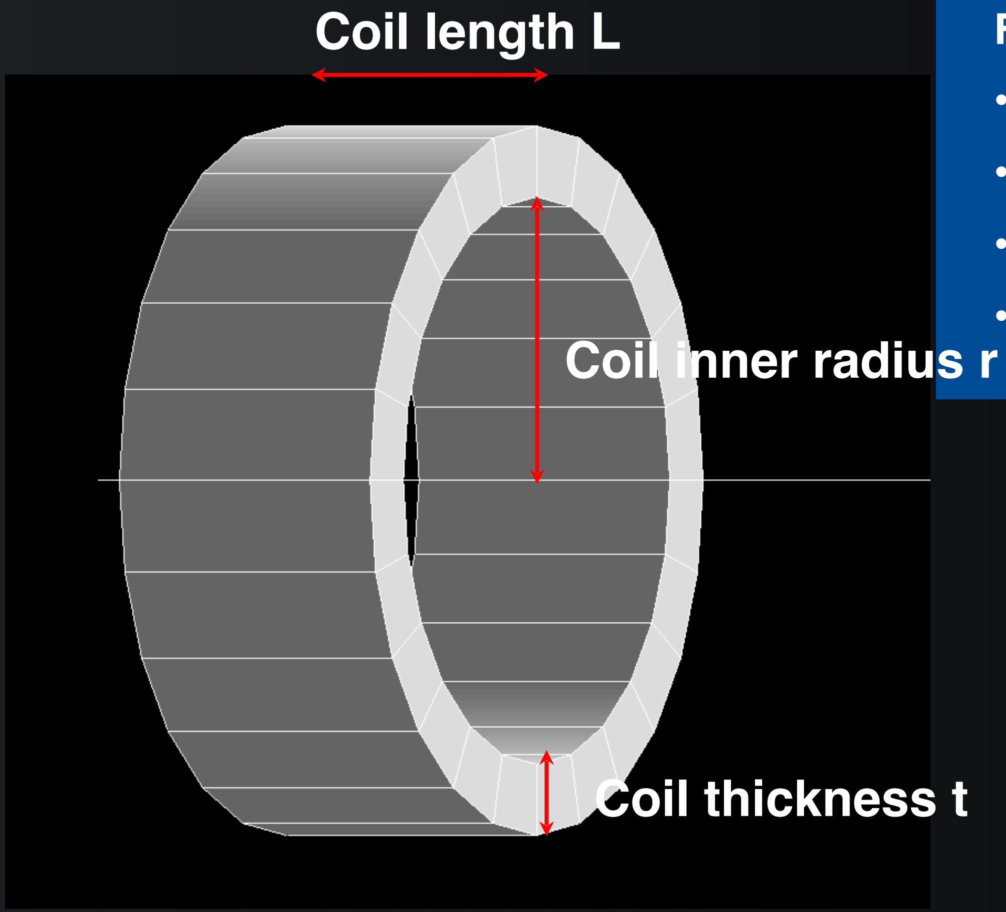
Scope of this talk

- Understand solenoid-based cooling channel using Toy model
  - Single Solenoid Coil
  - Multiple Solenoid Coil
  - Multiple Solenoid Coil and polarity flip for every other coil

Single Solenoid Coil



Fernow coil (referred from PRSTAB 10, 064001 (2007)

- Coil length = 400 mm
- Coil inner radius = 400 mm
- Coil thickness = 100 mm
- Current = 100 Amp/mm2

Single Solenoid Coil

- Conventional Solenoid Focusing Model

$$\frac{1}{f} = \frac{eB_z}{2m\gamma}$$

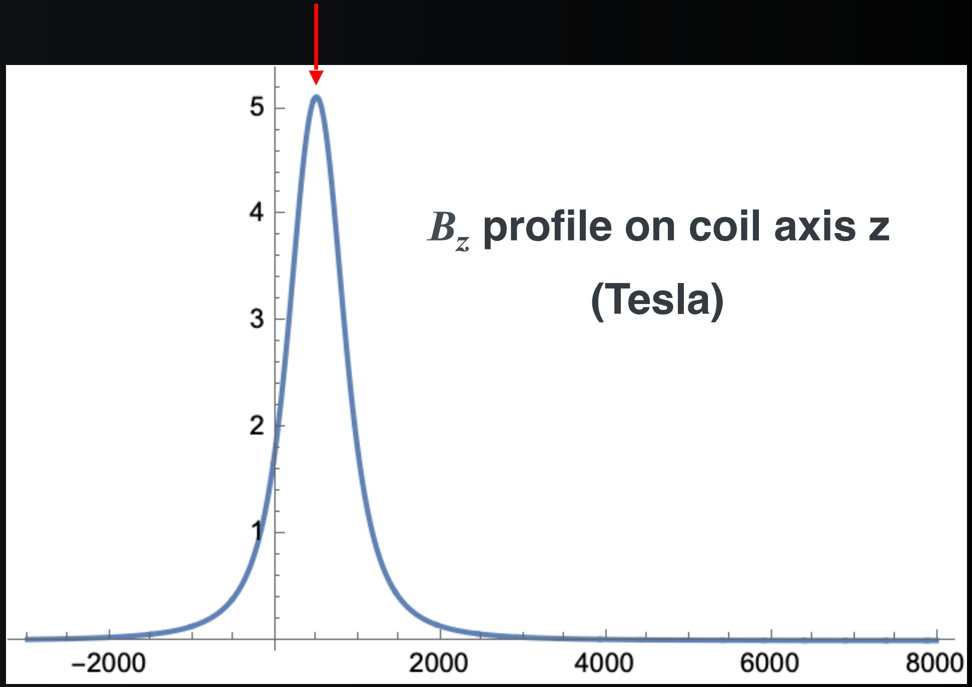
- As the formula shows, the focusing length is independent from the coil geometry

