HFOFO PROJECT UPDATES

Week of July 7-11, 2025

https://github.com/criggall/muon-cooling/tree/main

COMPARING EMITTANCE CALCULATIONS

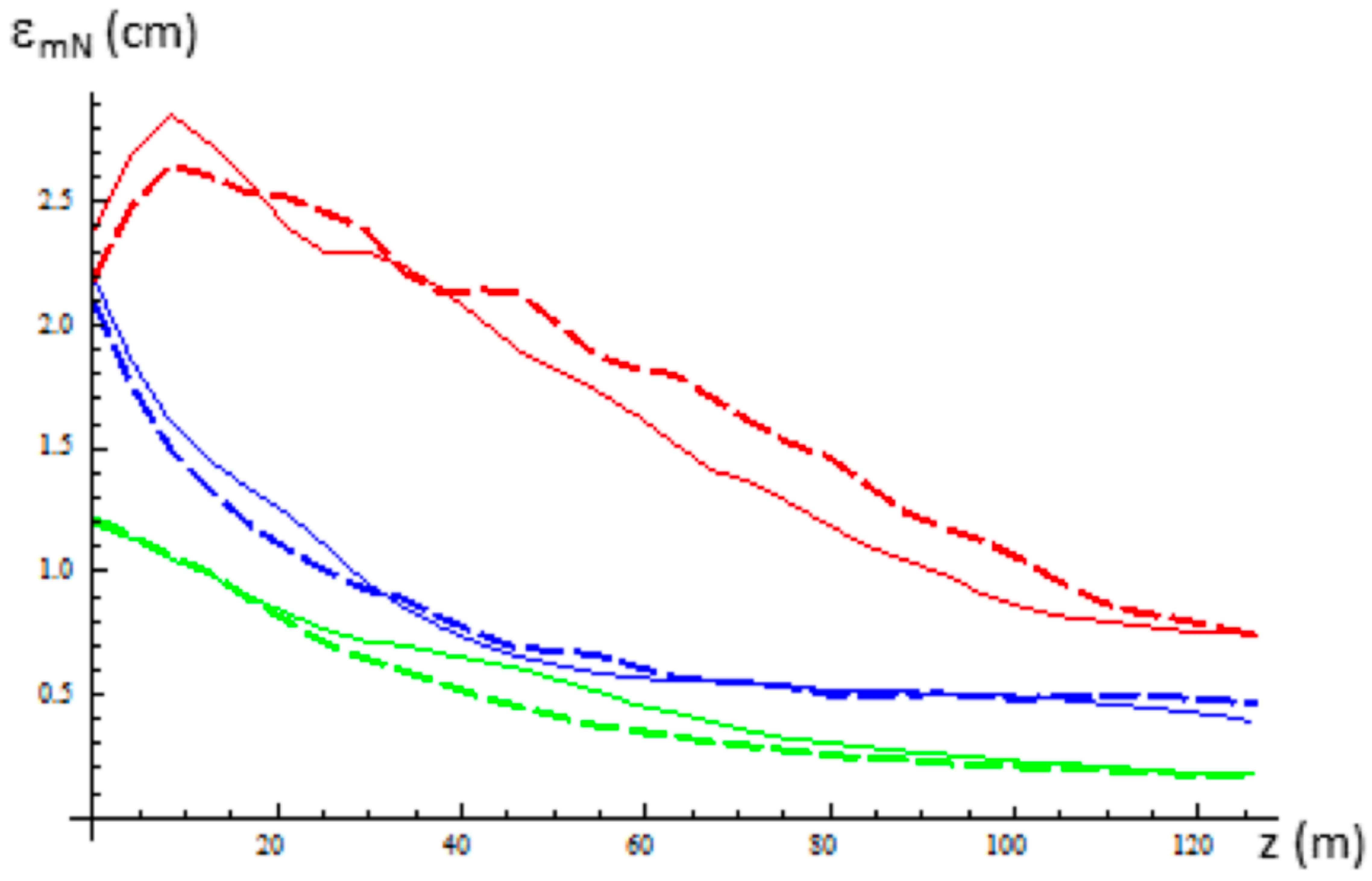
REFERENCE

- All tests included in these slides were performed with **Yuri's g4bl input file** (i.e., the original HFOFO design, with cooling) and the associated input beam file

