Muon Cooling Project Updates

Week of June 2-6, 2025

https://github.com/criggall/muon-cooling/tree/main/Solenoid-Study

Single solenoid

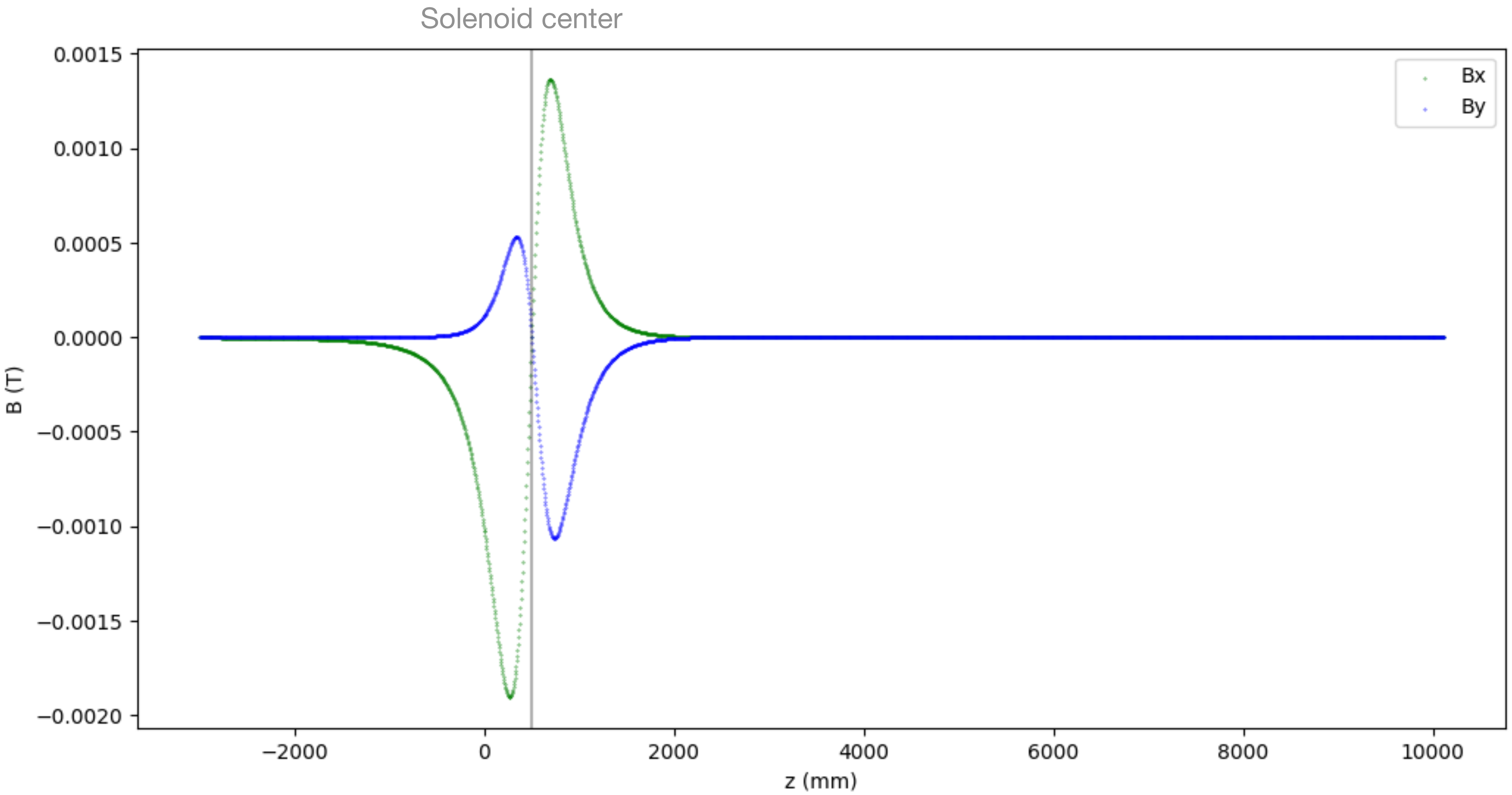
Simulation parameters:

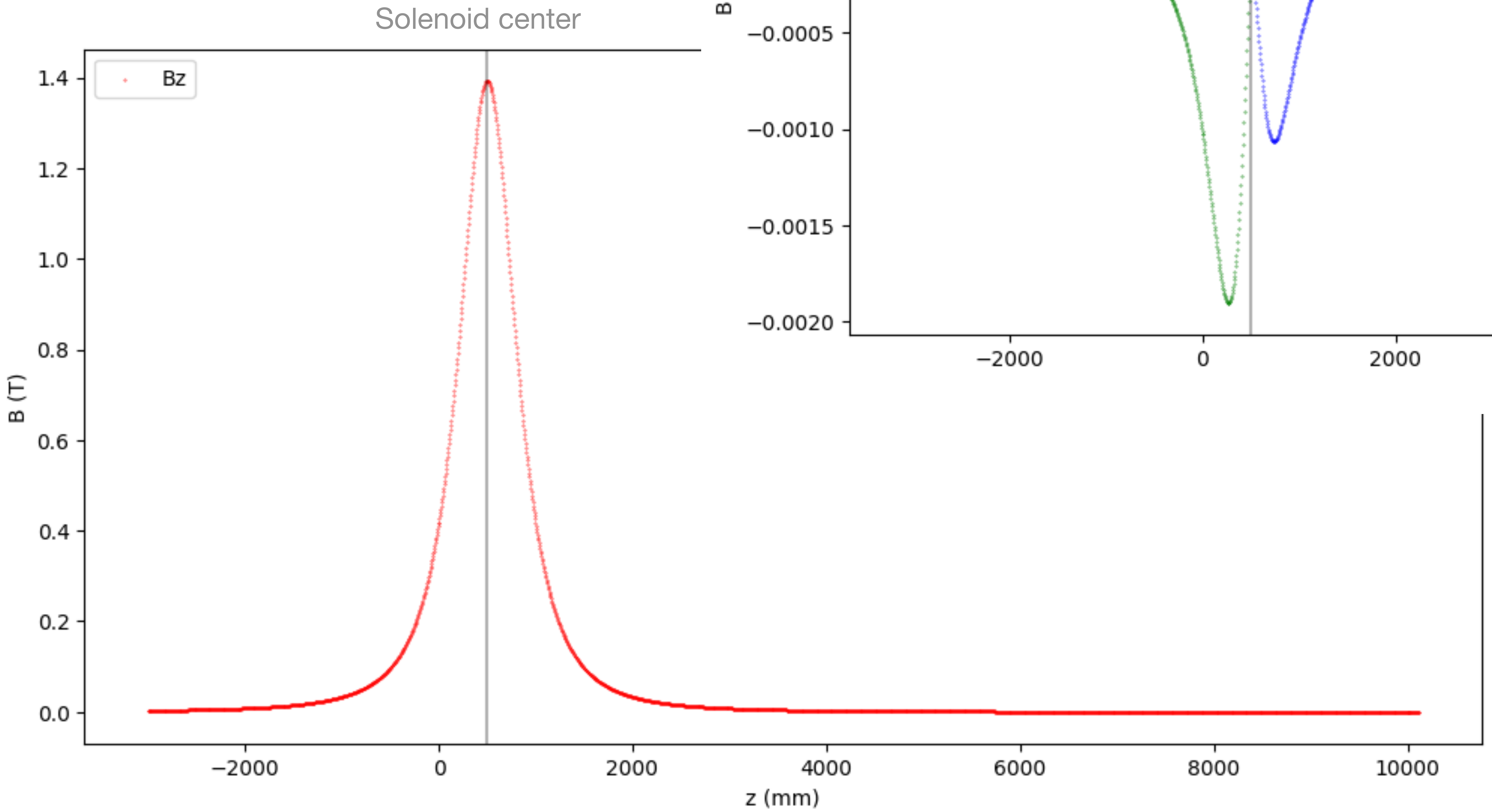
Coil length = 100mm

Coil centered at z = 500mm

Initial conditions
(x_0, y_0, x'_0, y'_0) = (1.5mm, 0, 0, 0)

Magnetic field

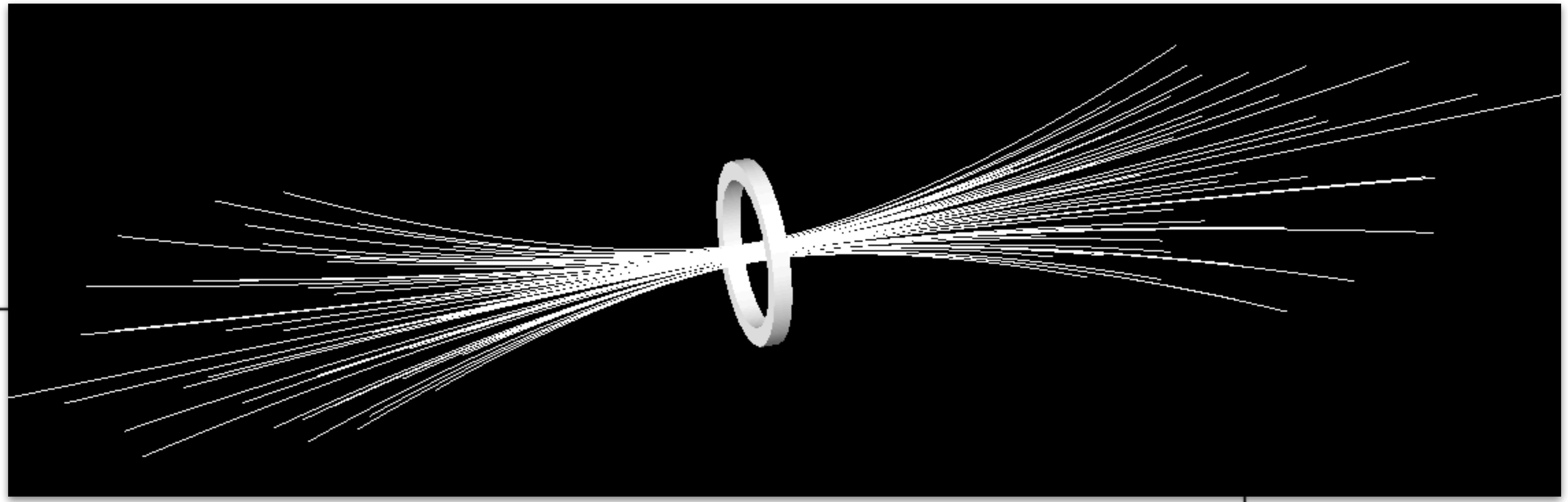


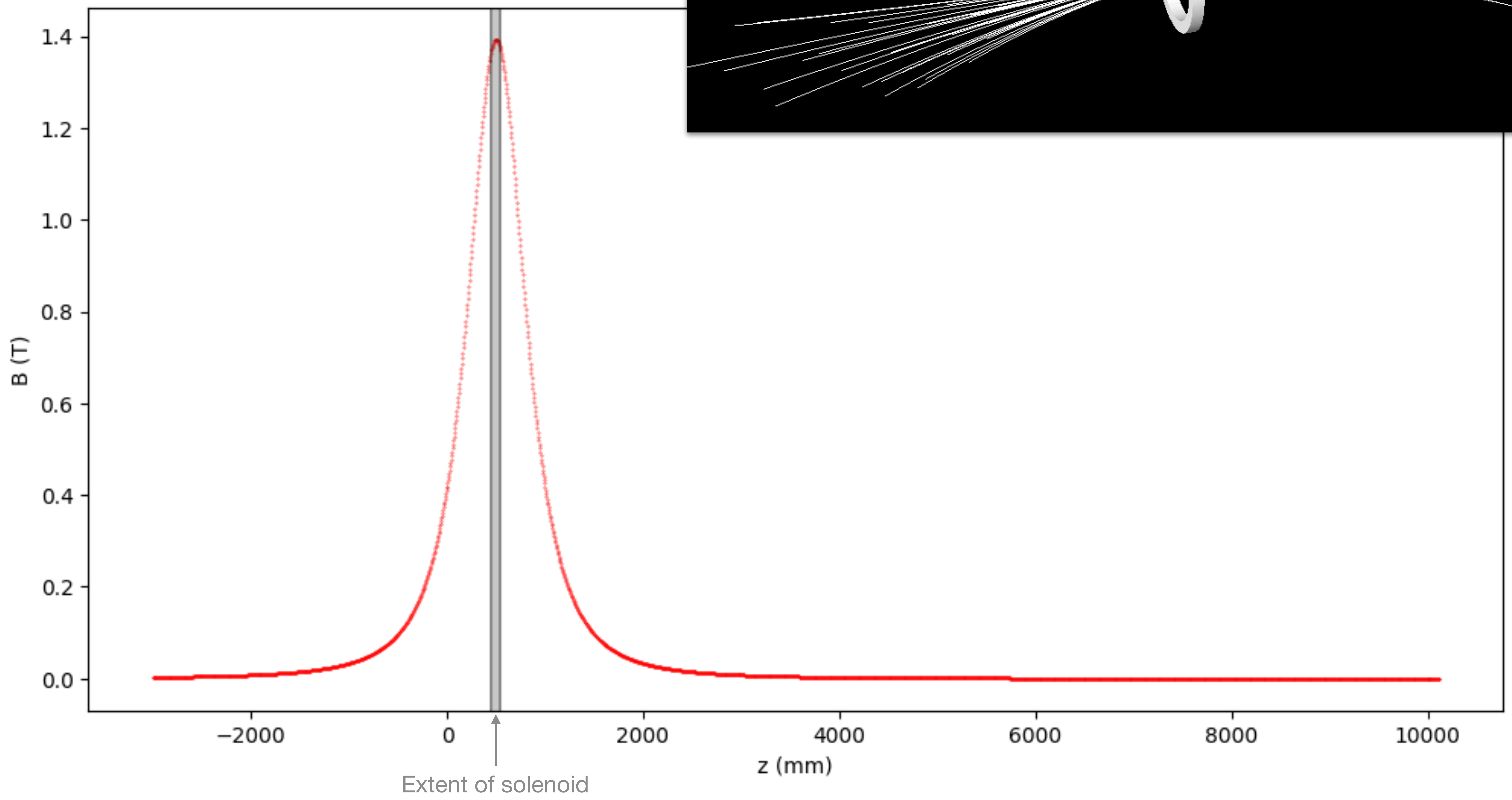


Starting μ with an initial x offset $\Longrightarrow$ larger amplitude B_x with opposite sign compared to B_y

Not symmetric w.r.t. solenoid center

Magnetic field





Magnetic field

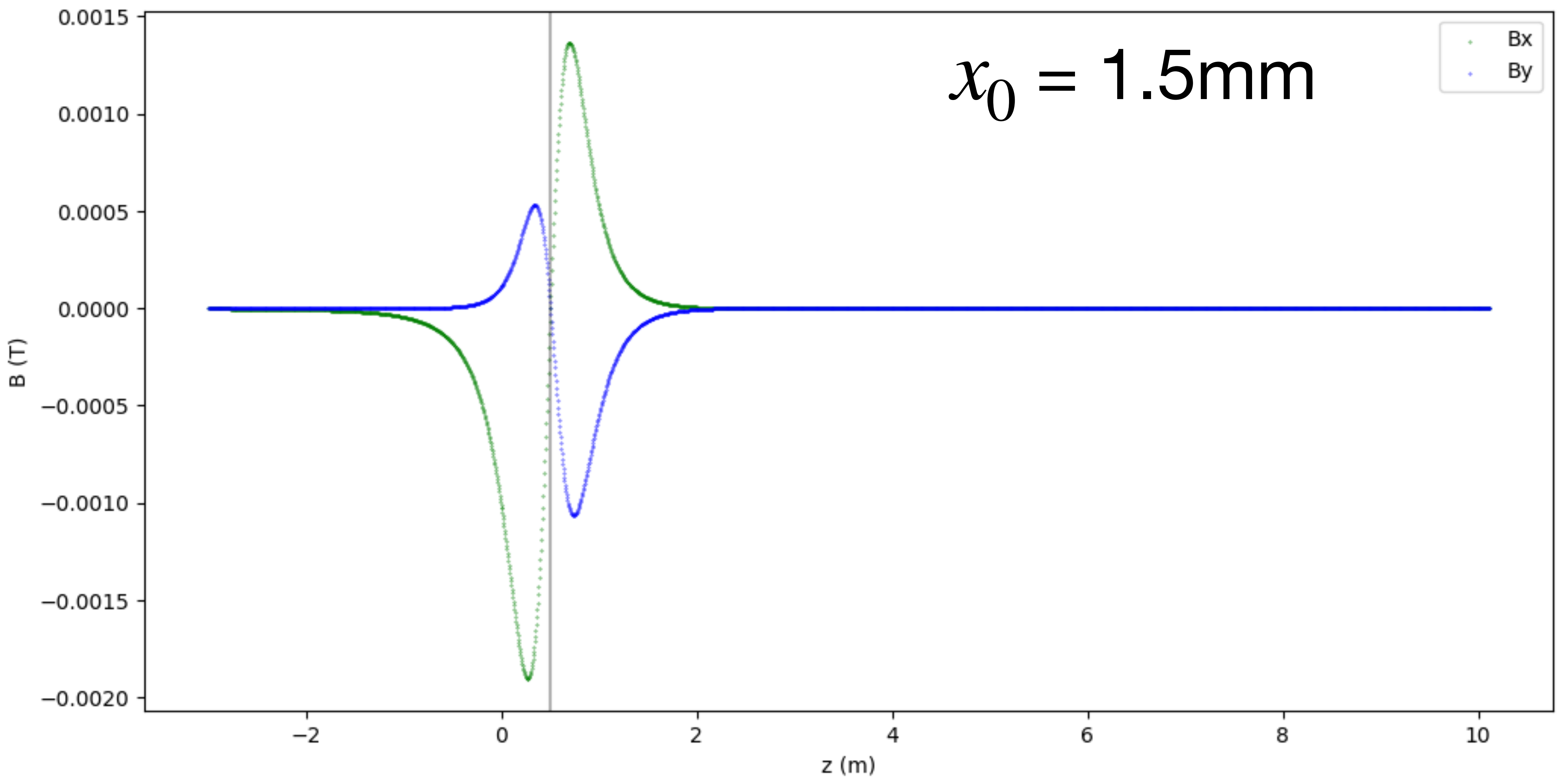
Adjusting sign of initial spatial offset mirrors field components ✓

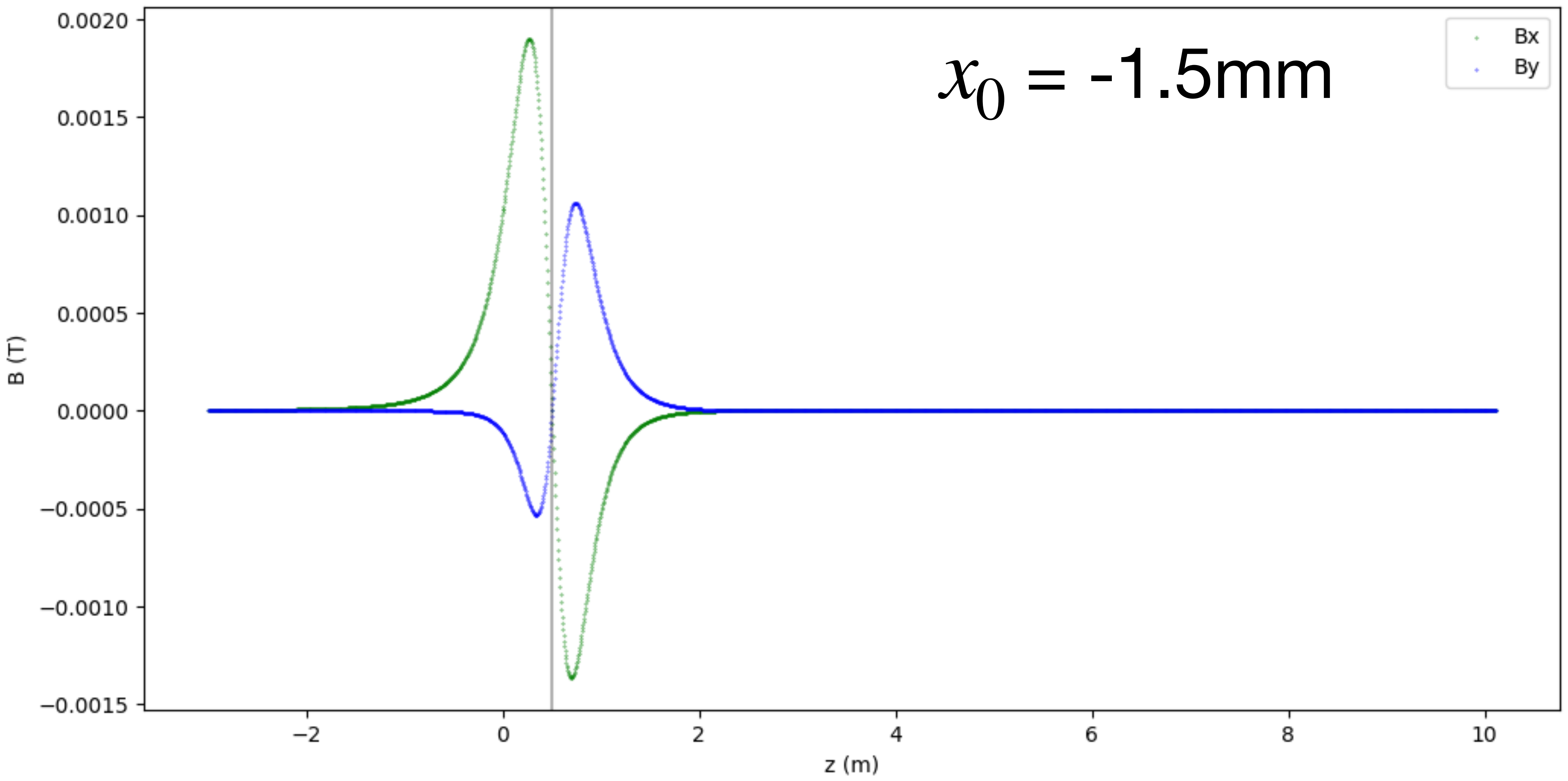
Other observations:

Applying *y offset* only also sees larger amplitude in respective component, but both components have *same sign*, unlike here with x offset

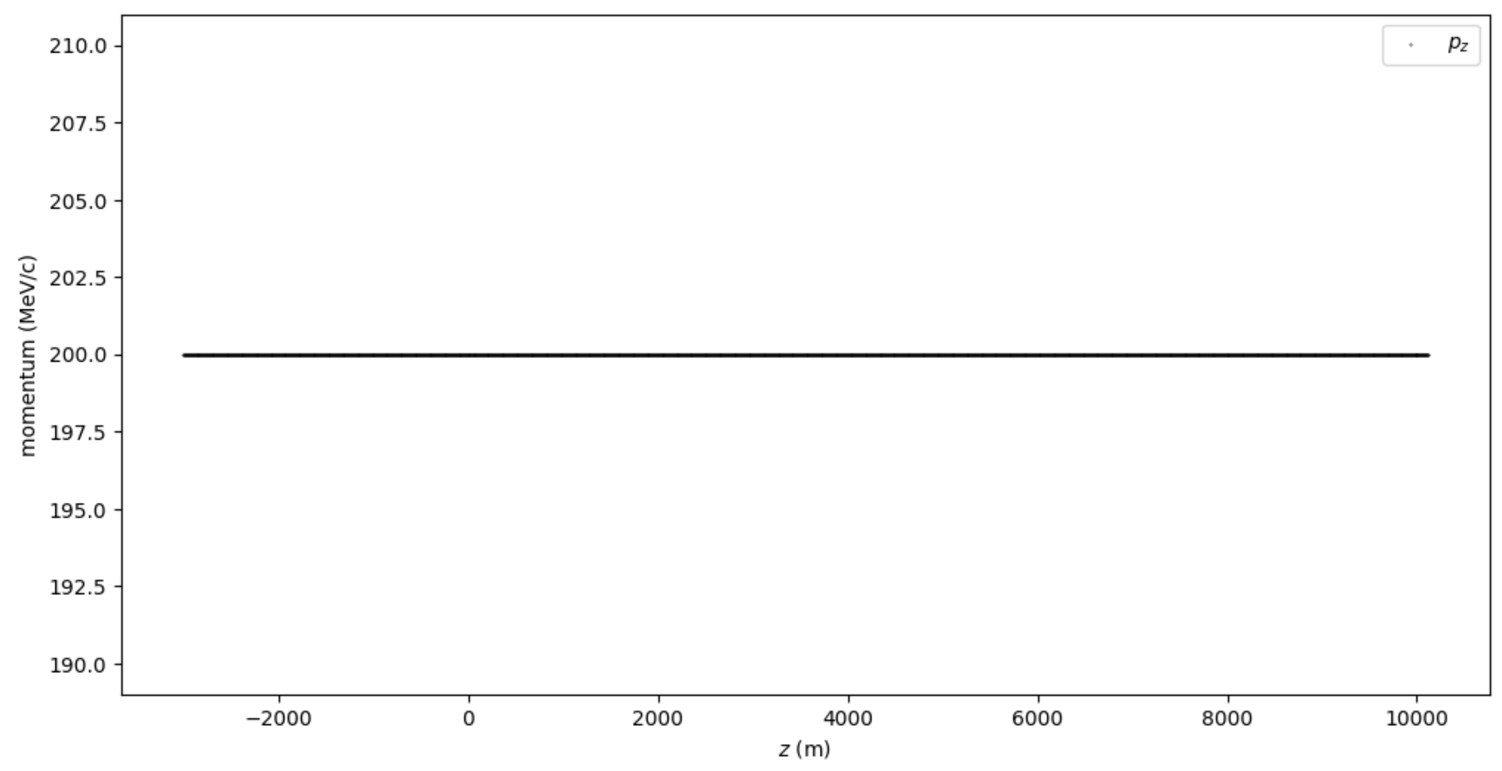
Applying offset to *both* x and y does not restore symmetry of components w.r.t. each other (preserved w.r.t. solenoid center for all tested spatial offsets)

Even small perturbations to x' (order 0.1 radians) wildly affect transverse dynamics — e.g., 0.1 seems stable; 0.2 diverges

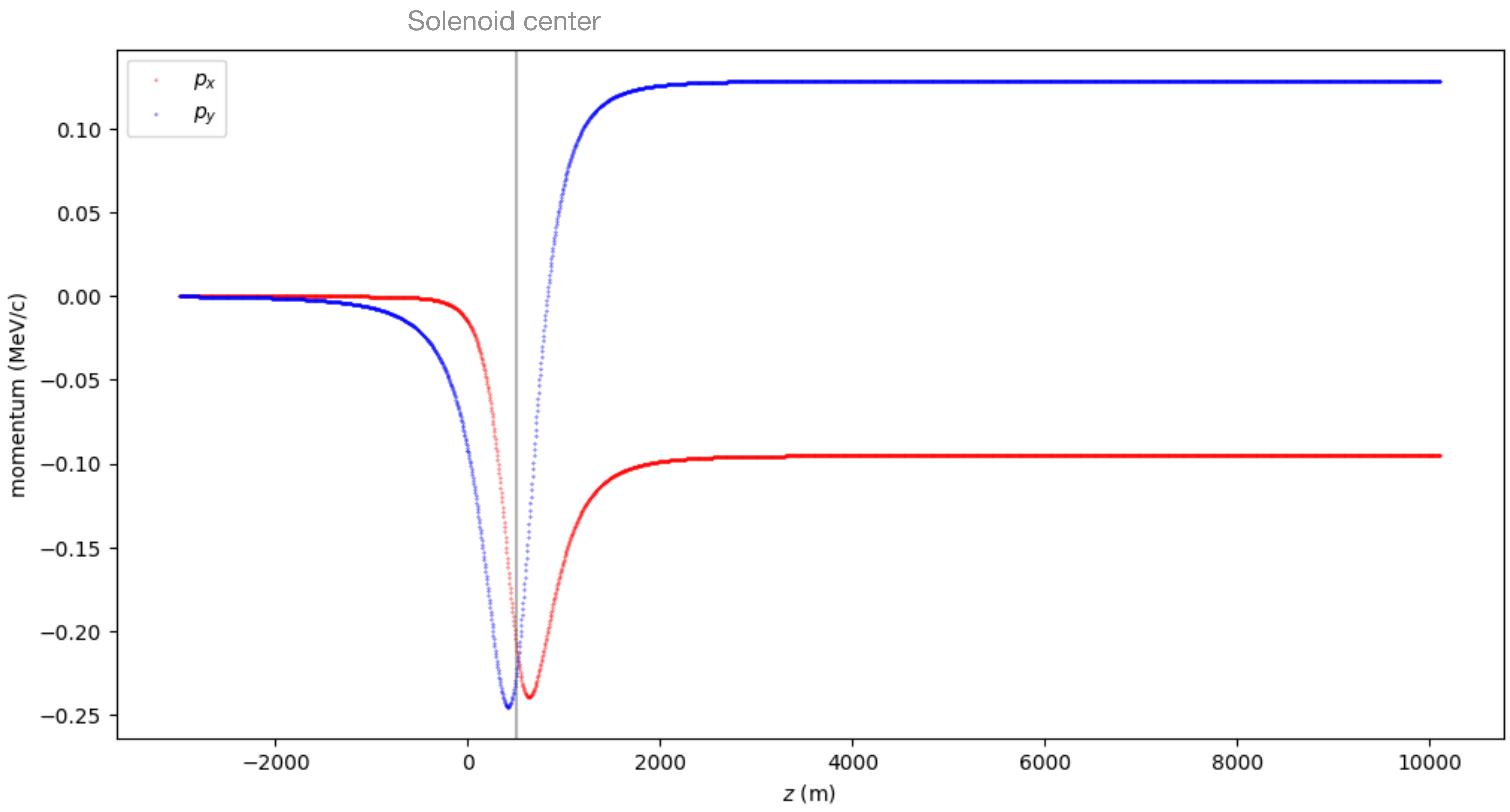




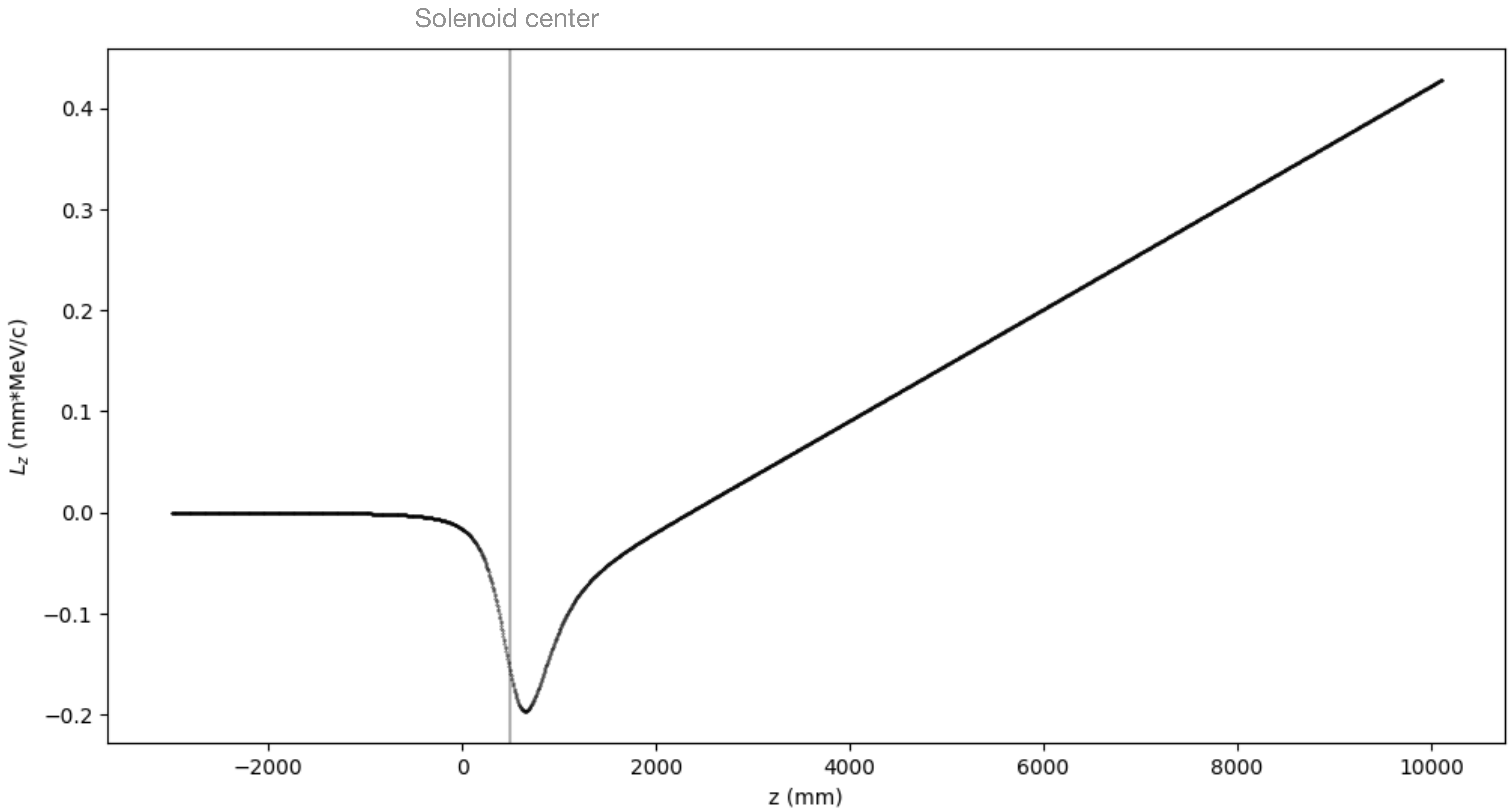
Transverse momentum



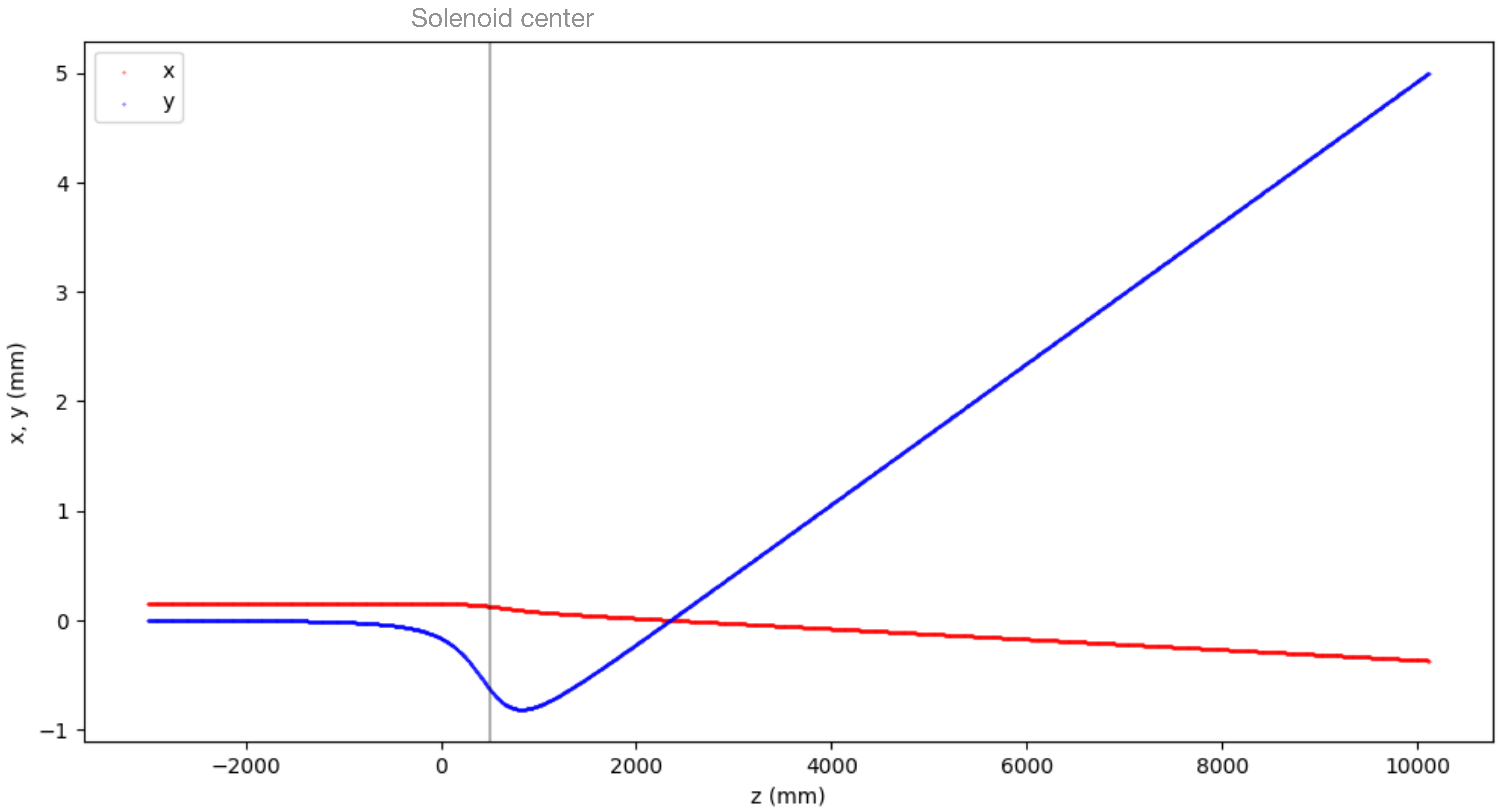
Longitudinal momentum is effectively constant