→ Thin lens approximation

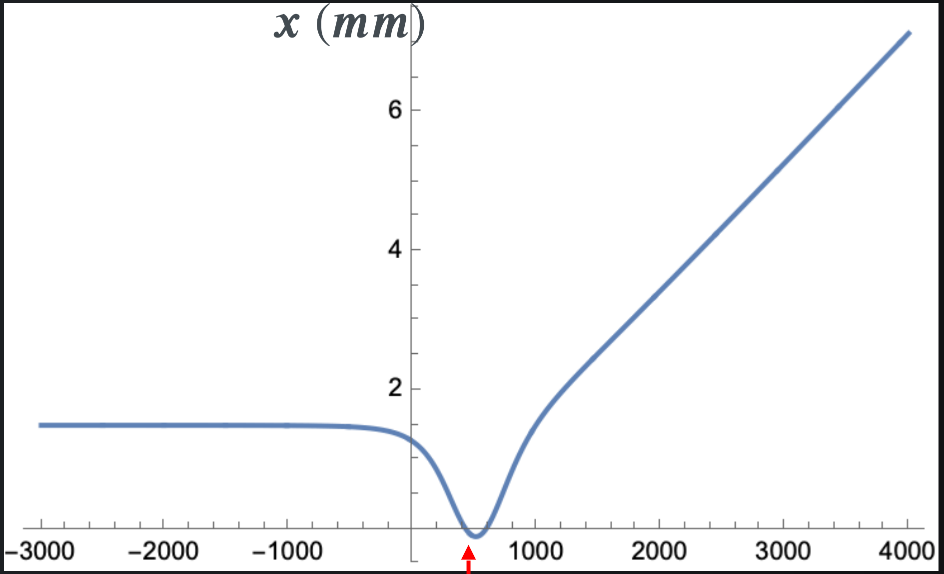
- The estimated focusing length is

$$f = 0.3\ \text{m}$$

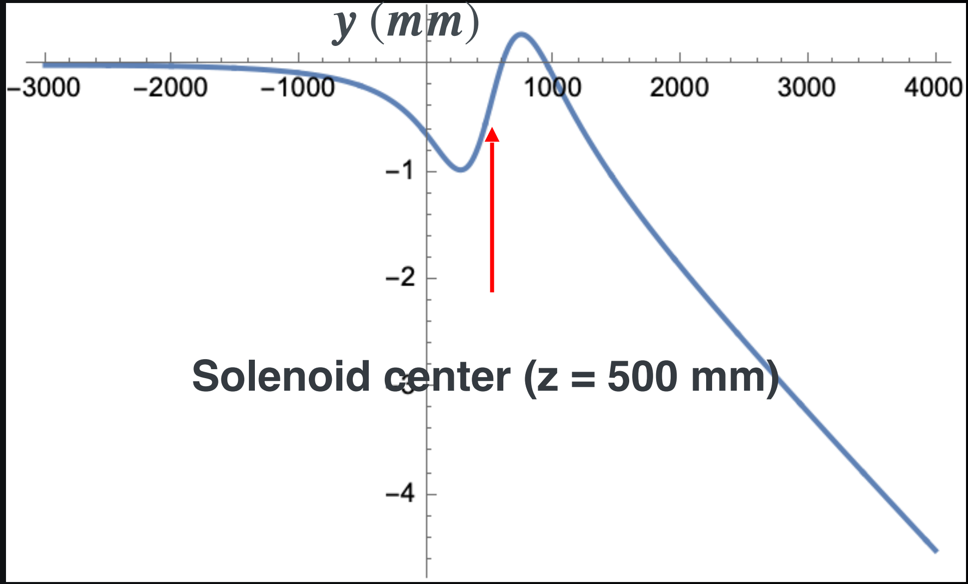
Solenoid center (z = 500 mm)



Single Solenoid Coil



Solenoid center (z = 500 mm)



Focusing strength seems too strong to focus particle

Probably, focusing length should be longer than coil length ($f > L$)

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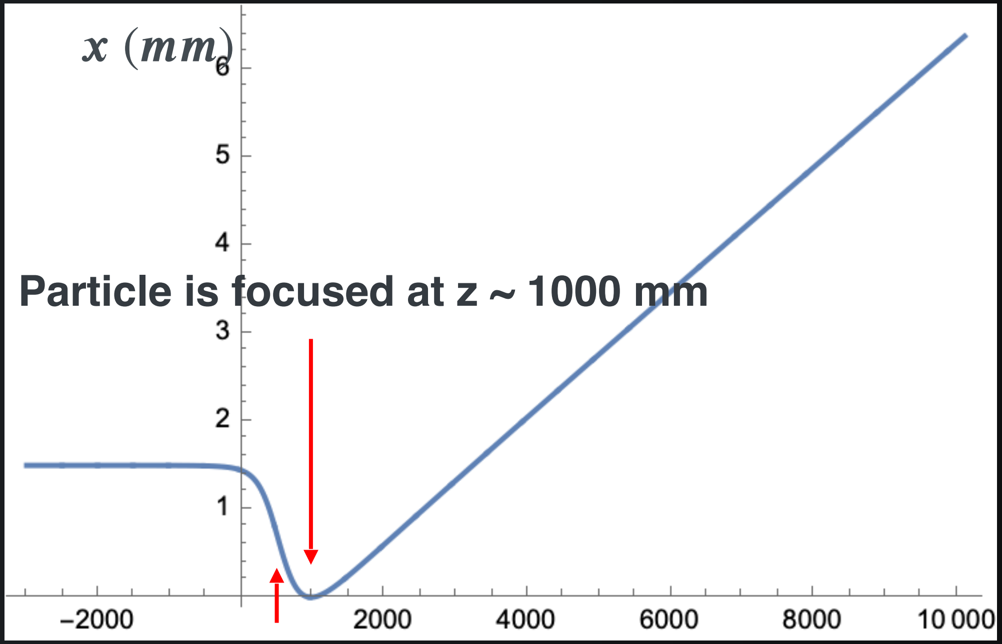
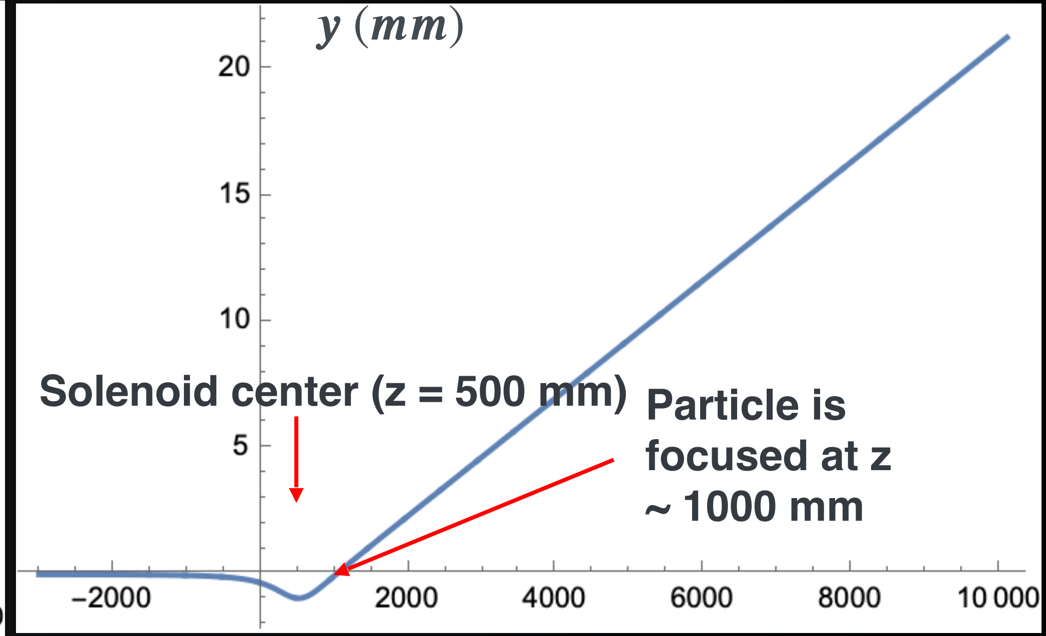
Single Solenoid Coil