- Thus we expect to see agreement with this plot from his paper →

- Note that the sampling rate is set by detector placement, where last week's ecalc9/ICOOL results came from reference particle tracking

- Number of events in input file is O(10,000)

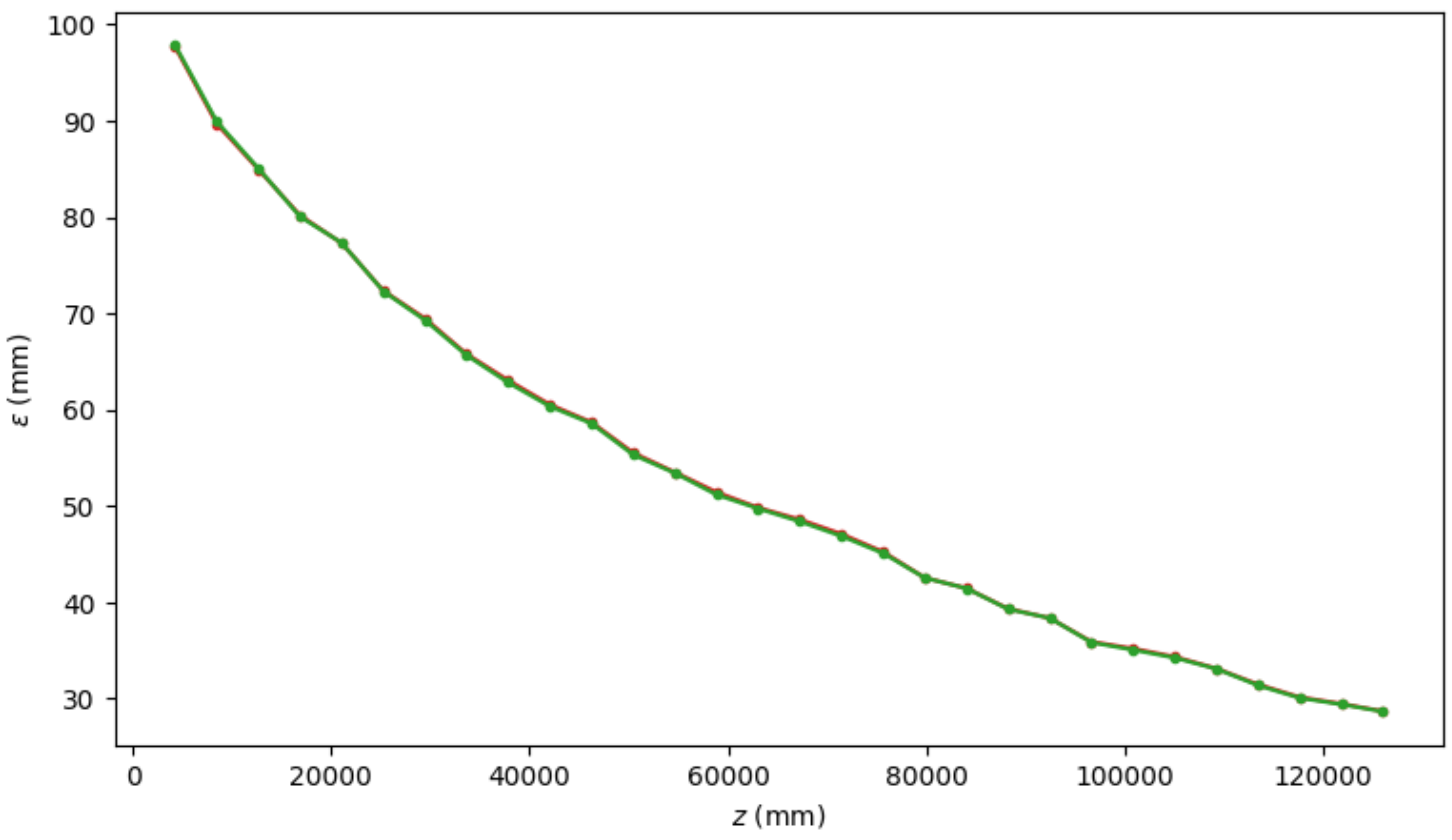


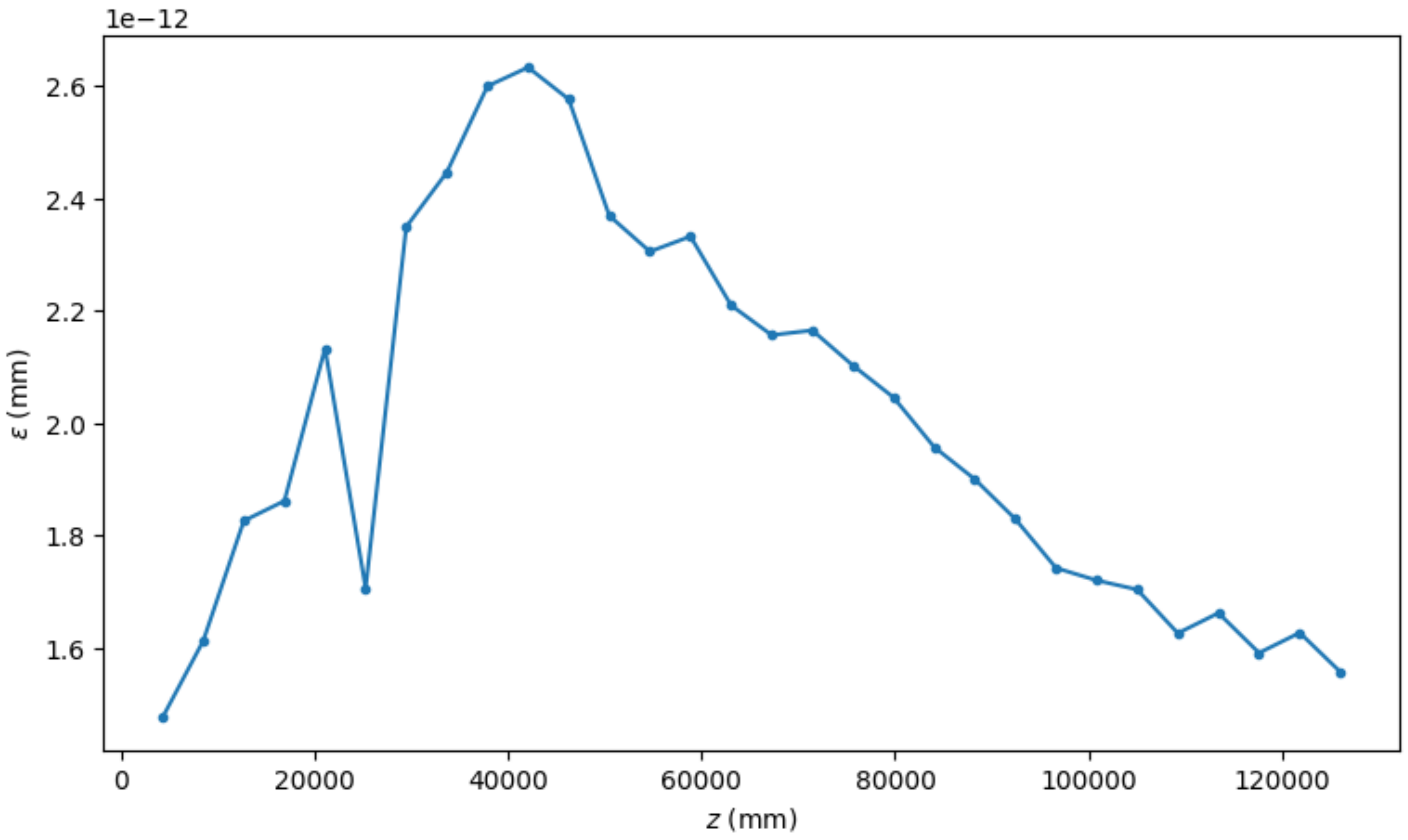
EMITTANCES FROM MATHEMATICA

with RMS covariance matrix

EMITTANCES (MATHEMATICA)