Angular momentum

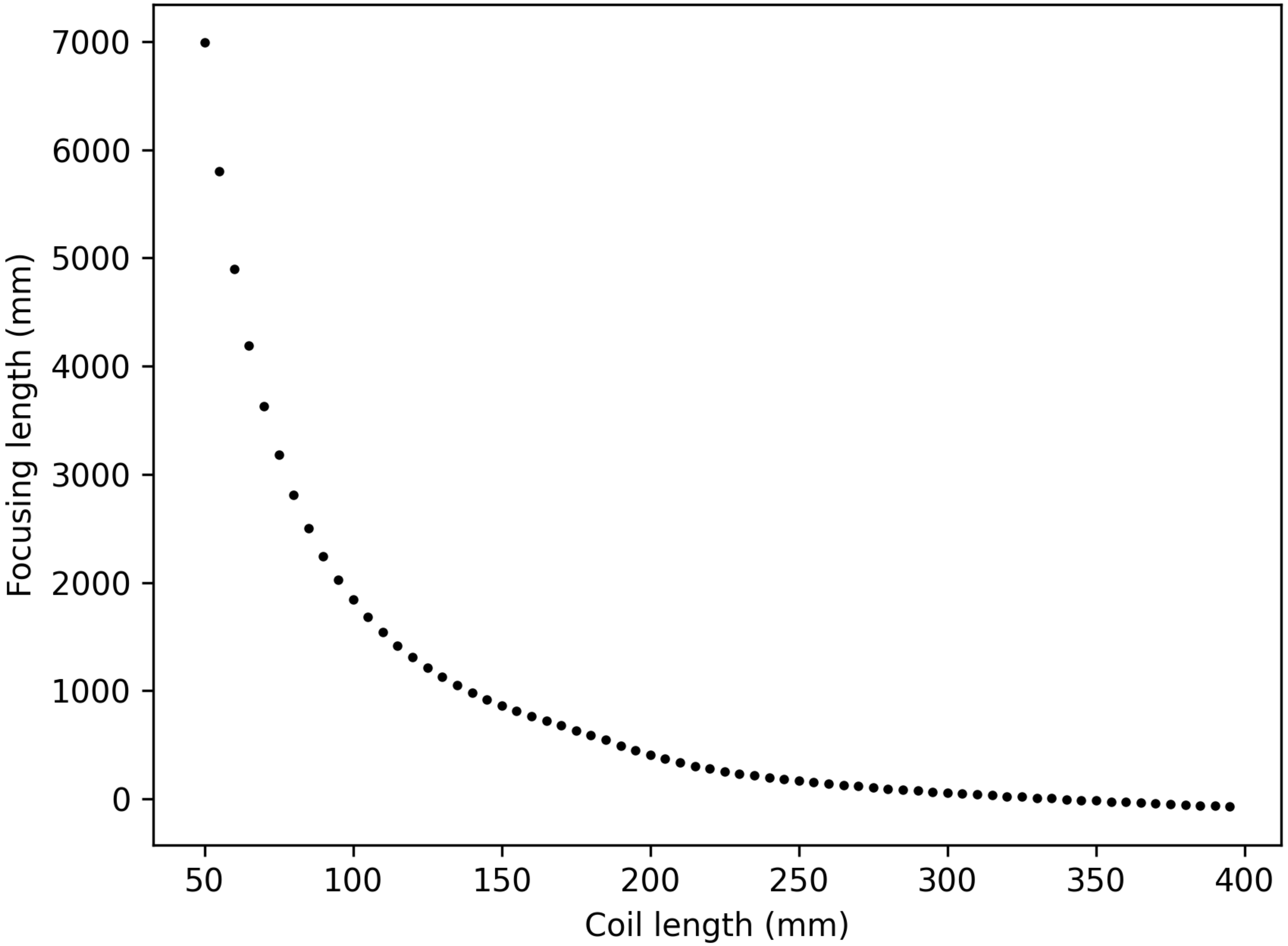


Transverse dynamics



Coil length scan

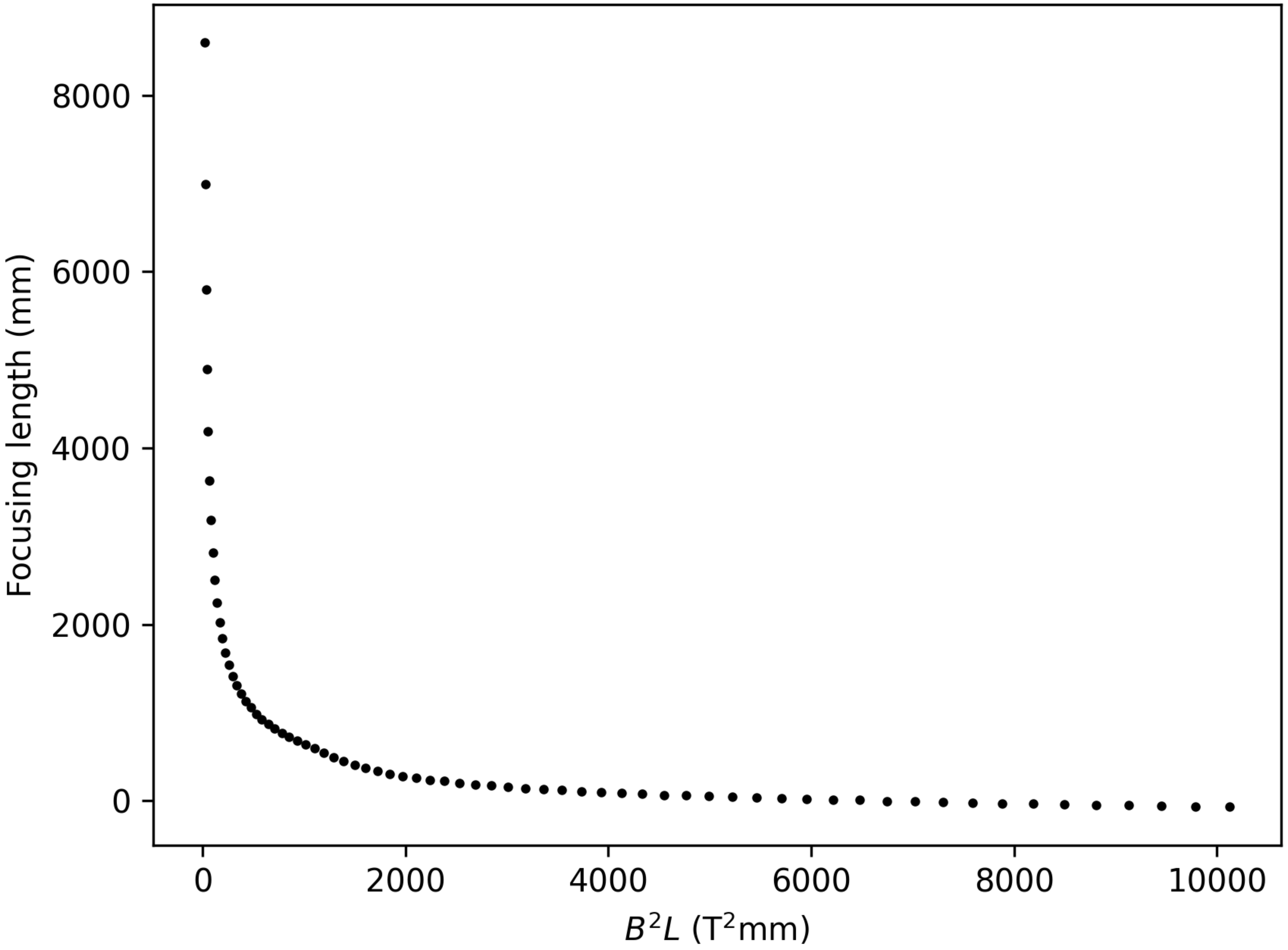
f dependence on coil length



Analytical form given by Fernow:

$$f = \frac{4p^2}{e^2B^2L}$$

f dependence on coil length



Analytical form given by Fernow:

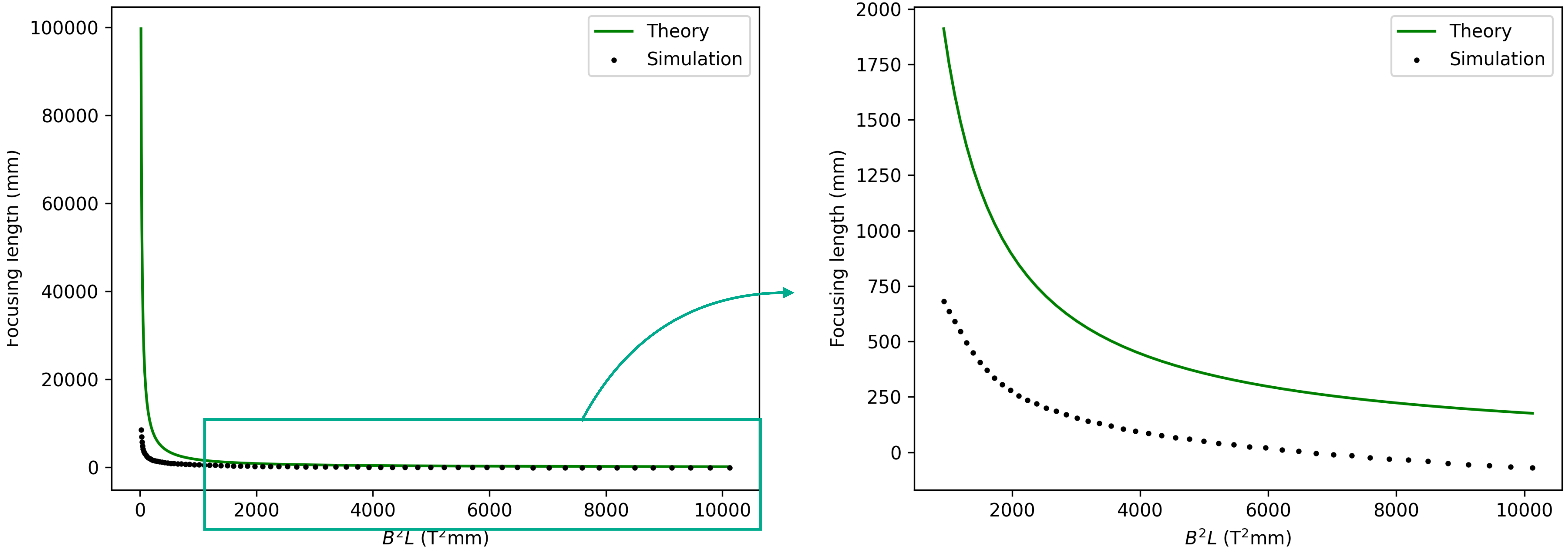
$$f = \frac{4p^2}{e^2 B^2 L}$$

B implicitly depends on L
$\implies$ more enlightening to consider both

f dependence on coil length

Analytical form given by Fernow:

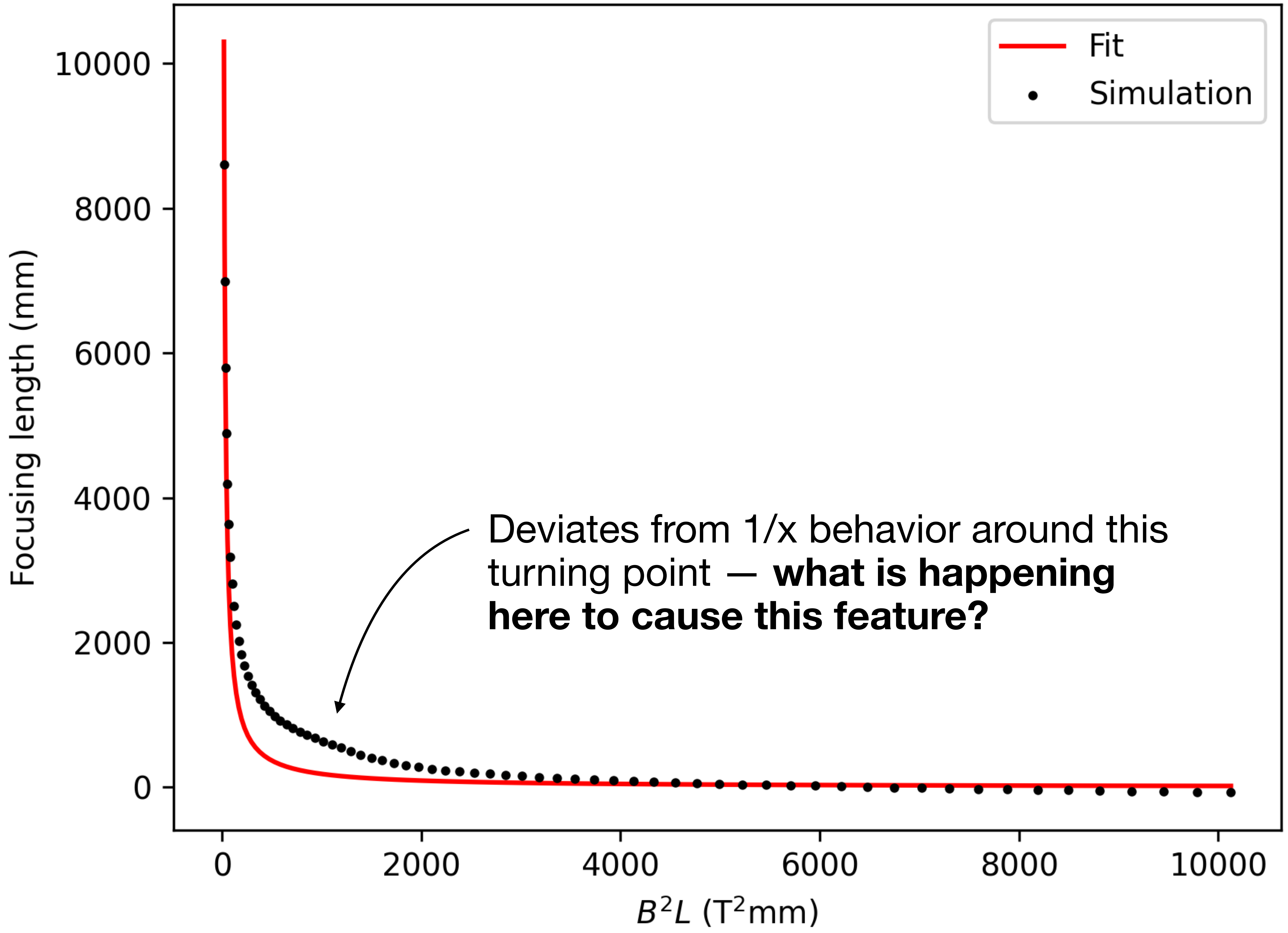
$$f = \frac{4p^2}{e^2 B^2 L}$$



Significant deviations from simulation at low B^2L values; no convergence even at large values

$\Longrightarrow$ Possible error in my code?

f dependence on coil length



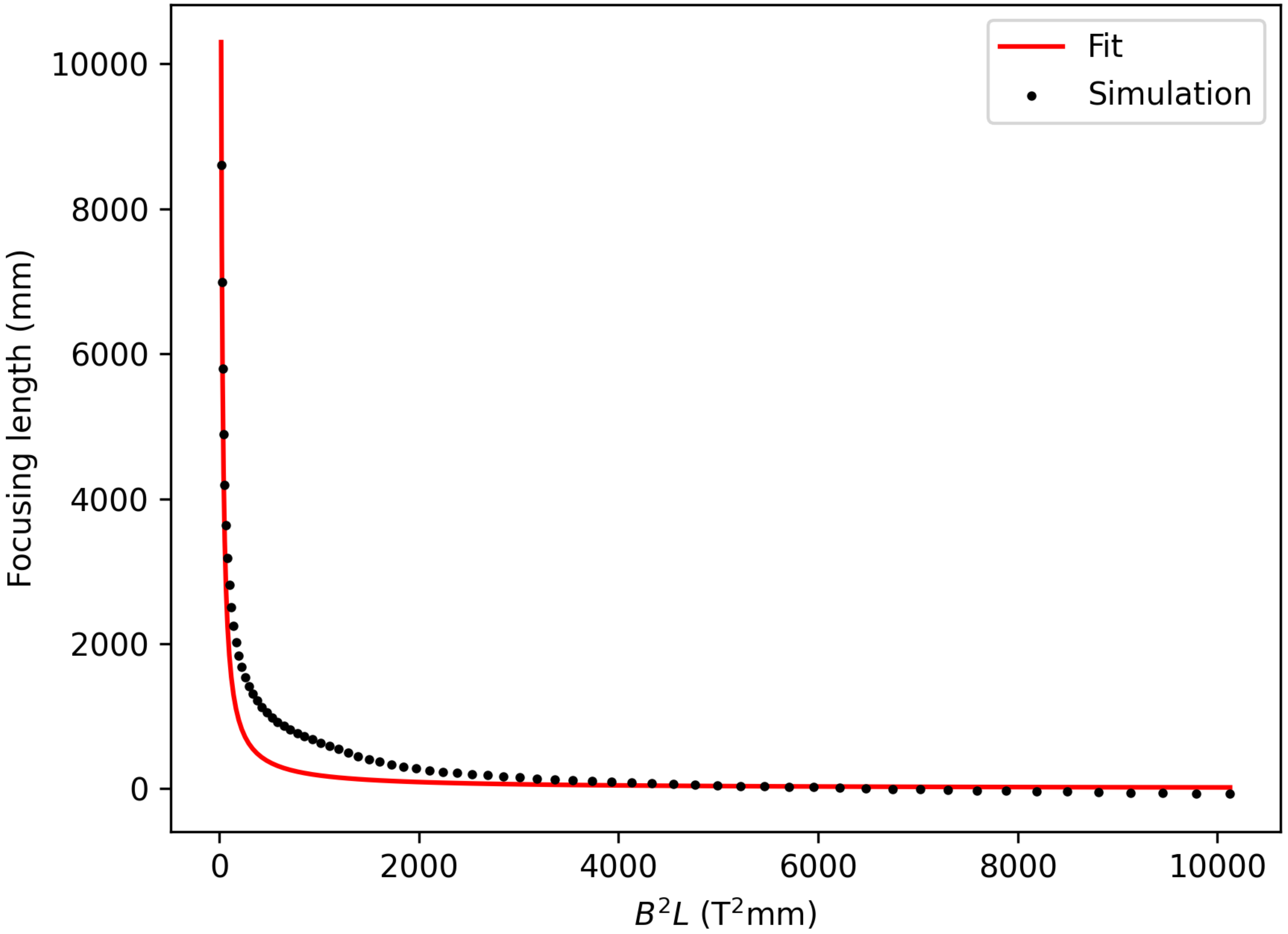
Analytical form given by Fernow:

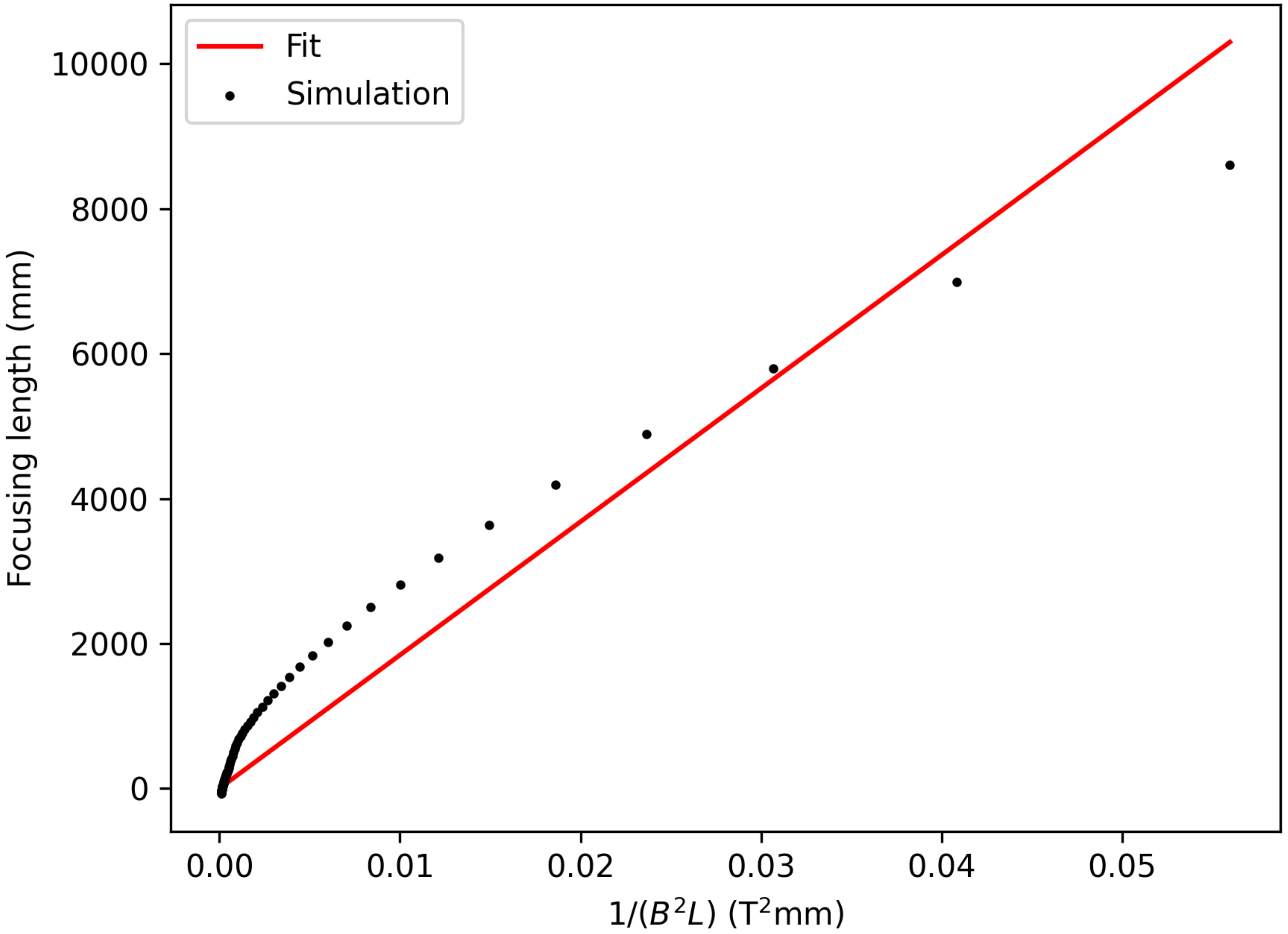
$$f = \frac{4p^2}{e^2B^2L}$$

Result from fit:

$$f = \frac{184010.8}{B^2L}$$

f dependence on coil length





+ Second solenoid

w/ **same** polarity

Simulation parameters:

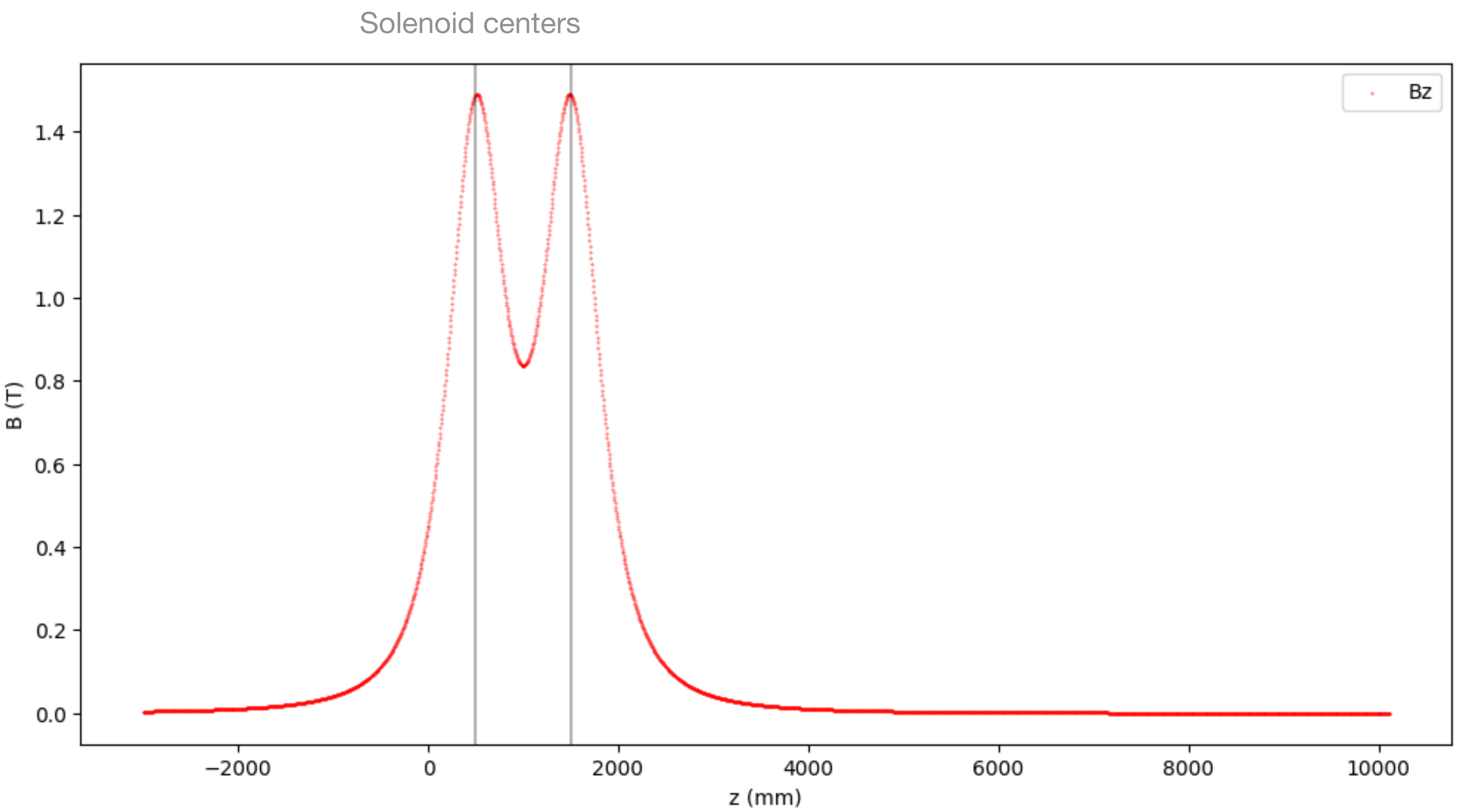
Coil length = 100mm

Initial coil centered at z = 500mm

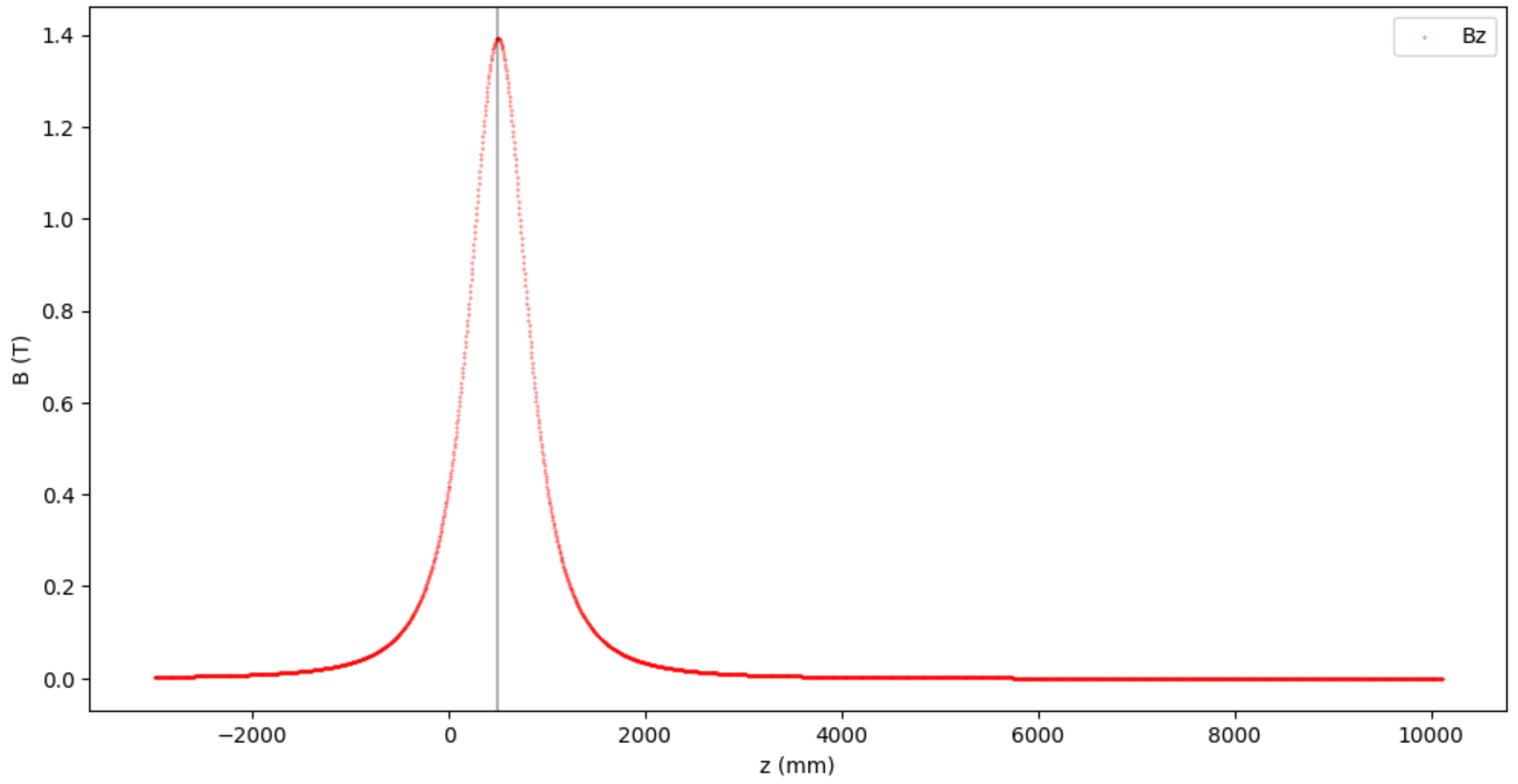
Initial conditions
(x_0, y_0, x'_0, y'_0) = (1.5mm, 0, 0, 0)

Spacing between coil centers = 1000mm

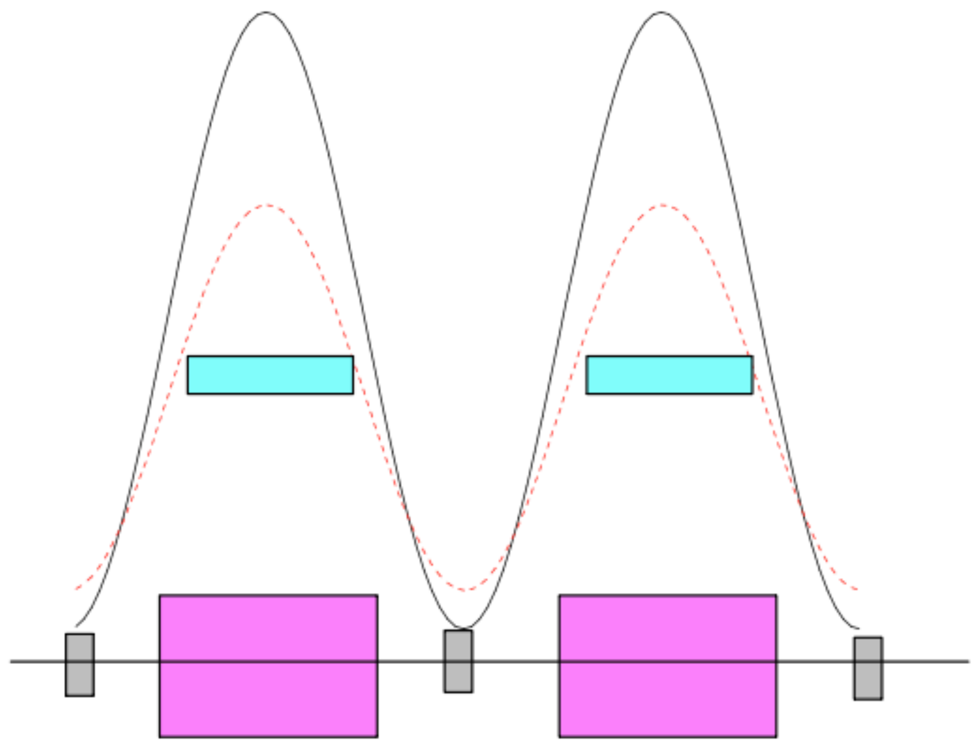
Longitudinal B field



Compare to single solenoid:



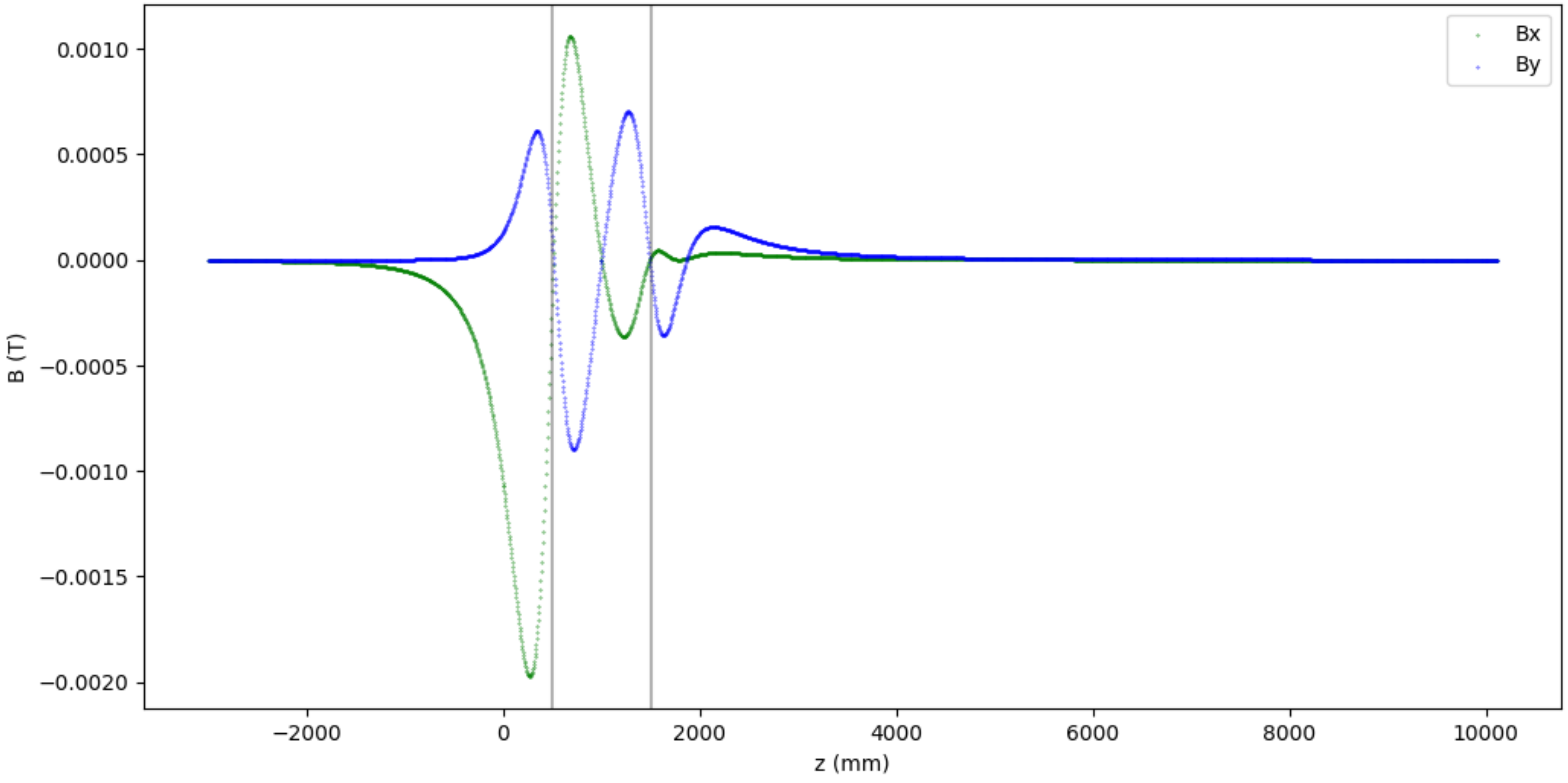
Consistent with expectations from Fernow paper ✓

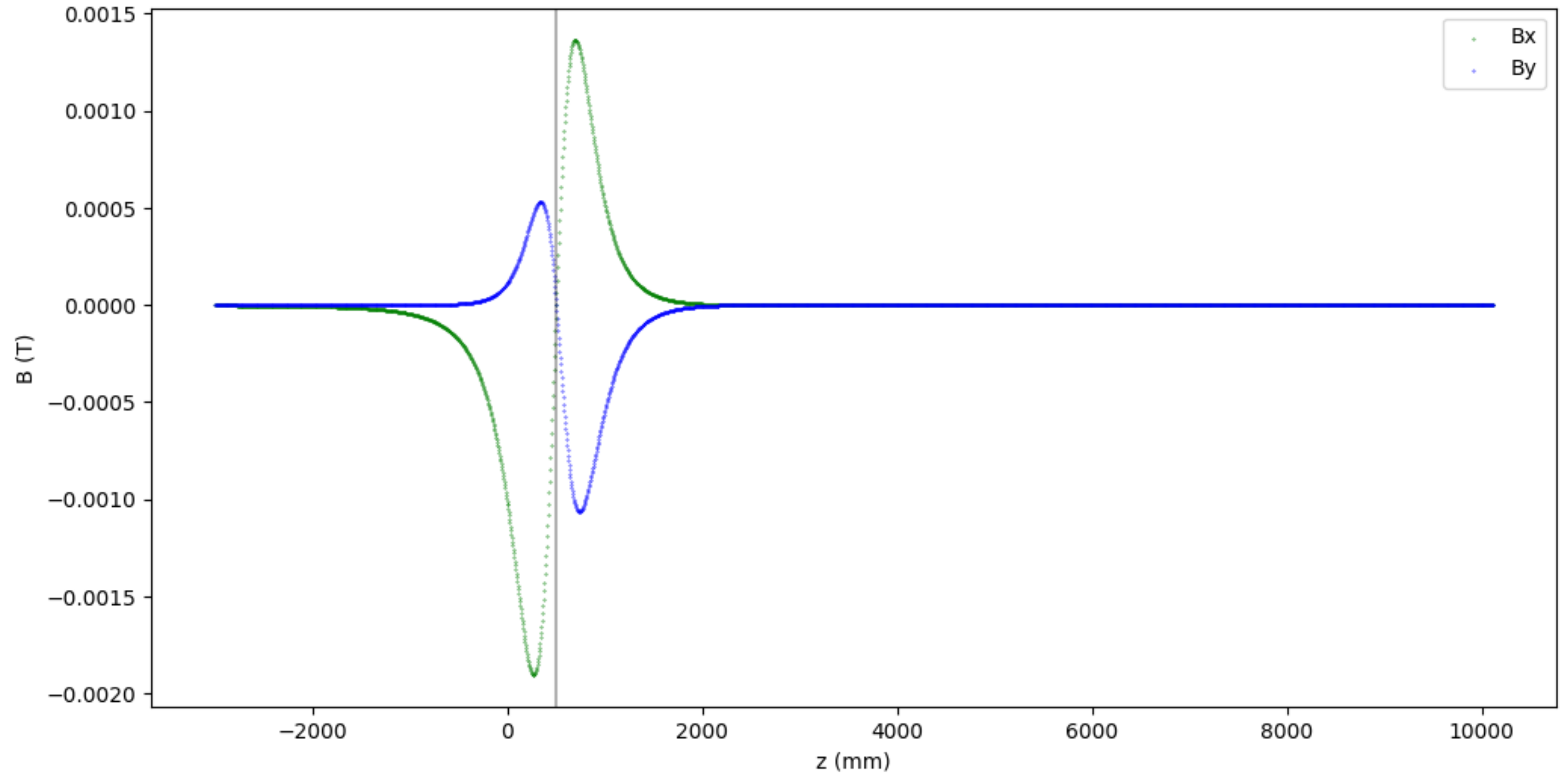


Transverse B field

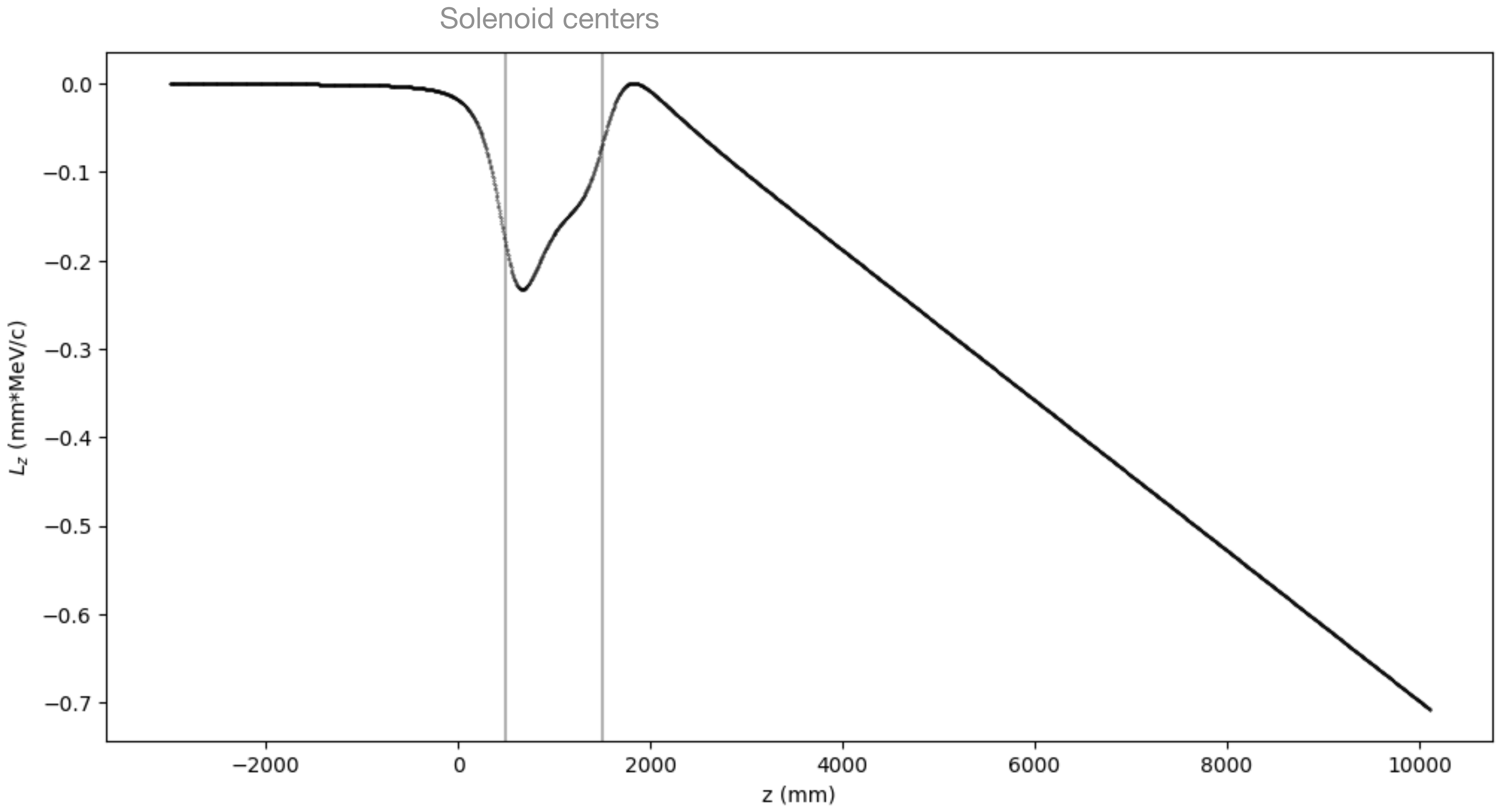
Compare to single solenoid:

Solenoid centers

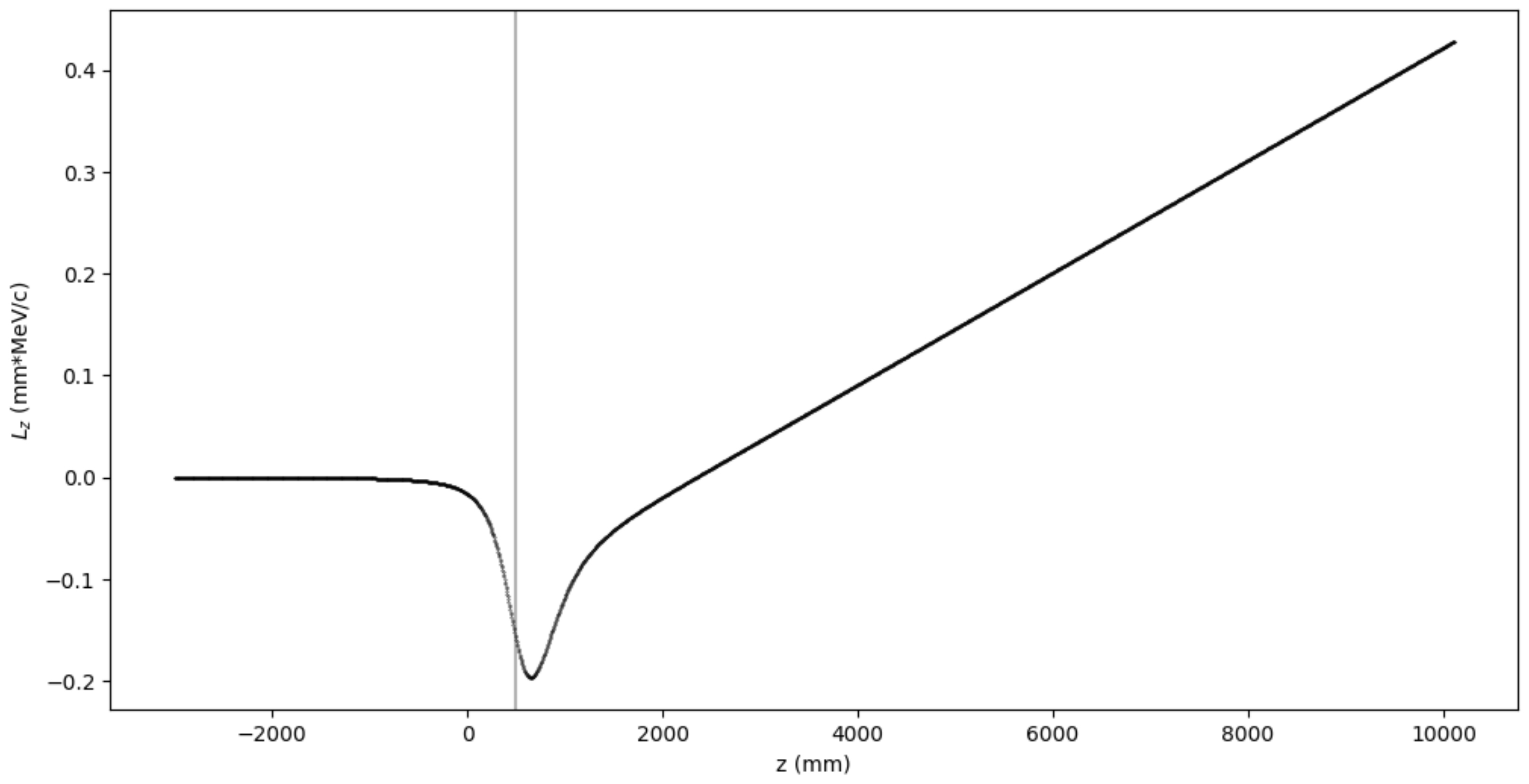




Angular momentum



Compare to single solenoid:



+ Second solenoid

w/ **flipped** polarity

Simulation parameters:

Coil length = 100mm

Initial coil centered at z = 500mm

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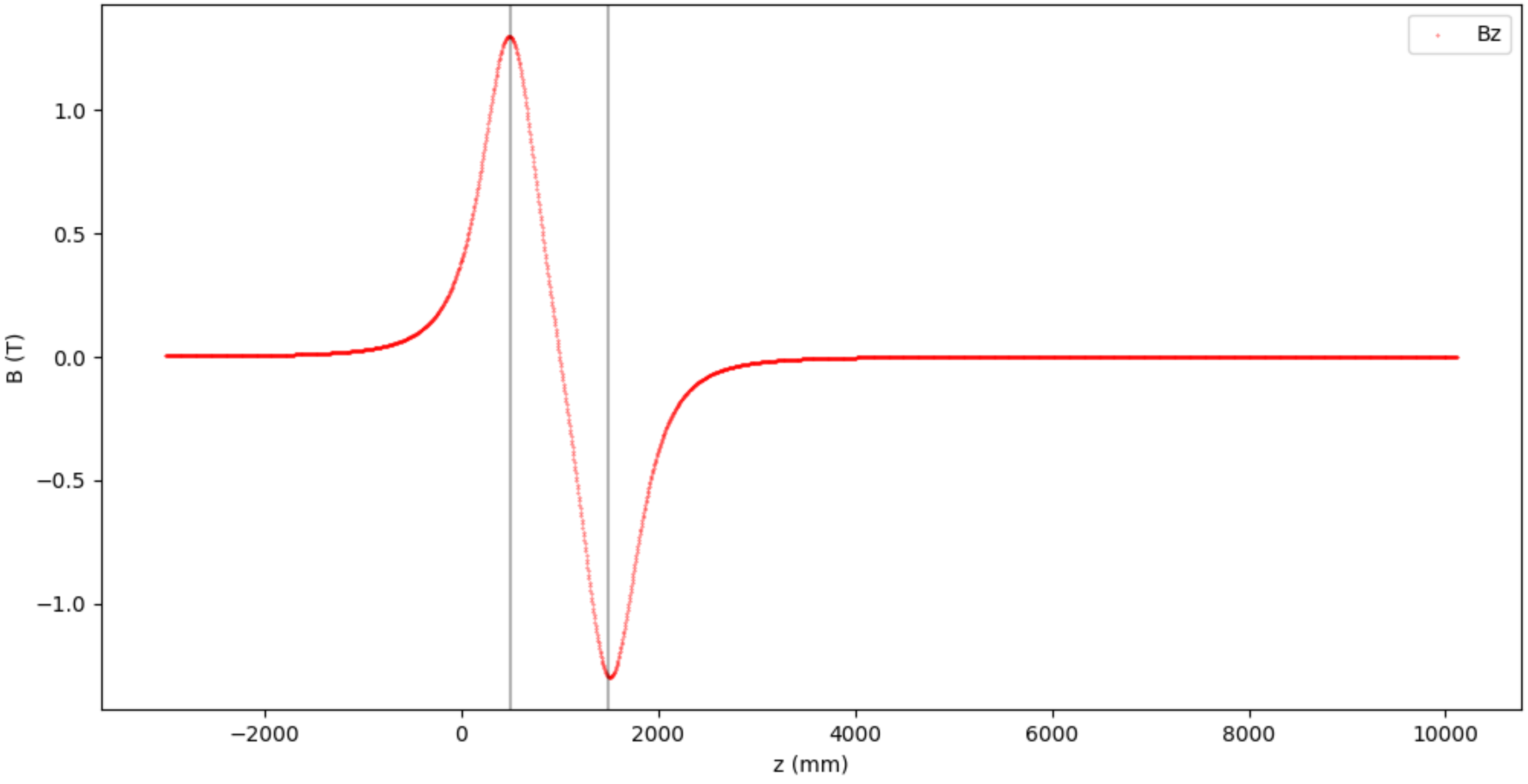
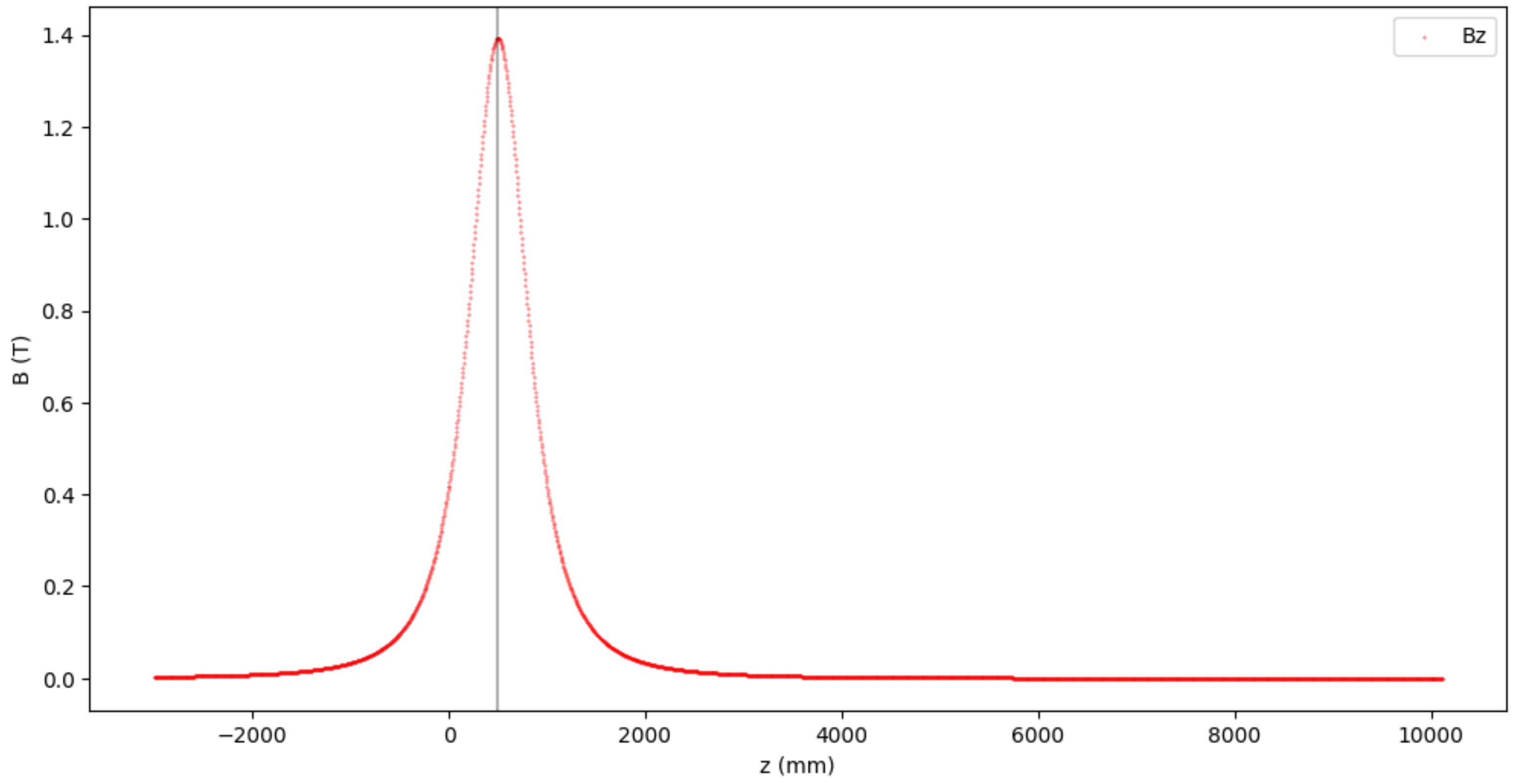
Spacing between coil centers = 1000mm

Longitudinal B field

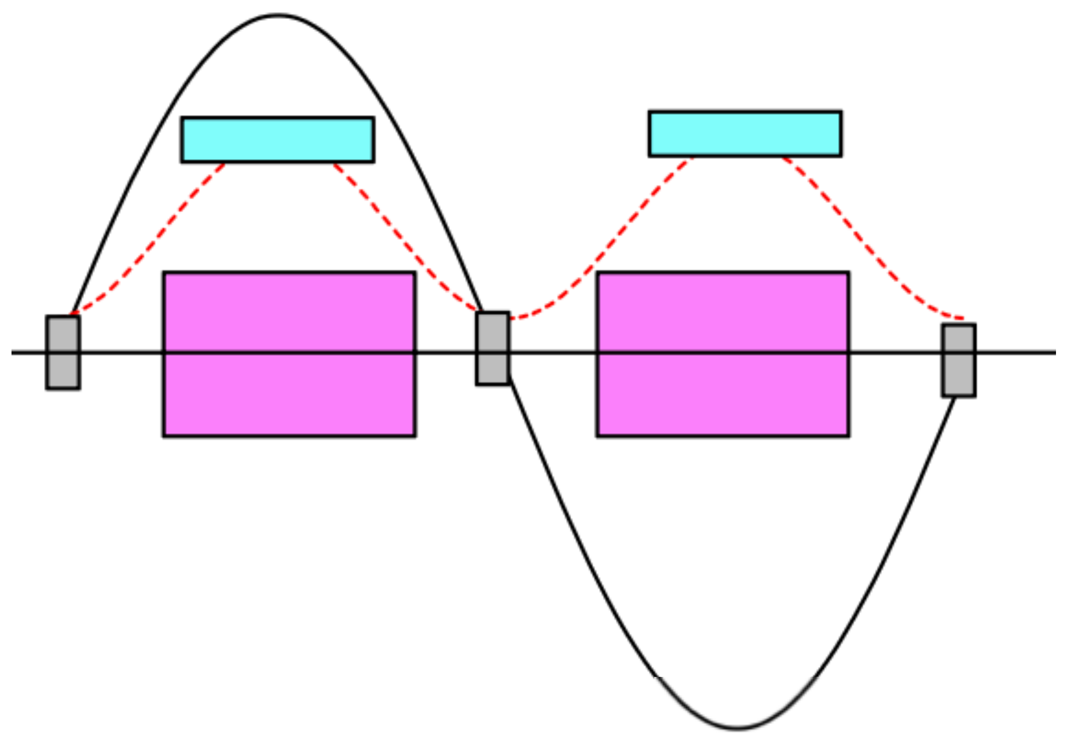
Solenoid centers



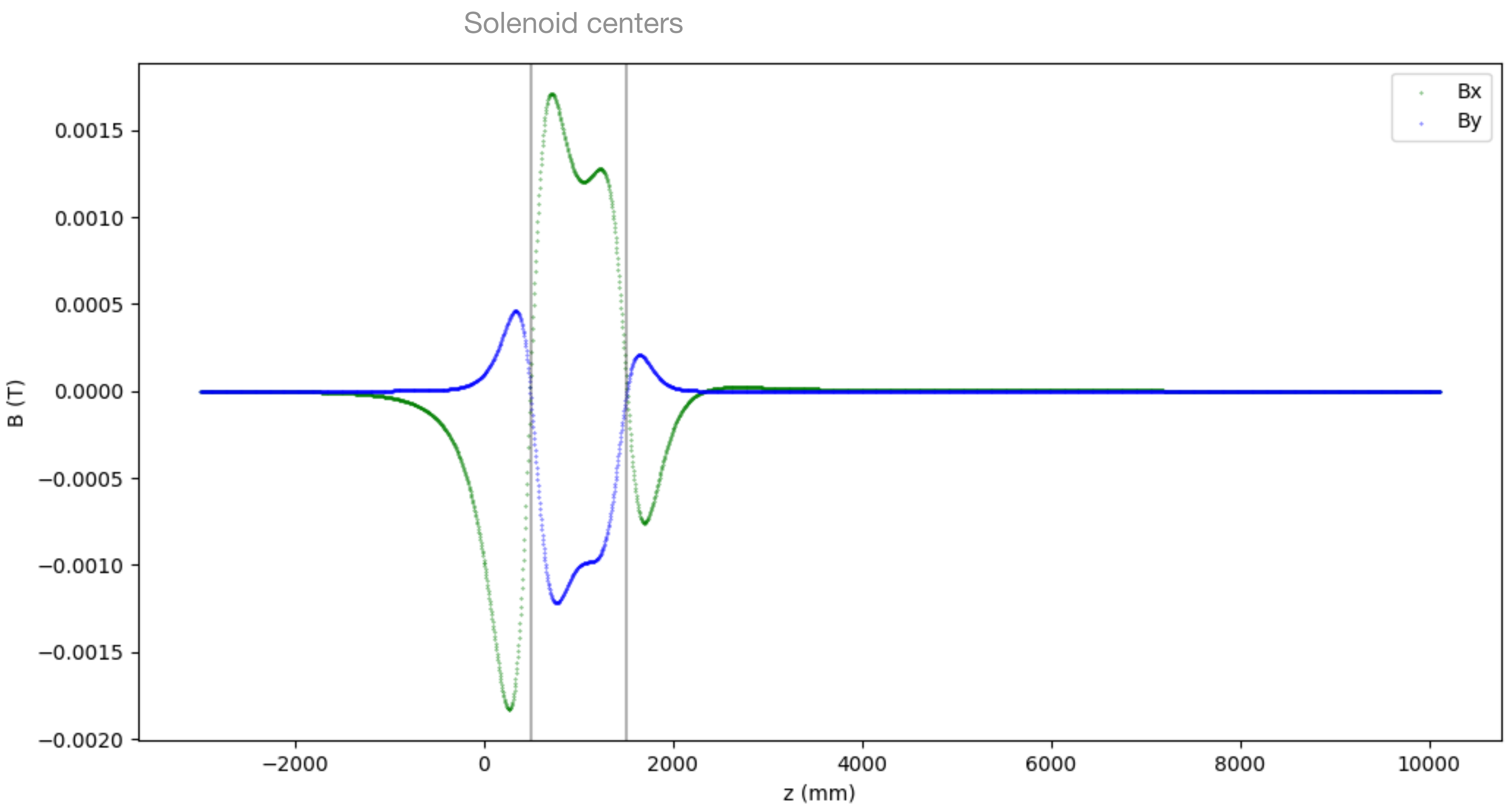
Compare to single solenoid:



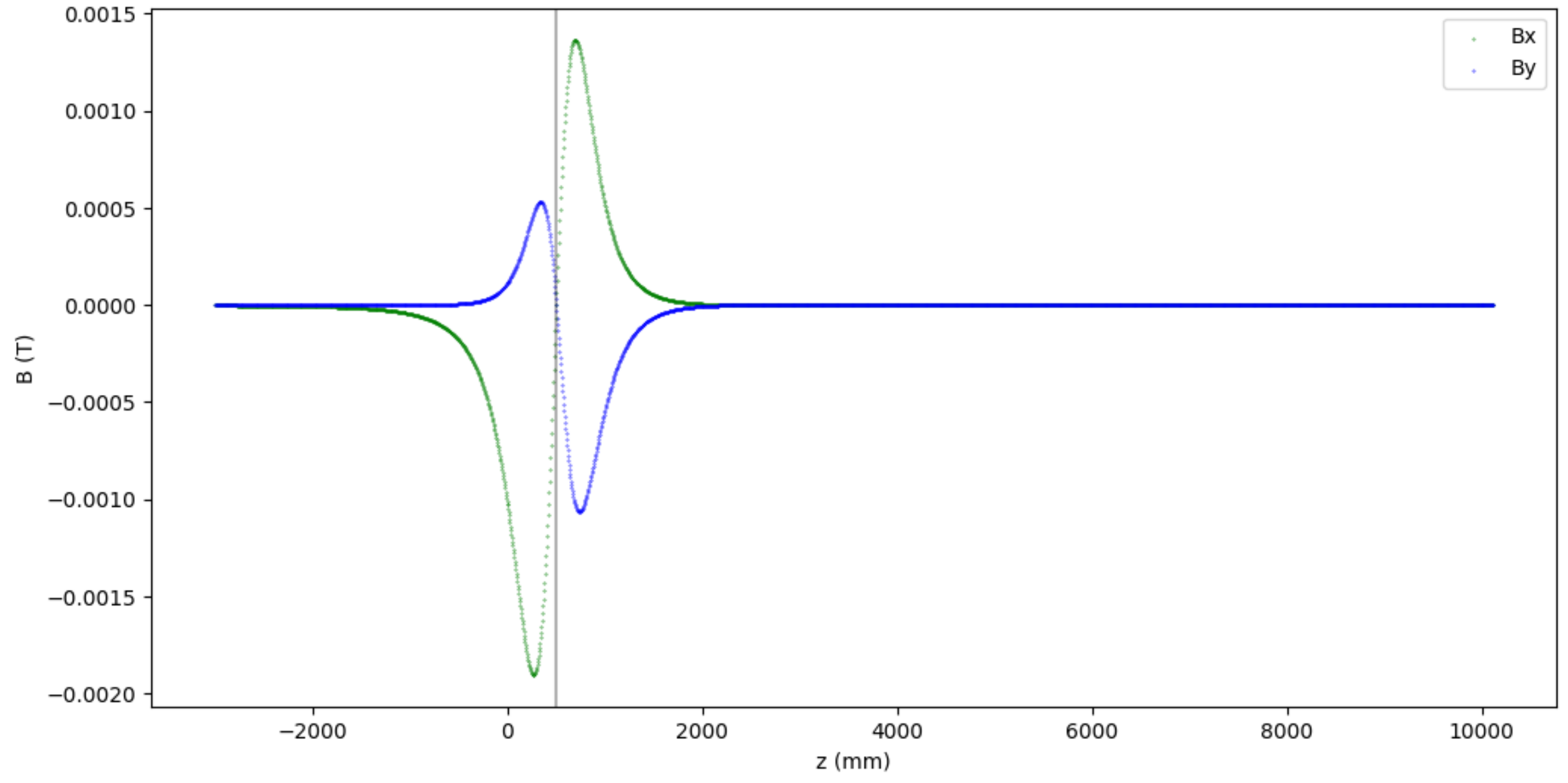
Consistent with expectations from Fernow paper ✓



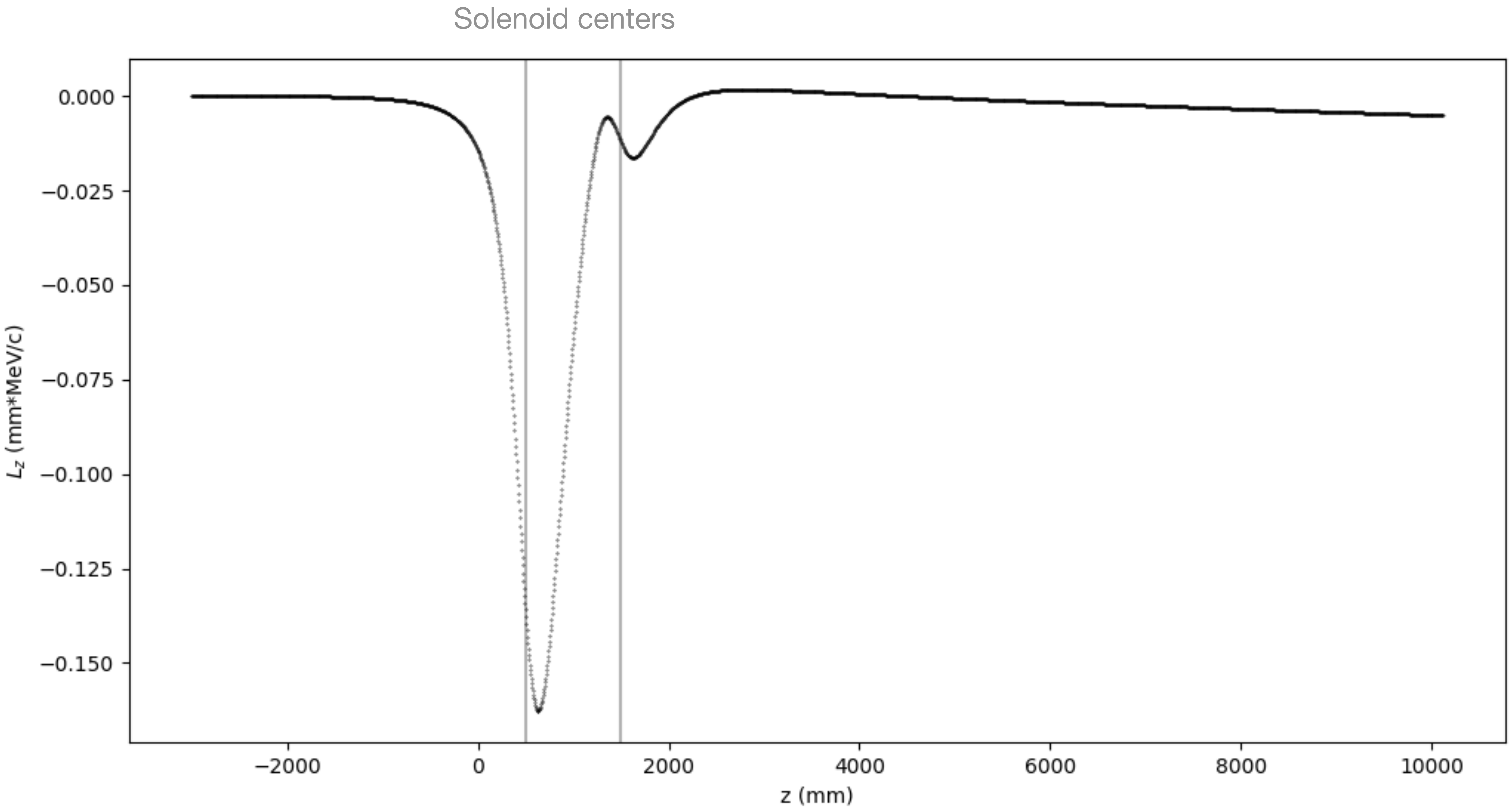
Transverse B field



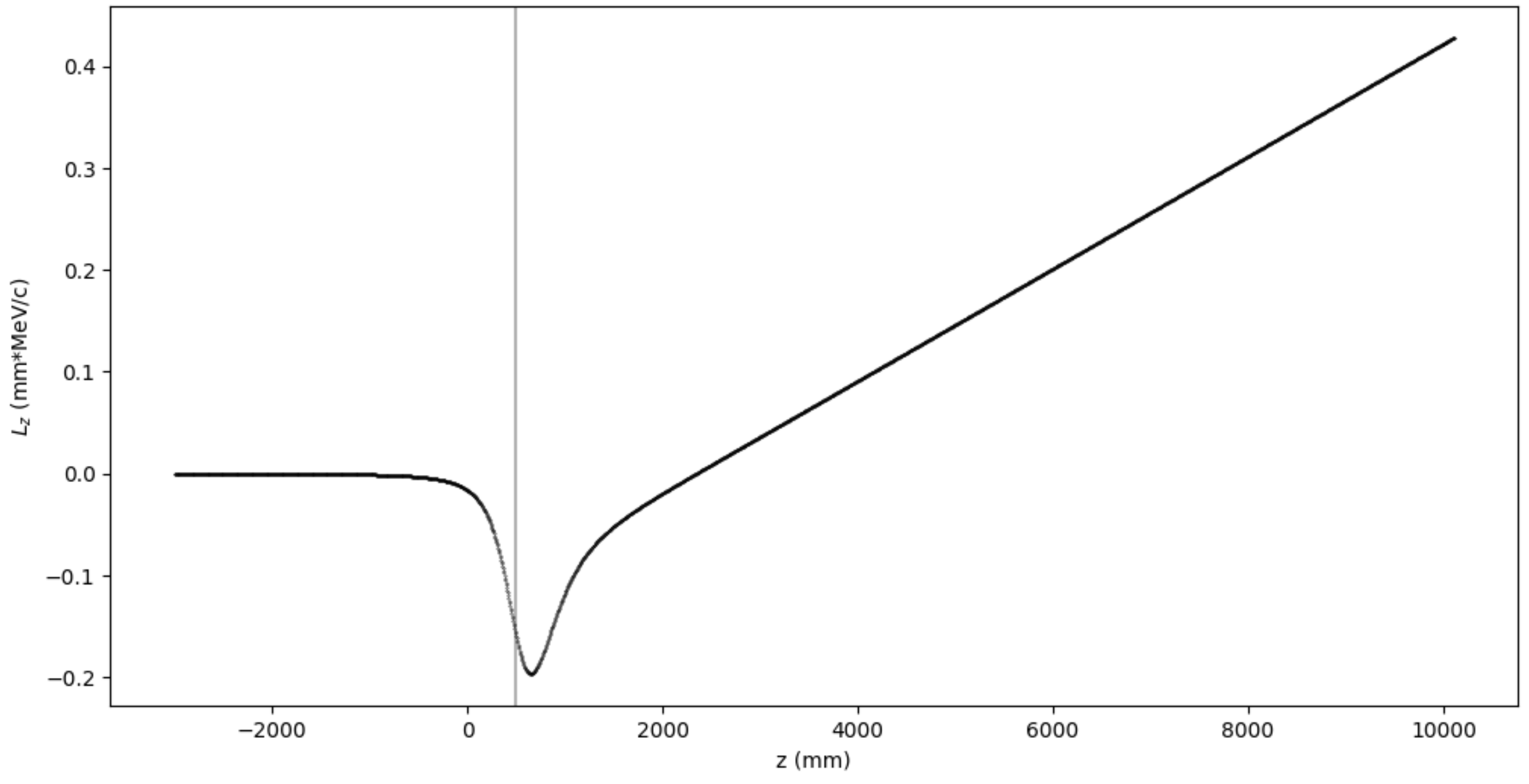
Compare to single solenoid:



Angular momentum



Compare to single solenoid:



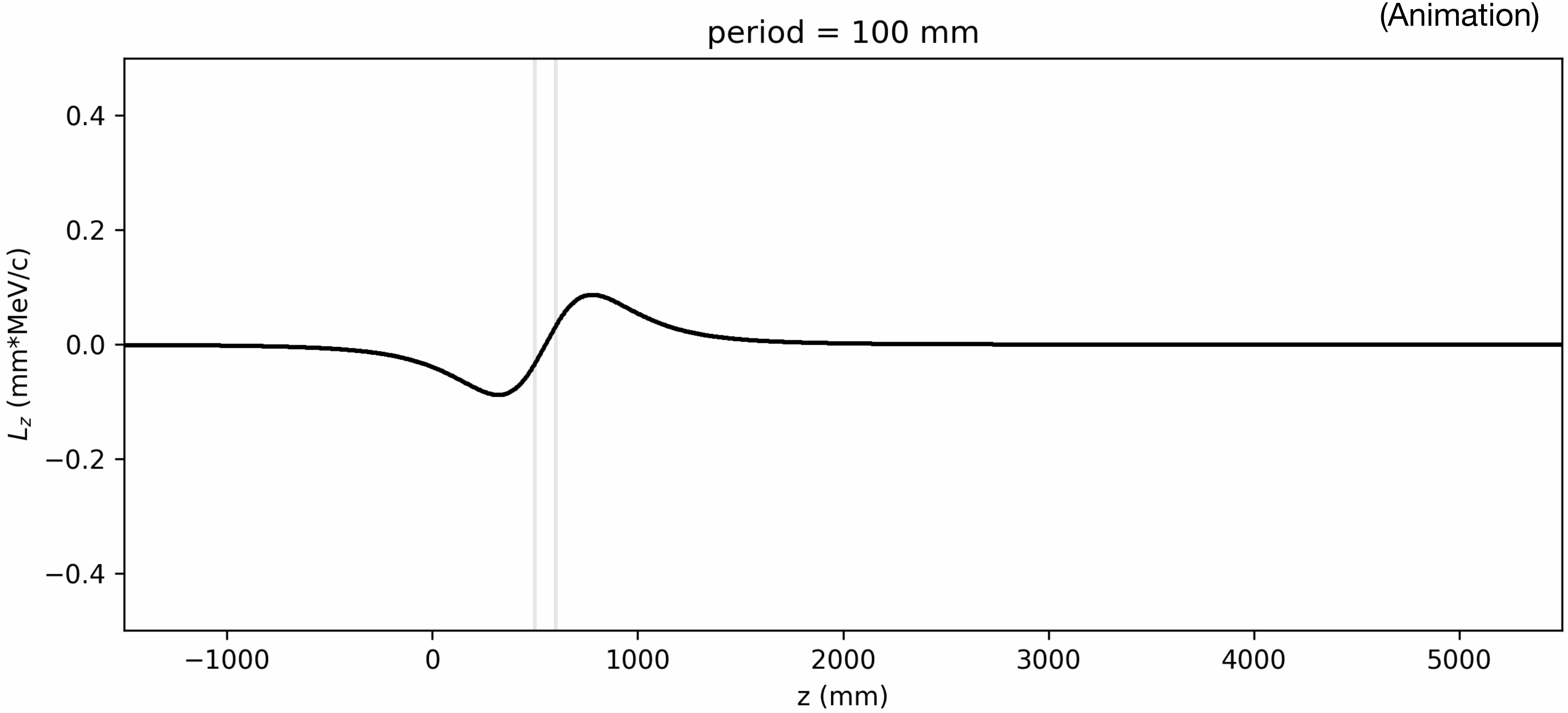
Anisotropy w.r.t. zero $\Longrightarrow$ need to adjust spacing to achieve symmetry

Solenoid spacing scan

w/ **flipped** polarity

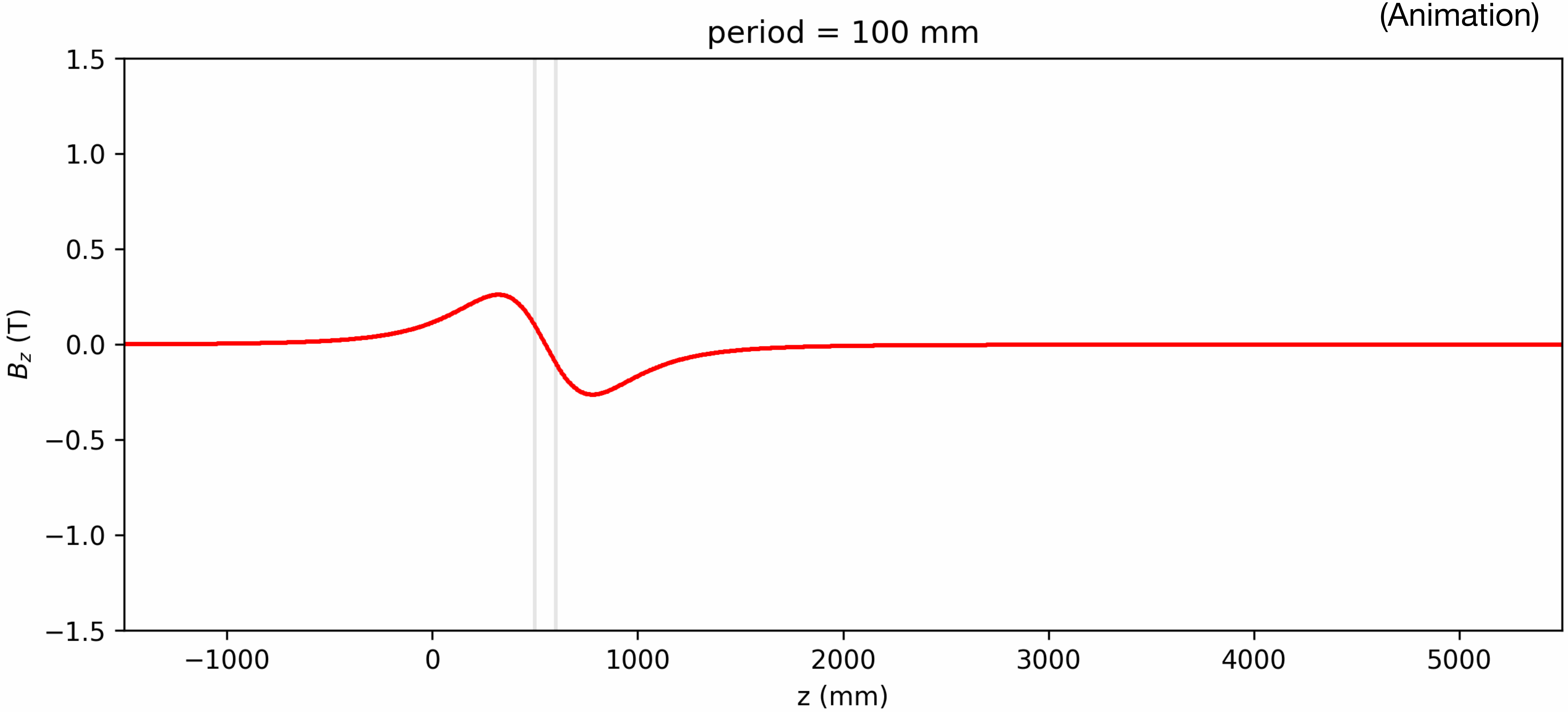
Placing the second solenoid

(Animation)

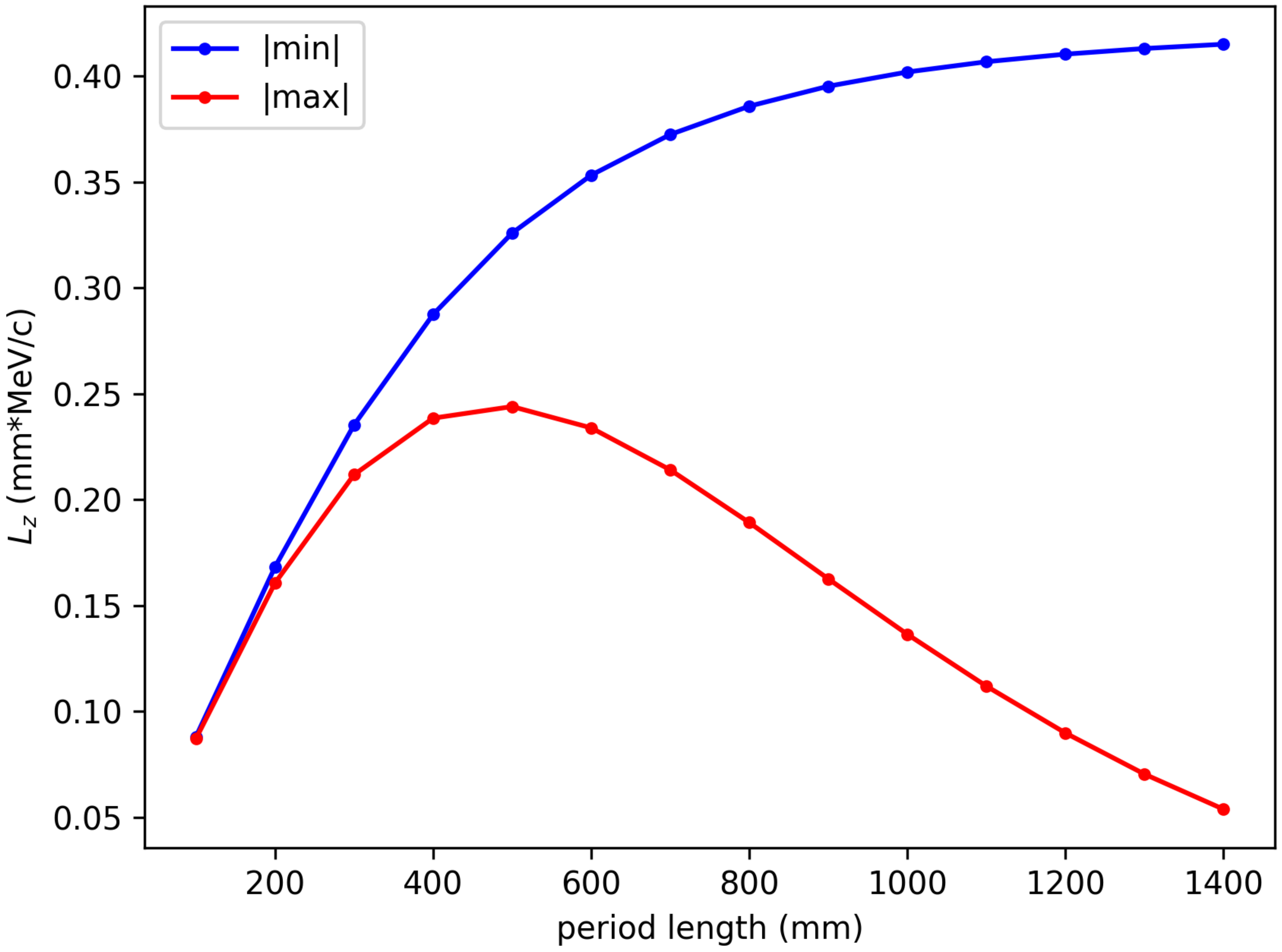


Placing the second solenoid

(Animation)