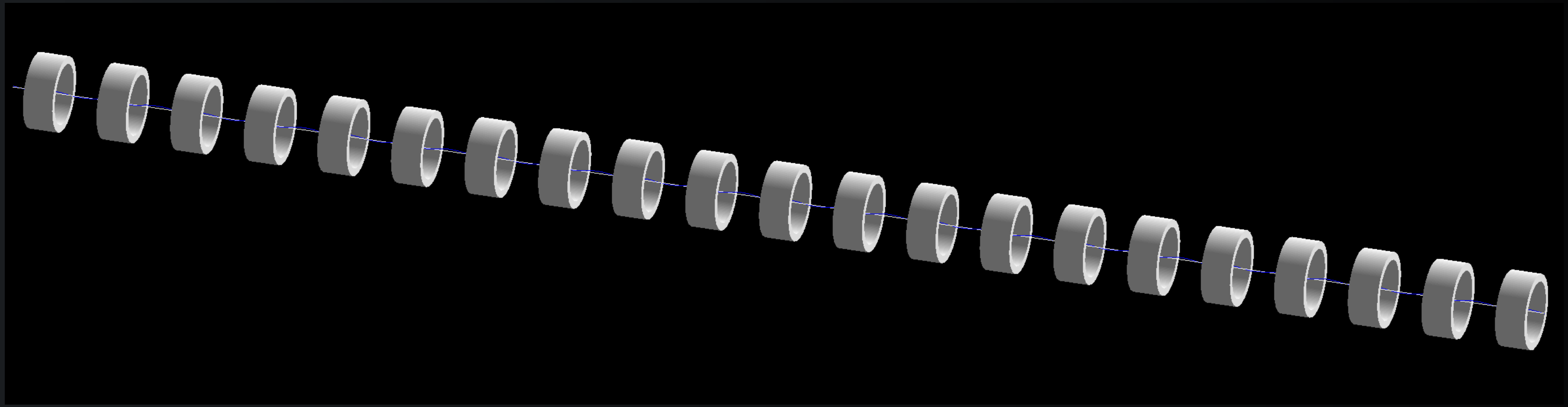
Solenoid center (z = 500 mm)



When the coil length $L =$ 200 mm (half from original length)

$f =$ 0.5 m

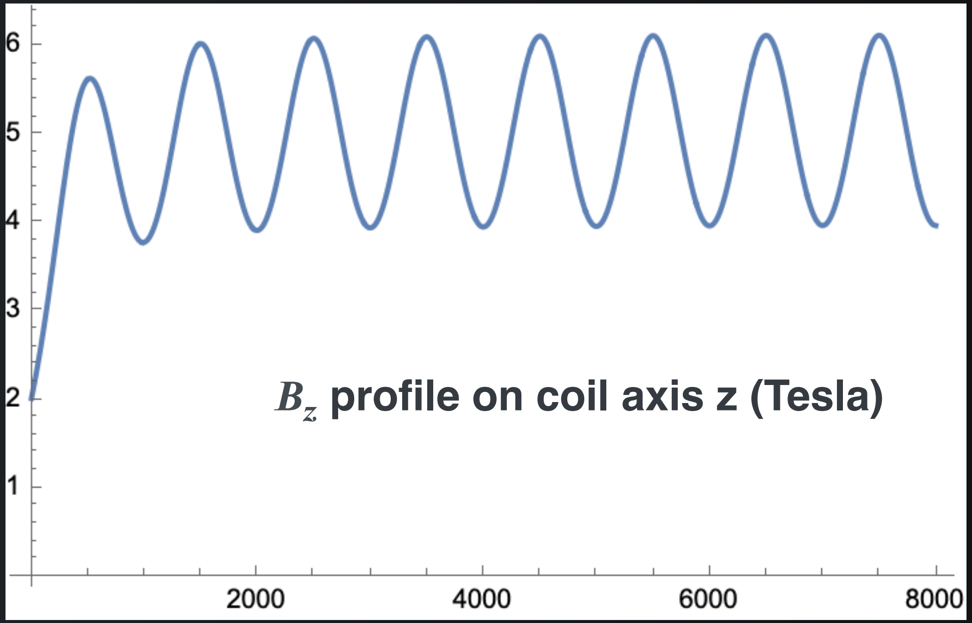
Multiple Solenoid Coil



Fernow coil (referred from PRSTAB 10, 064001 (2007))