Three emittance components, as calculated from the RMS covariance matrix:





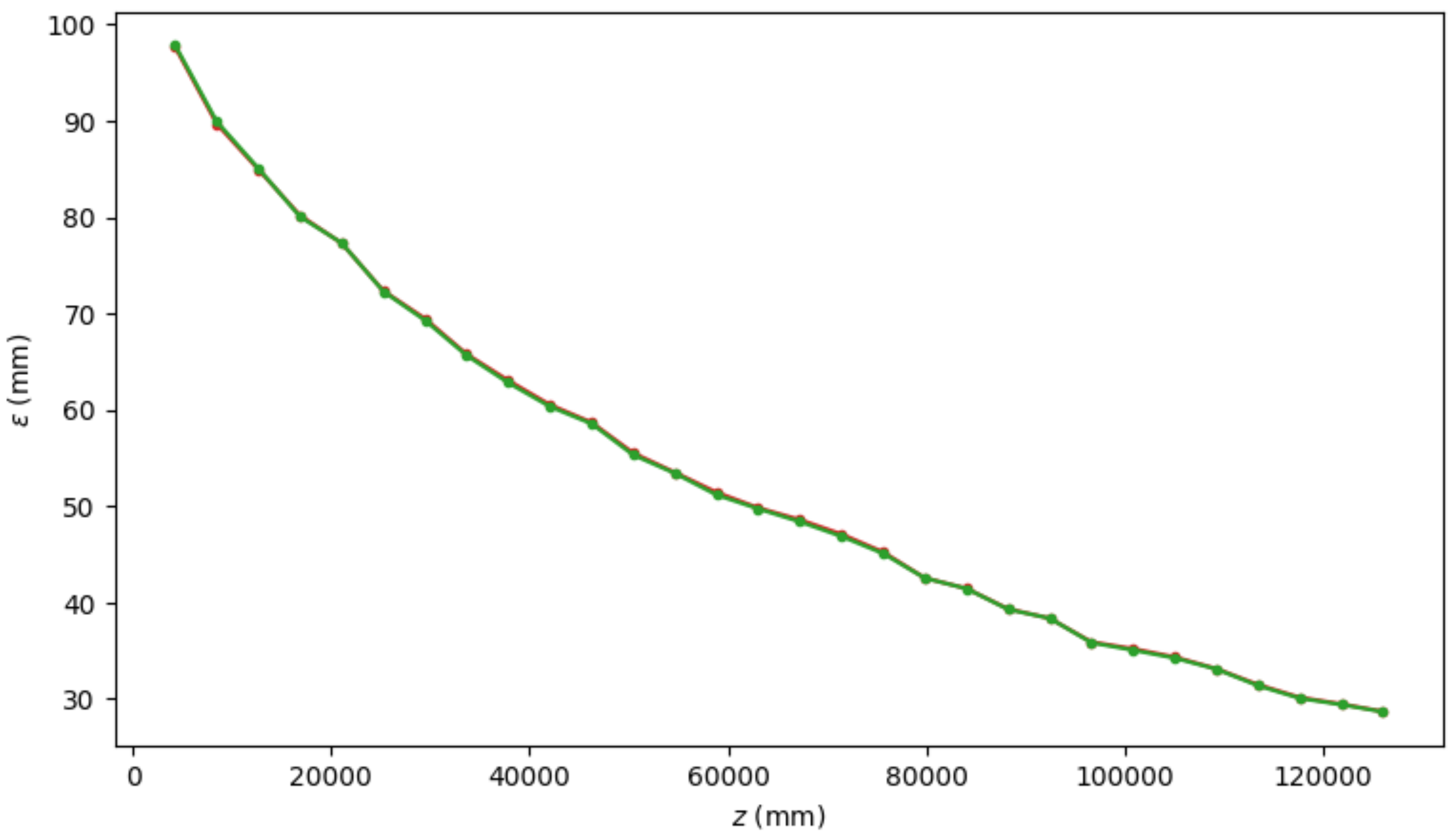