Placing the second solenoid



Magnitude of minimum and maximum L_z equivalent $\Longrightarrow L_z$ symmetric

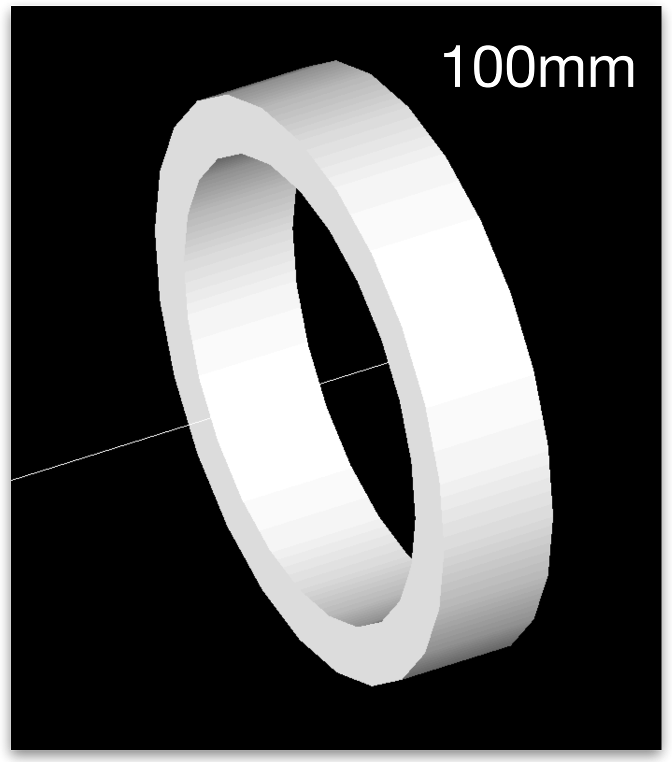
∴ Conclude that **smaller spacings** achieve better results

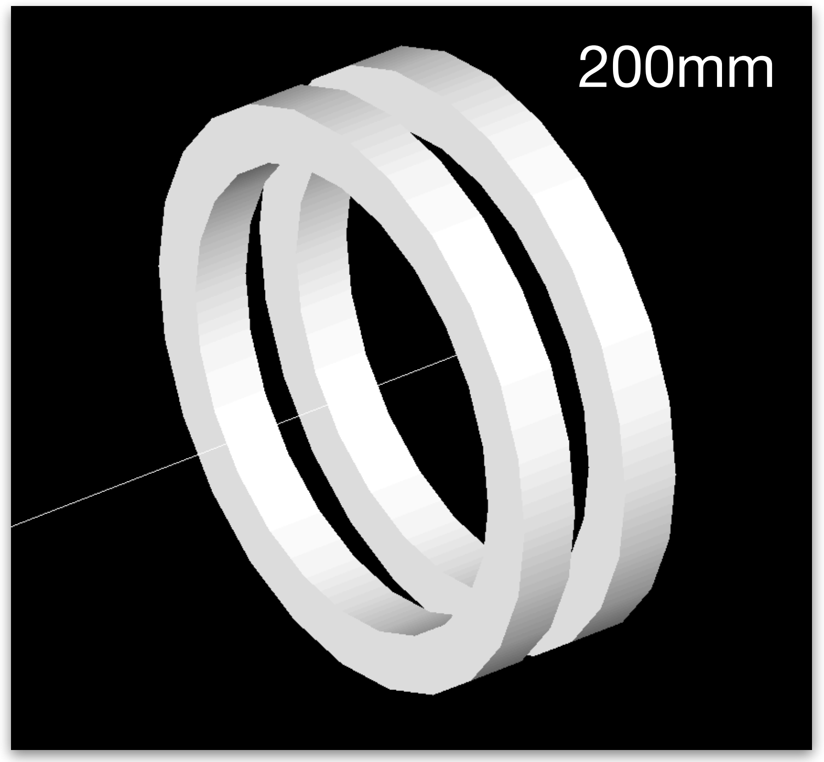
(From previous slide, B field is also sinusoidal in this regime ✓)

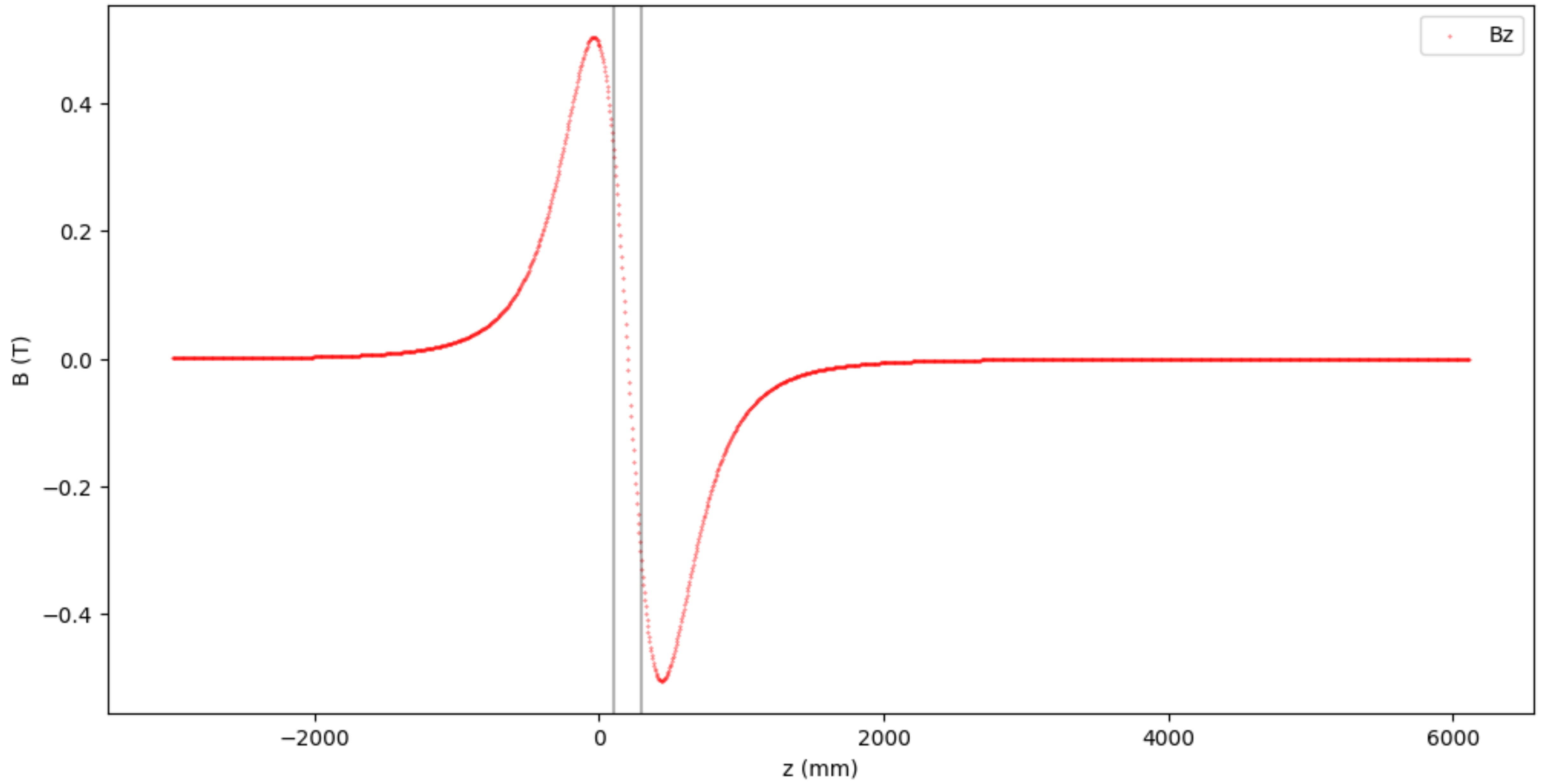
Placing the second solenoid

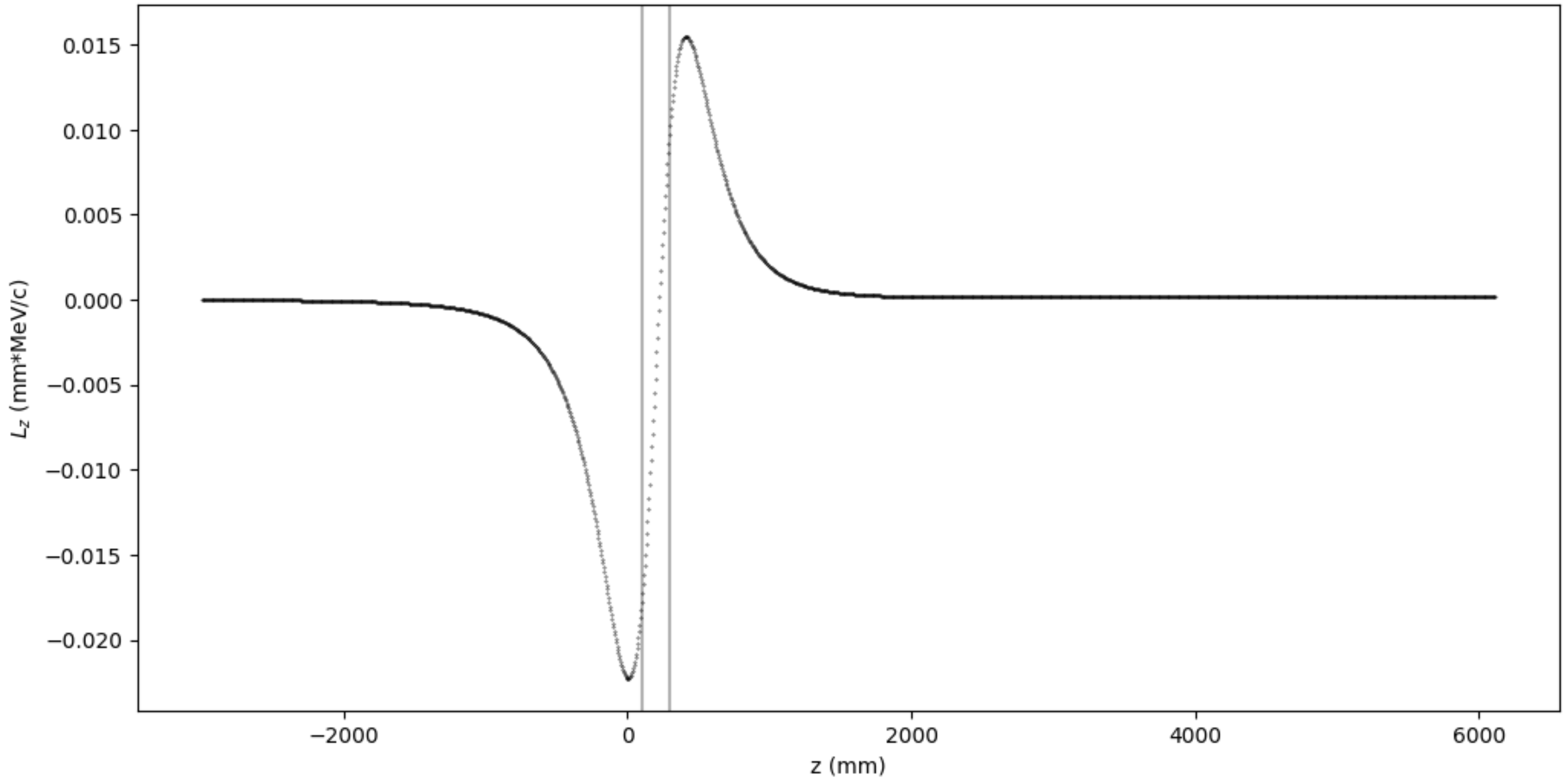
Revisiting the geometry: 100mm spacing
$\Longrightarrow$ solenoids are *immediately next to each other*

∴ Try with spacing of 200mm to avoid a trivial solution:









Should e.g. the initial conditions be adjusted to avoid a solution where L_z and field are non-zero outside the solenoids?

Placing the second solenoid

Some definitions for clarity:

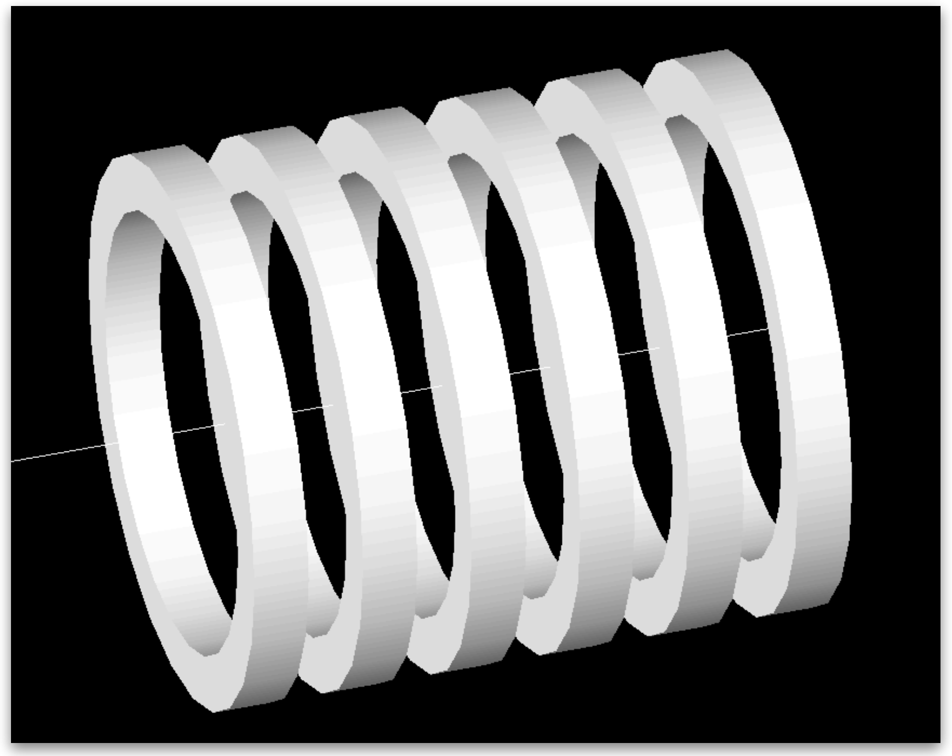
Period ≡ distance between centers of solenoids

```
# FOFO channel
param solinner=400.0
param solthick=100.0
param sollength=100.0
param period=1000.0
param solcurrent=100.0
param maxStep=5.0
coil Coil1 innerRadius=$solinner outerRadius=$solinner+$solthick length=$sollength maxR=5000 maxZ=10000.0
solenoid FOFOCoil coilName=Coil1 current=$solcurrent
solenoid FOFOposCoil coilName=Coil1 current=$solcurrent
solenoid FOFOnegCoil coilName=Coil1 current=-$solcurrent
```

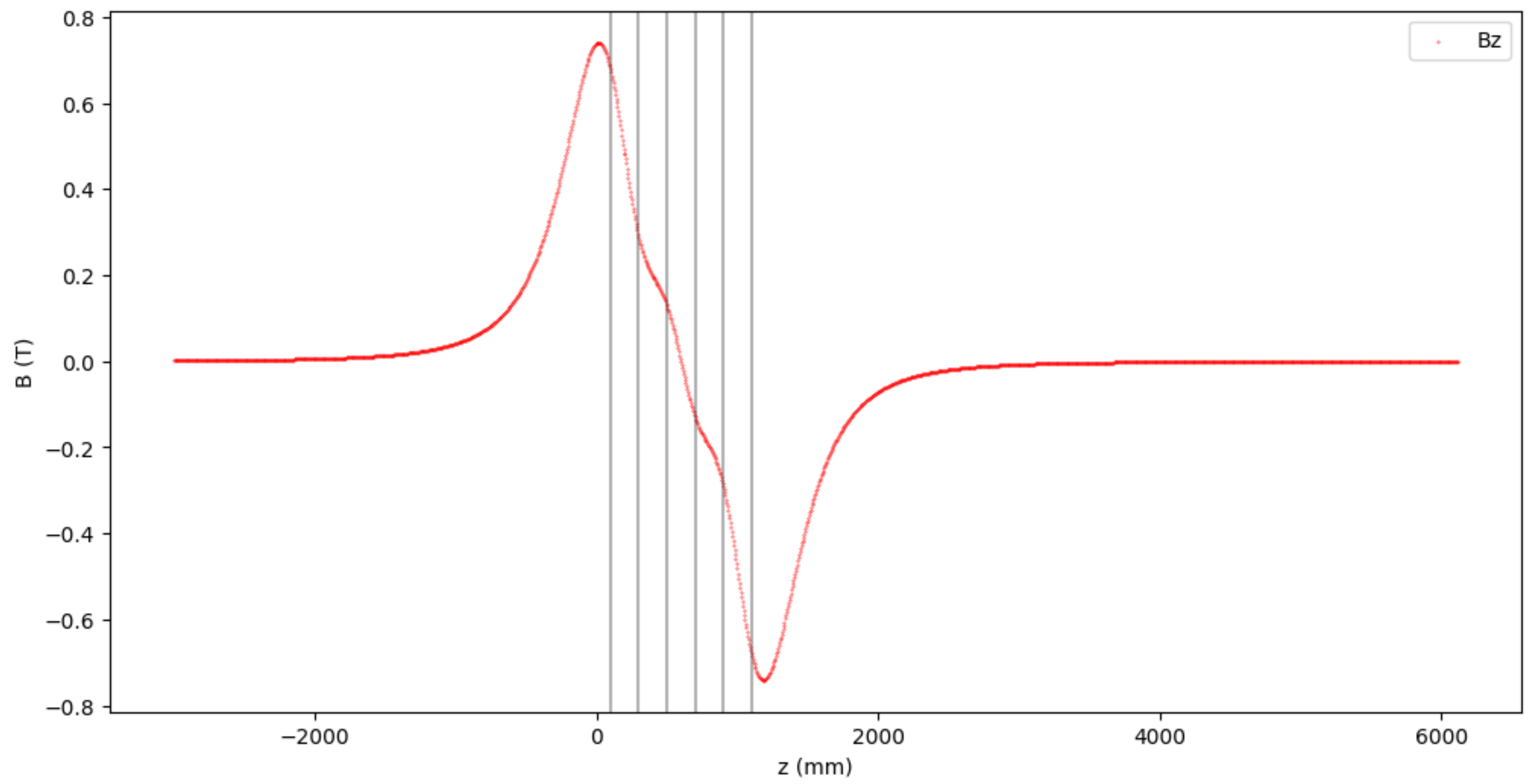
```
do i 0 0

    place FOFOposCoil x=0 z=500+2*$i*$period  y=0
    place FOFOnegCoil x=0 z=500+2*$i*$period+$period y=0

enddo
```