- Coil length = 400 mm
- Coil inner radius = 400 mm
- Coil thickness = 100 mm
- Current = 100 Amp/mm2
- Period length = 1000 mm

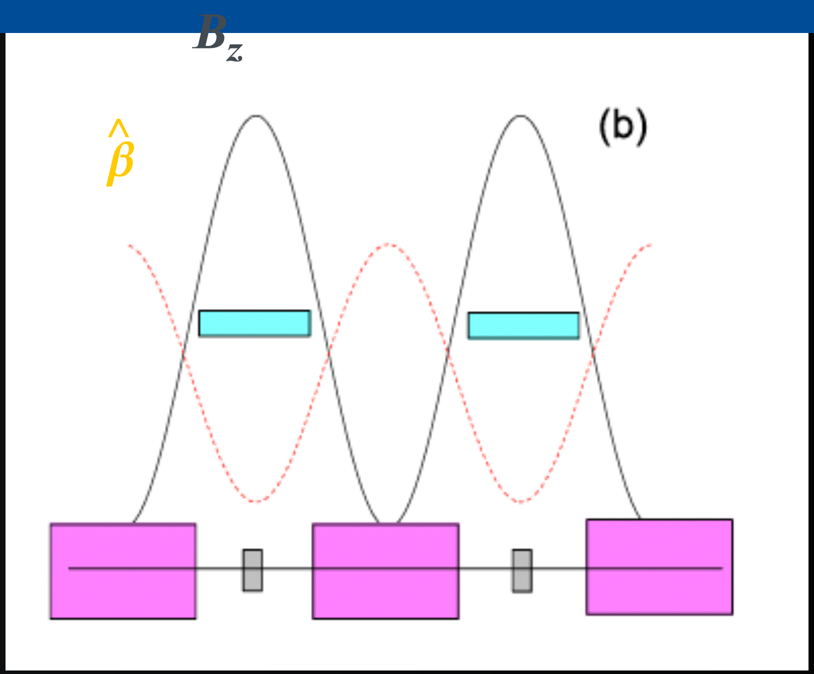
Multiple Solenoid Coil



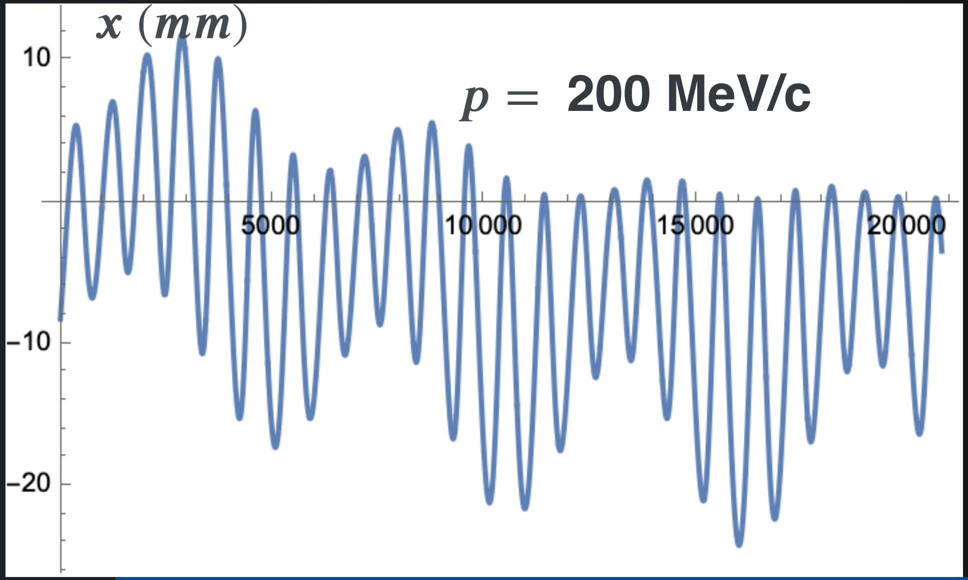
B_z profile on coil axis z (Tesla)

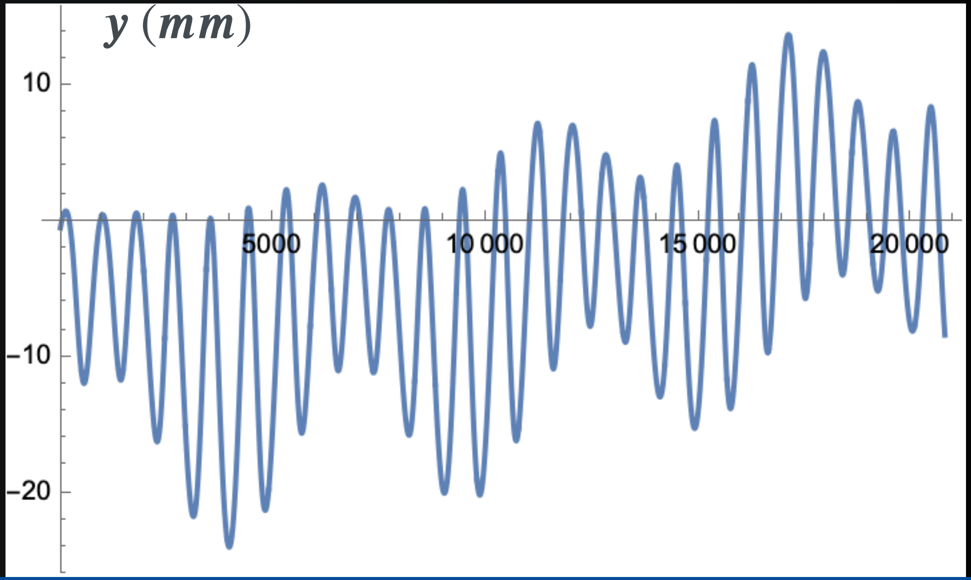
Each peak appears at the coil center

FIG.1 in PRSTAB 10, 064001 (2007)