Downward trend = cooling ✓

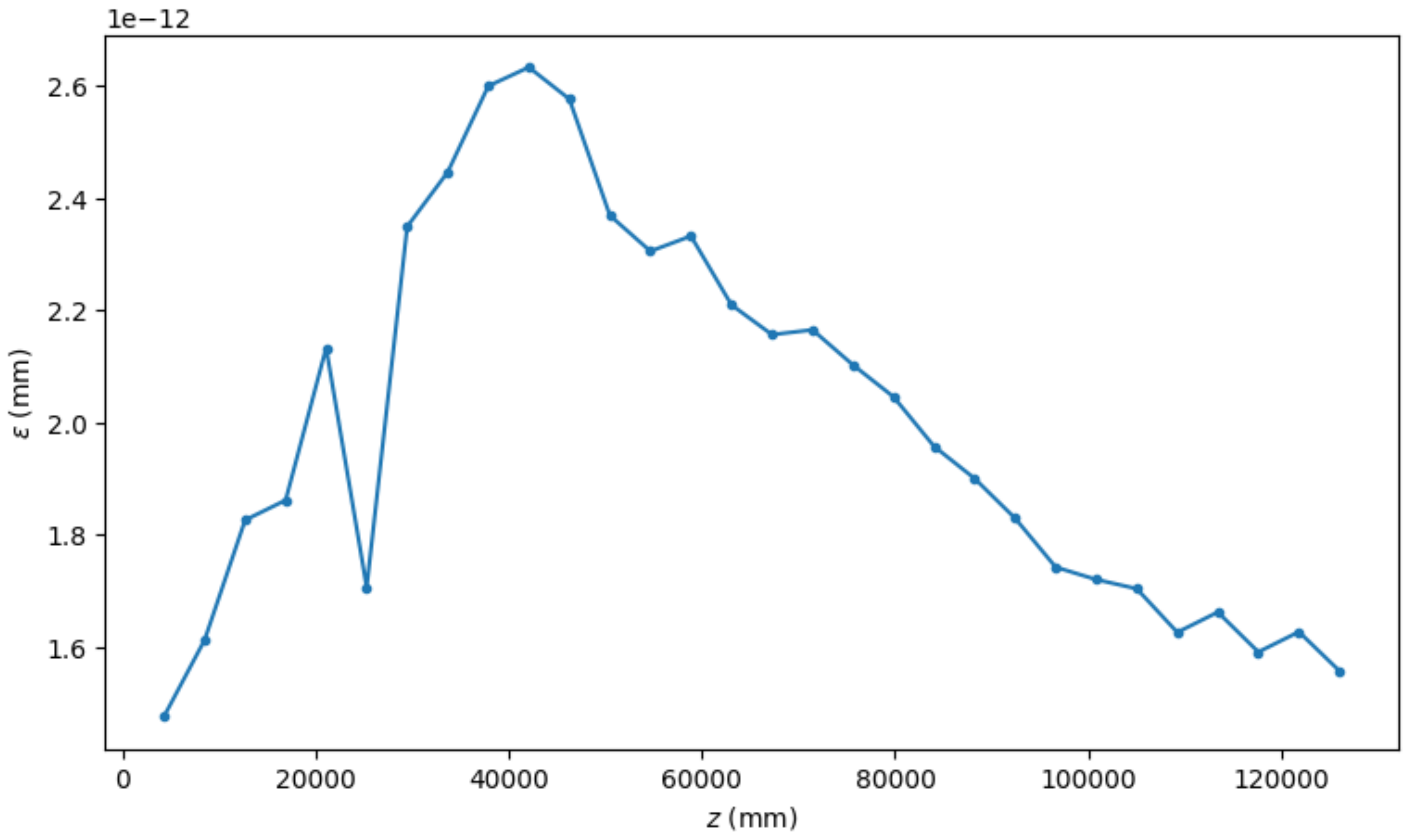
Observe two maxima, around 20m and 40m —