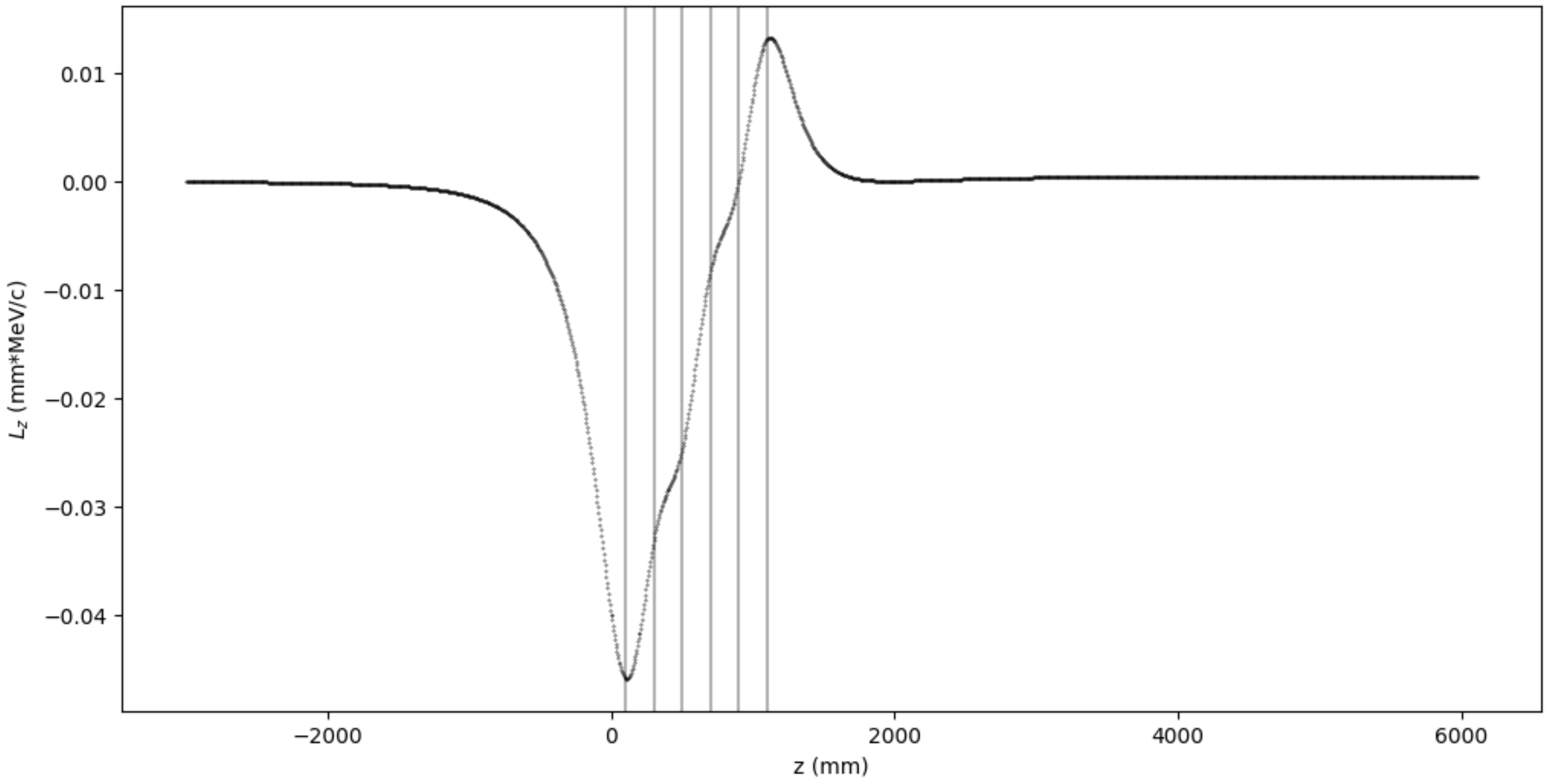
See https://github.com/criggall/muon-cooling/blob/main/Solenoid-Study/build-channel/morecoils.in

Placing additional solenoids



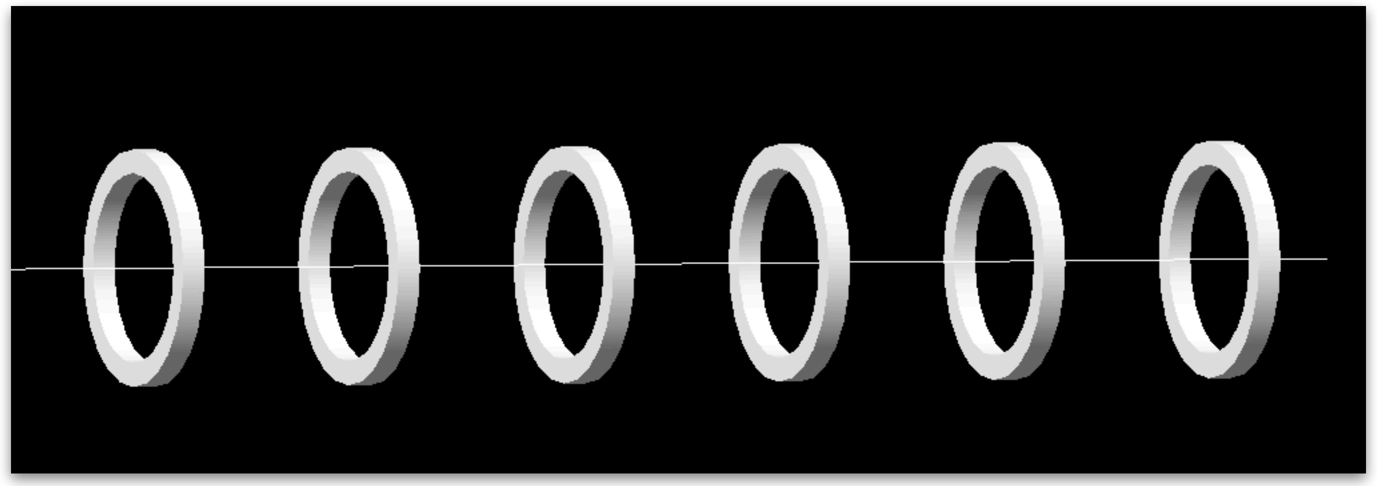
Continuing with the 200mm spacing, extending to a **six**-solenoid channel:



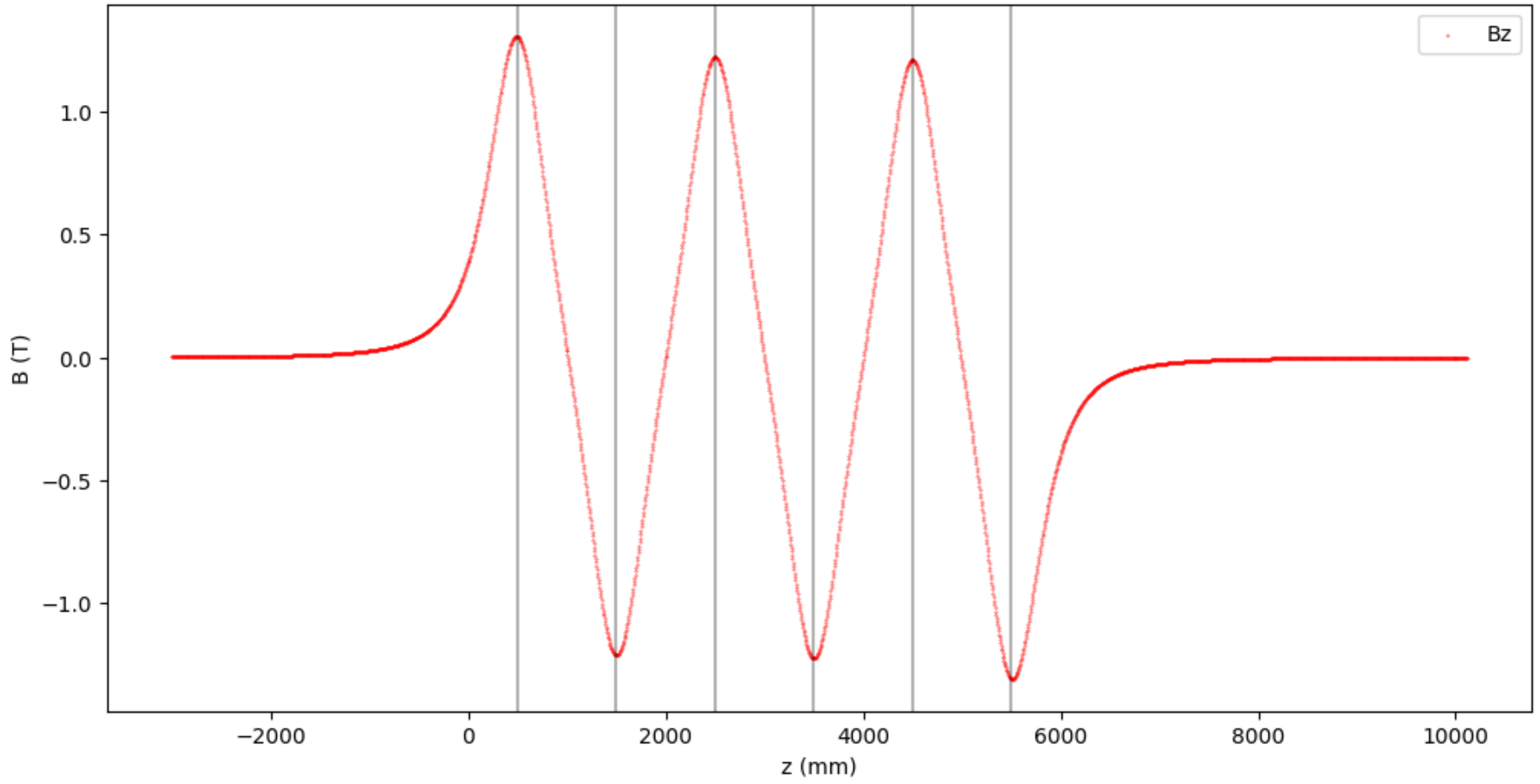


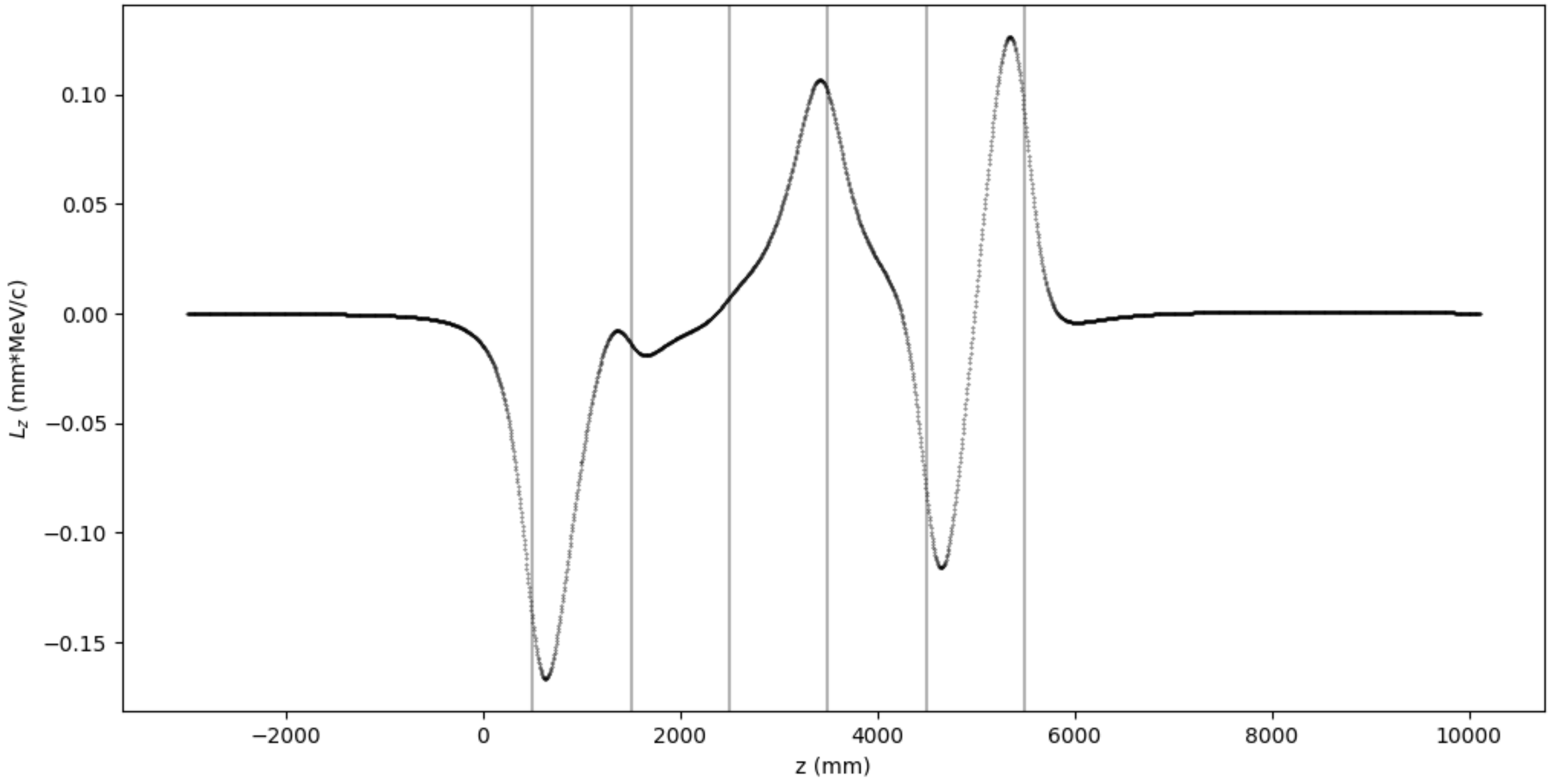
The longitudinal B field and L_z become **non-uniform**!

Placing additional solenoids



Switching back to 1000mm spacing (nearly) **restores the sinusoidal behavior** of B_z

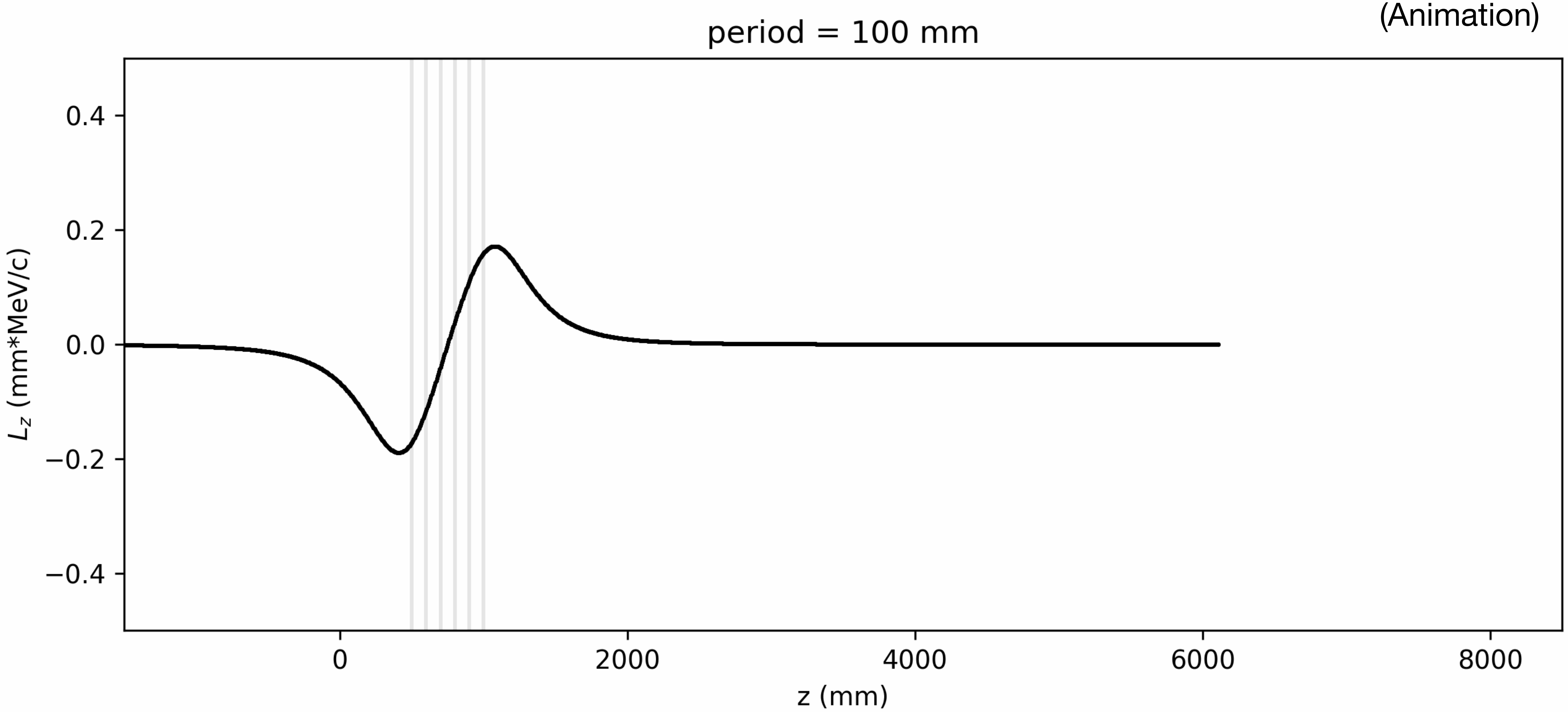




…But we do not see isotropy in L_z behavior

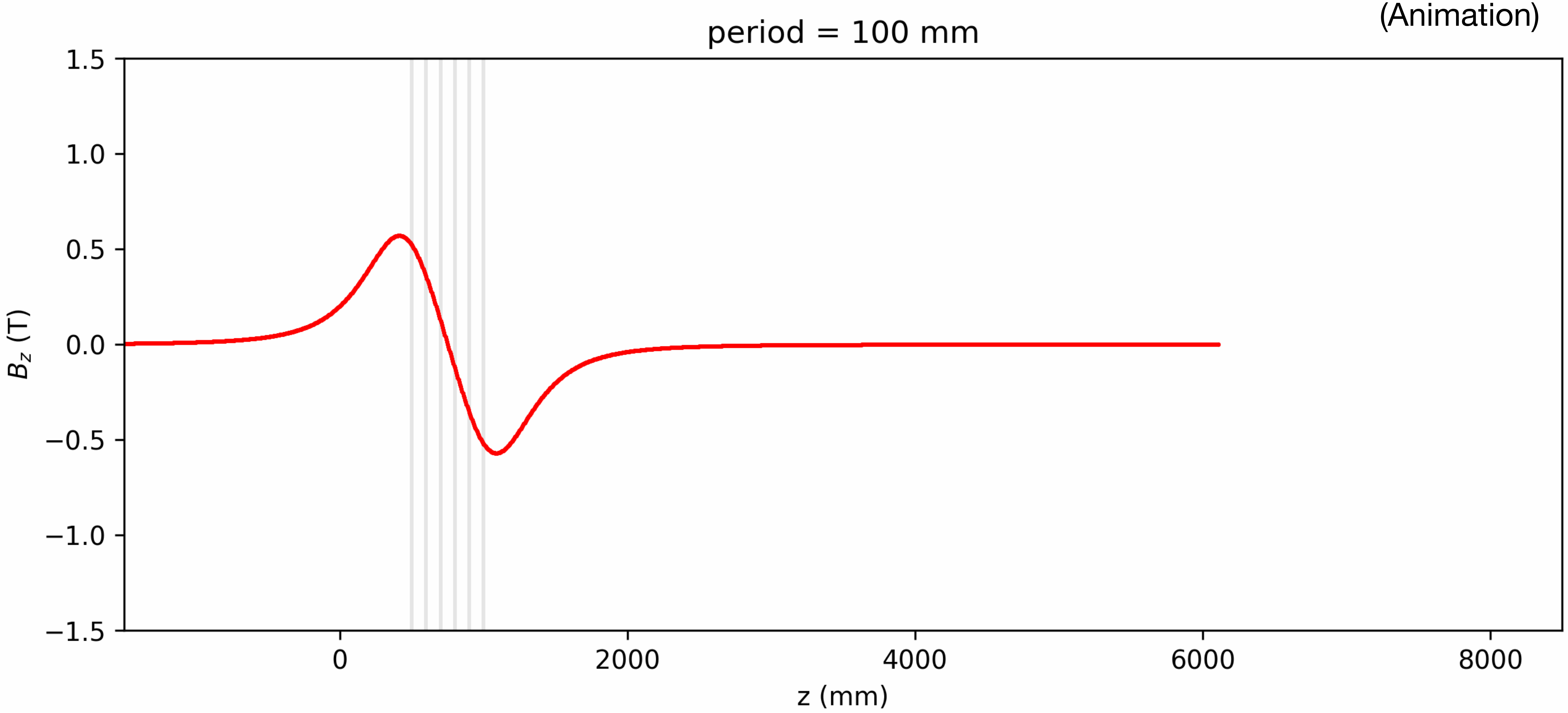
Placing additional solenoids

(Animation)



Placing additional solenoids

(Animation)



Placing additional solenoids

Qualitatively, spacings **around 400mm** yield best results, approaching consistency with both:

- Sinusoidal B field

- Symmetric L_z behavior

Though clearly neither of these properties are wholly fulfilled

Which should be prioritized?