Multiple Solenoid Coil

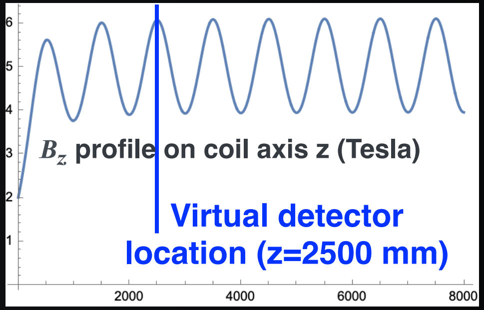


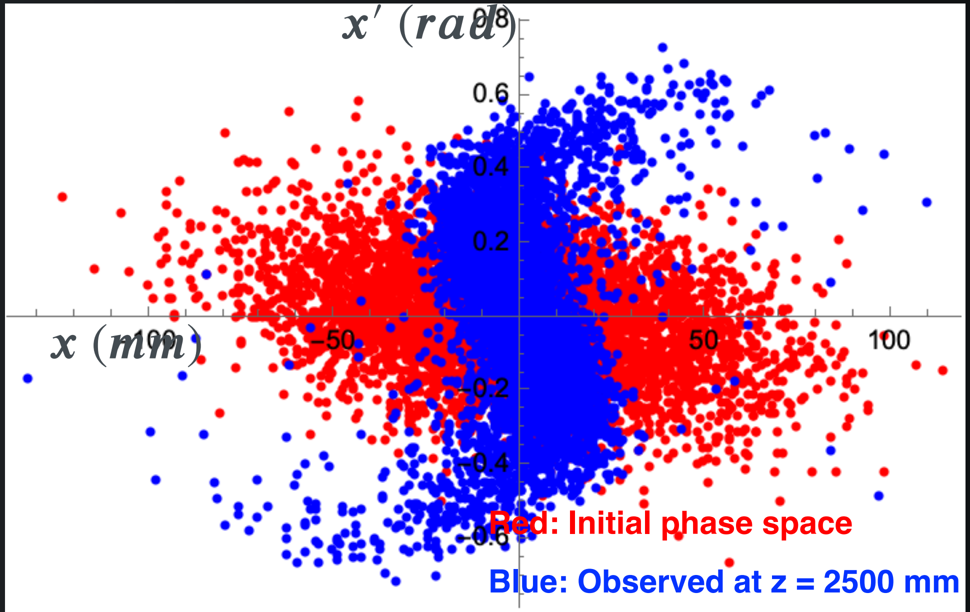


Tracking of single particle which has position offset

- **Oscillate around the magnet center (reference orbit)**
- **Oscillation not stable even though initial angular momentum is included**
  - $L = xp_y - yp_x$
- **Mismatching is not critical issue for Twiss parameter study though we will figure out matching issue later**

Multiple Solenoid Coil





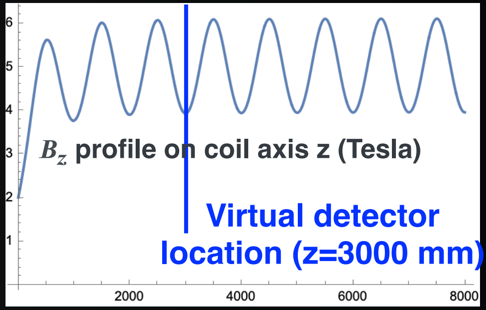
- **Evolution of phase space in Fernow channel**
- **Although the phase space is filamented due to a non-linear motion, the Twiss parameters can be extracted**

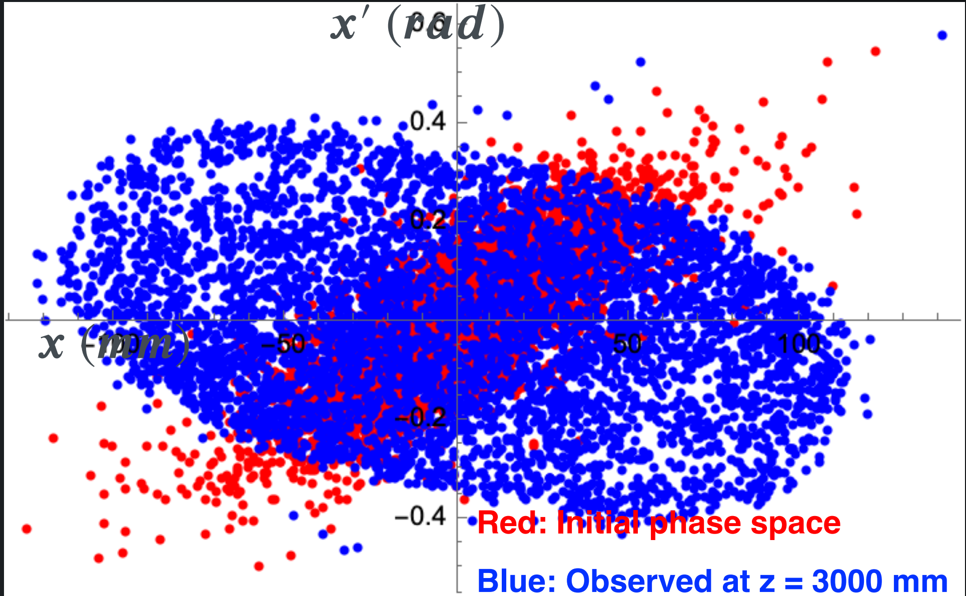
$$\tilde{M} = \begin{pmatrix} 0.000220647 & 0.000359925 & -2.70331\times10^{-6} & -0.000431054 & 7.74032\times10^{-11} & 0.000225893 \\ 0.000359925 & 0.0678051 & 0.000389266 & -0.0022377 & 7.26931\times10^{-10} & 0.00150732 \\ -2.70331\times10^{-6} & 0.000389266 & 0.000214141 & 0.000302741 & -4.31112\times10^{-11} & 0.000659453 \\ -0.000431054 & -0.0022377 & 0.000302741 & 0.0691676 & -2.04229\times10^{-10} & 0.0101021 \\ 7.74032\times10^{-11} & 7.26931\times10^{-10} & -4.31112\times10^{-11} & -2.04229\times10^{-10} & 5.97259\times10^{-14} & 2.46894\times10^{-9} \\ 0.000225893 & 0.00150732 & 0.000659453 & 0.0101021 & 2.46894\times10^{-9} & 0.58094 \end{pmatrix}$$

$$Det(\Sigma_4)^{1/4} = 0.0382\ \text{m}$$

$$\hat{\beta}_{\varphi} \sim \frac{\sigma_{1,1}}{Det(\Sigma_4)^{1/4}} = 0.057\ \text{m}$$