Why here? No unique lattice features at these locations

COMPARING DIRECTLY TO YURI'S NOTEBOOK

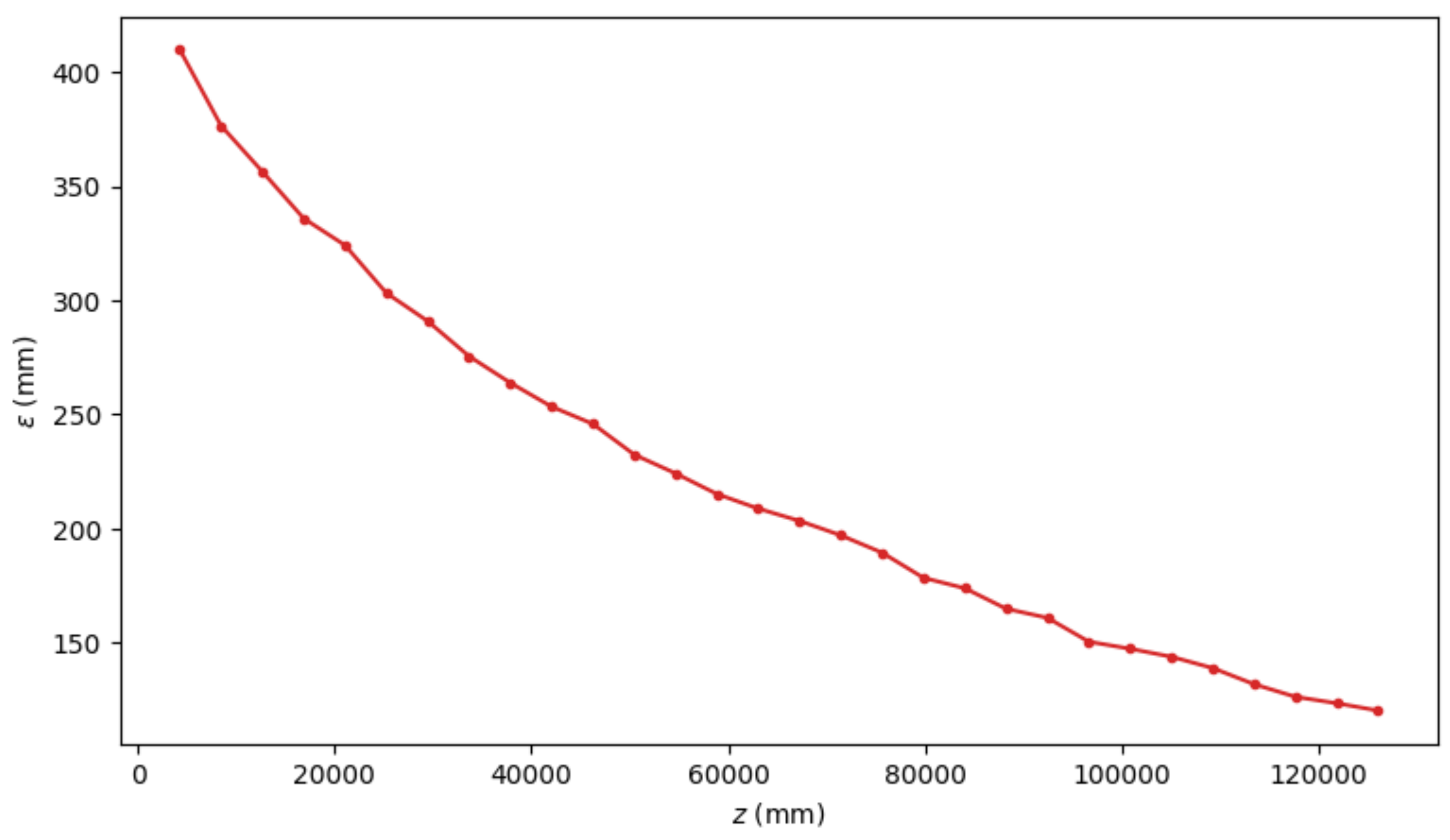
Values from Mathematica for first detector: {0.84044, 0.854529, 2.48799} cm