Multiple Solenoid Coil





- Evolution of phase space in Fernow channel
- Although the phase space is filamented due to a non-linear motion, the Twiss parameters can be extracted

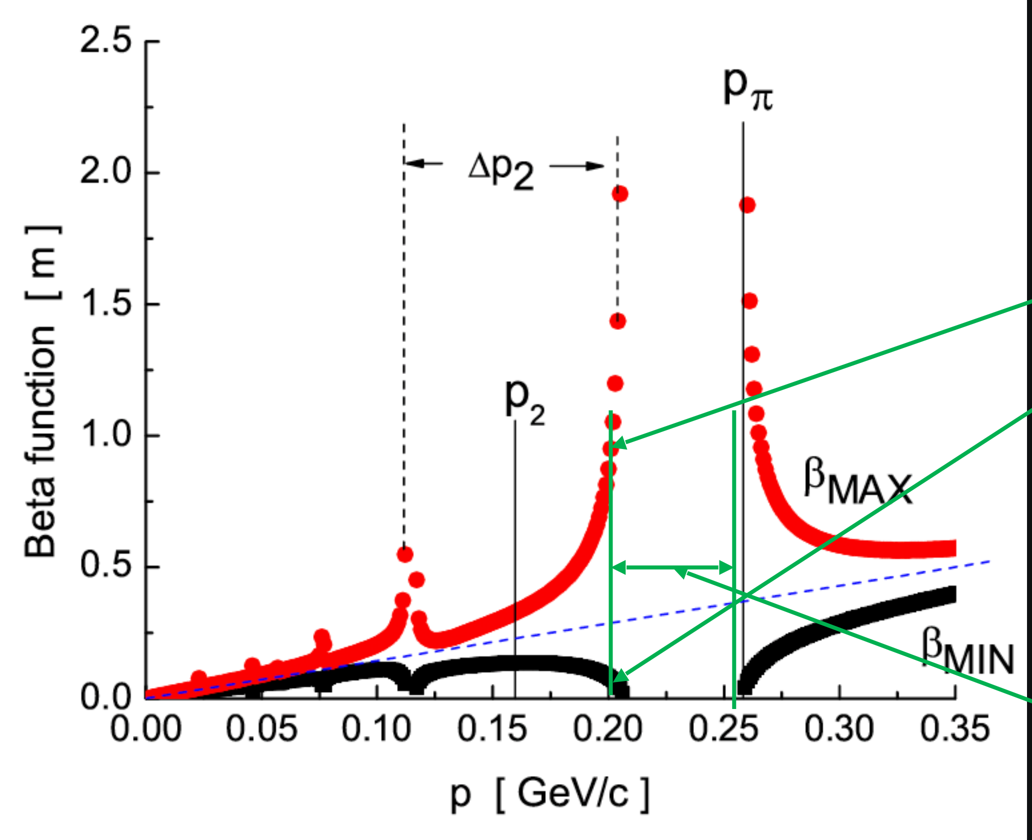
$$\tilde{M} = \begin{pmatrix} 0.00309927 & -0.00208256 & -0.0000297911 & -0.00930383 & 3.46366\times10^{-10} & 0.0000974241 \\ -0.00208256 & 0.0322217 & 0.00879677 & 0.00075964 & -6.19871\times10^{-10} & -0.000320508 \\ -0.0000297911 & 0.00879677 & 0.00291672 & -0.00179792 & -4.67382\times10^{-11} & -0.0000751696 \\ -0.00930383 & 0.00075964 & -0.00179792 & 0.0331989 & -8.56048\times10^{-10} & -0.0000714287 \\ 3.46366\times10^{-10} & -6.19871\times10^{-10} & -4.67382\times10^{-11} & -8.56048\times10^{-10} & 6.11443\times10^{-14} & -1.98643\times10^{-10} \\ 0.0000974241 & -0.000320508 & -0.0000751696 & -0.0000714287 & -1.98643\times10^{-10} & 0.0113122 \end{pmatrix}$$

$$Det(\Sigma_4)^{1/4} = 0.0356 \text{ m}$$

$$\hat{\beta}_\varphi \sim \frac{\sigma_{1,1}}{Det(\Sigma_4)^{1/4}} = 0.87 \text{ m}$$

Multiple Solenoid Coil

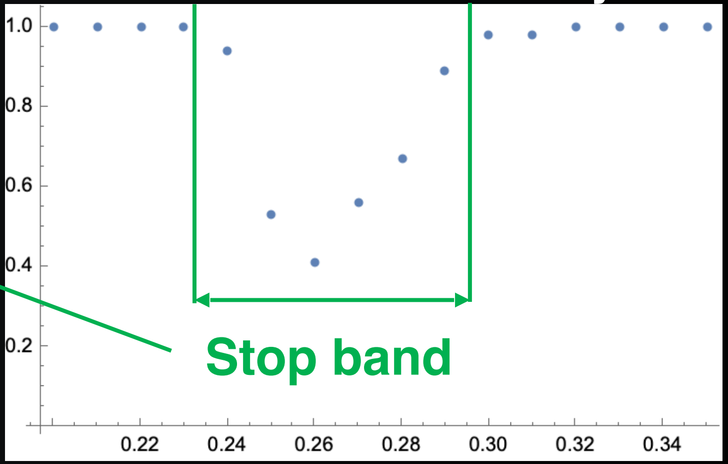
FIG.4 in PRSTAB 10, 064001 (2007)



Our analysis agrees well with Fernow's results

- $\hat{\beta}_{MAX} =$ **0.87 m @ 0.2 GeV/c**
- $\hat{\beta}_{MIN} =$ **0.057 m @ 0.2 GeV/c**

Transmission efficiency



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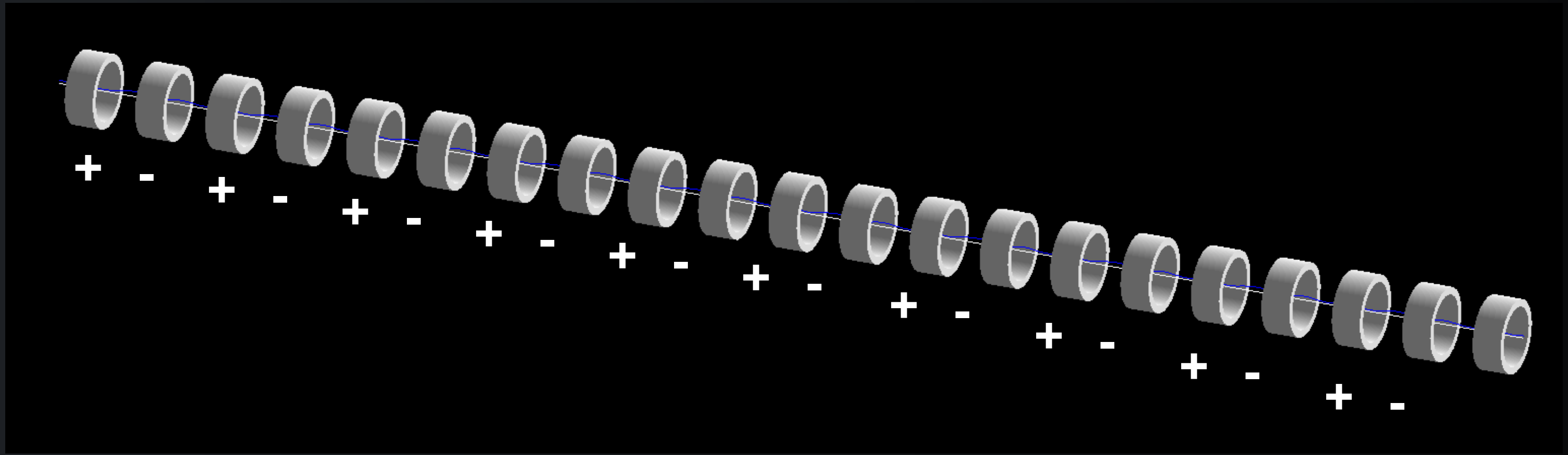
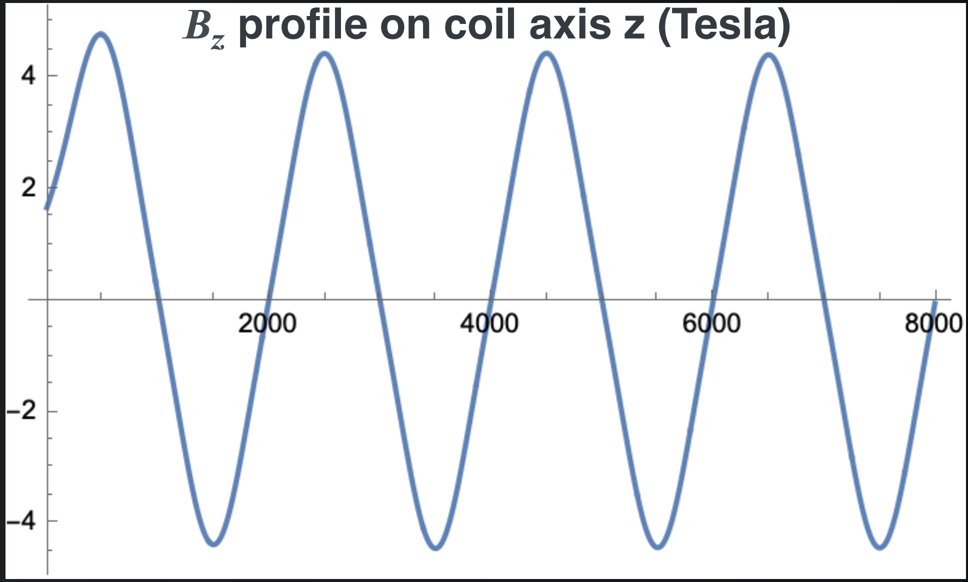
Multiple Solenoid Coil

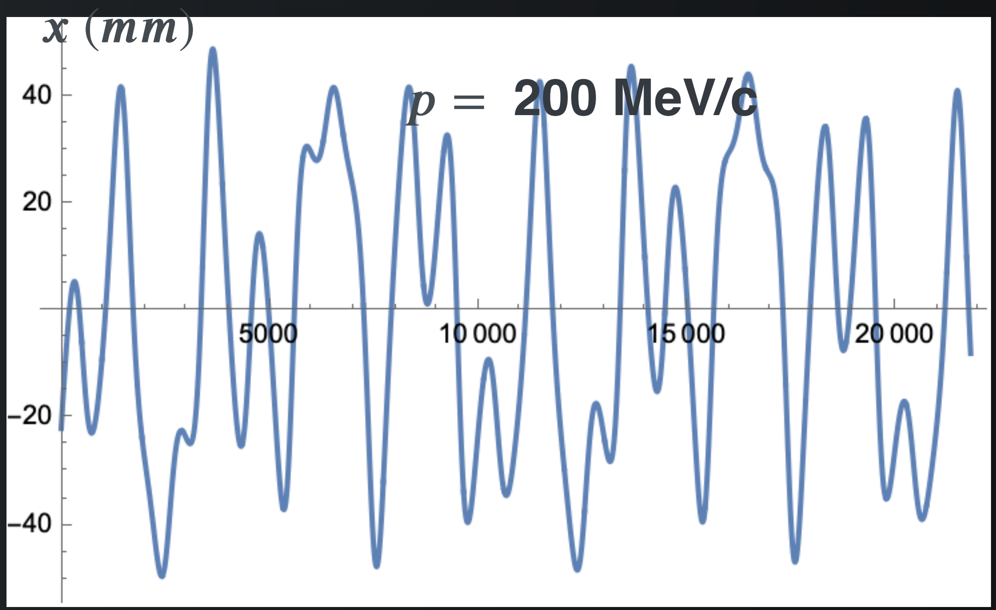
Flipping polarity of every other coil

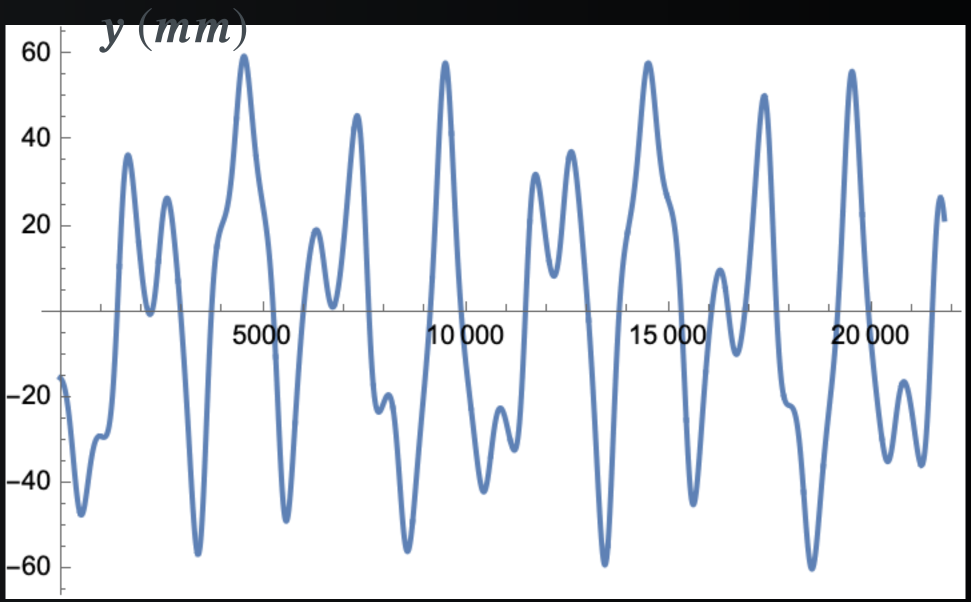




Multiple Solenoid Coil

Flipping polarity of every other coil

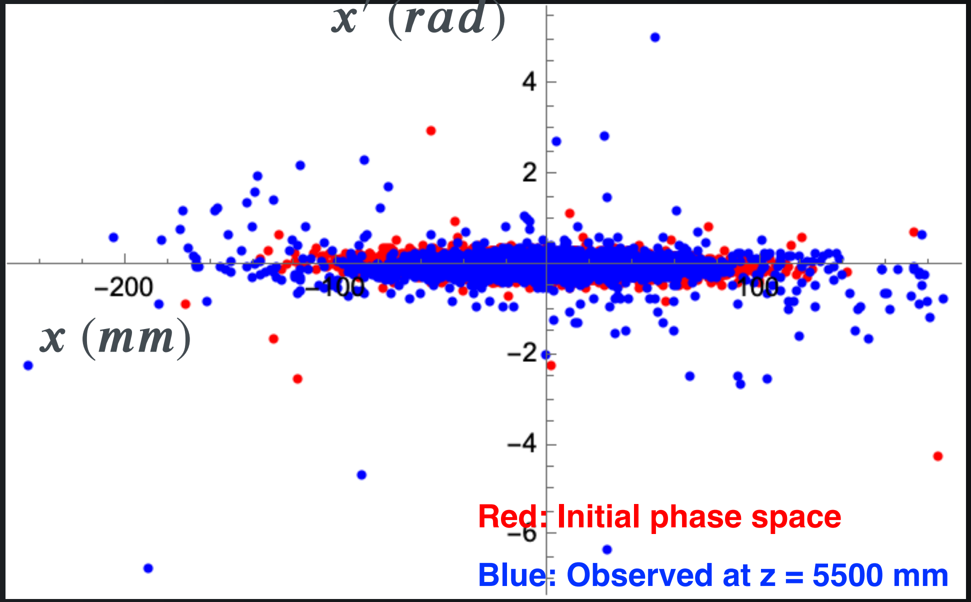




Tracking of single particle which has position offset

Multiple Solenoid Coil

Flipping polarity of every other coil



- **Evolution of phase space in flipped channel**
- **No filamentation appears (see slide 10 to compare with non-flip case)**

$$\tilde{M} = \begin{pmatrix} 0.00177919 & -0.00102959 & -2.00129\times10^{-6} & 0.00802567 & 4.45362\times10^{-11} & -0.000736594 \\ -0.00102959 & 0.0888213 & -0.00792747 & 0.00387642 & 1.08709\times10^{-11} & -0.0105706 \\ -2.00129\times10^{-6} & -0.00792747 & 0.00174341 & -0.000654134 & 1.9687\times10^{-10} & -0.000917076 \\ 0.00802567 & 0.00387642 & -0.000654134 & 0.0988695 & -1.04447\times10^{-9} & -0.00815233 \\ 4.45362\times10^{-11} & 1.08709\times10^{-11} & 1.9687\times10^{-10} & -1.04447\times10^{-9} & 6.28484\times10^{-14} & -1.47129\times10^{-9} \\ -0.000736594 & -0.0105706 & -0.000917076 & -0.00815233 & -1.47129\times10^{-9} & 0.247848 \end{pmatrix}$$

$$Det(\Sigma_4)^{1/4} = 0.010 \text{ m}$$

$$\hat{\beta}_\varphi \sim \frac{\sigma_{1,1}}{Det(\Sigma_4)^{1/4}} = 0.18 \text{ m}$$

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Conclusion & Next step

- Reproduce key results from the first part of Fernow's PRSTAB paper
  - Recalculate and verify beta functions
  - Reproduce stop band structure
  - Analyze both non-flipping and flipping solenoid configurations
- As the next step, introduce tilted coil configuration