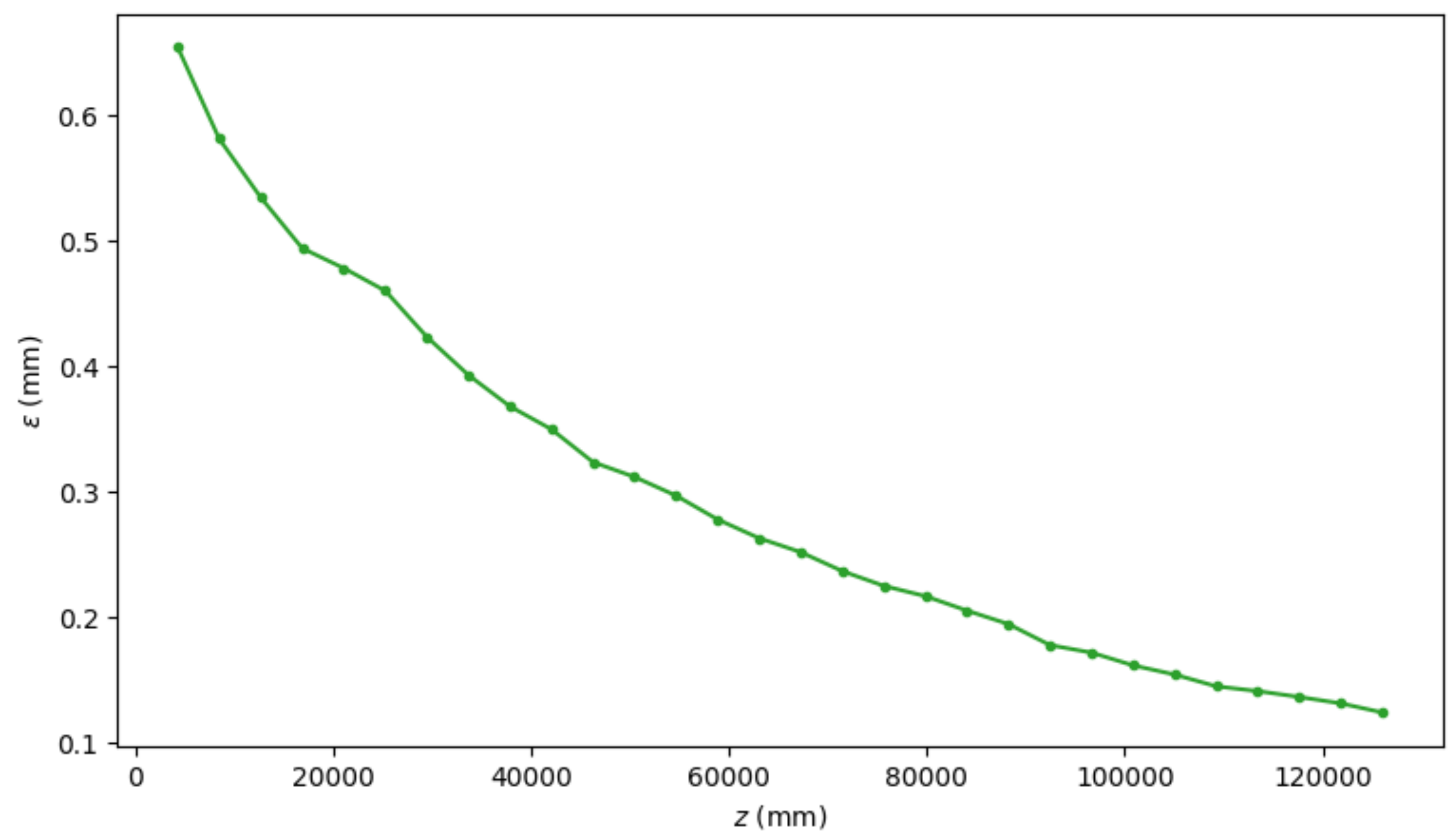


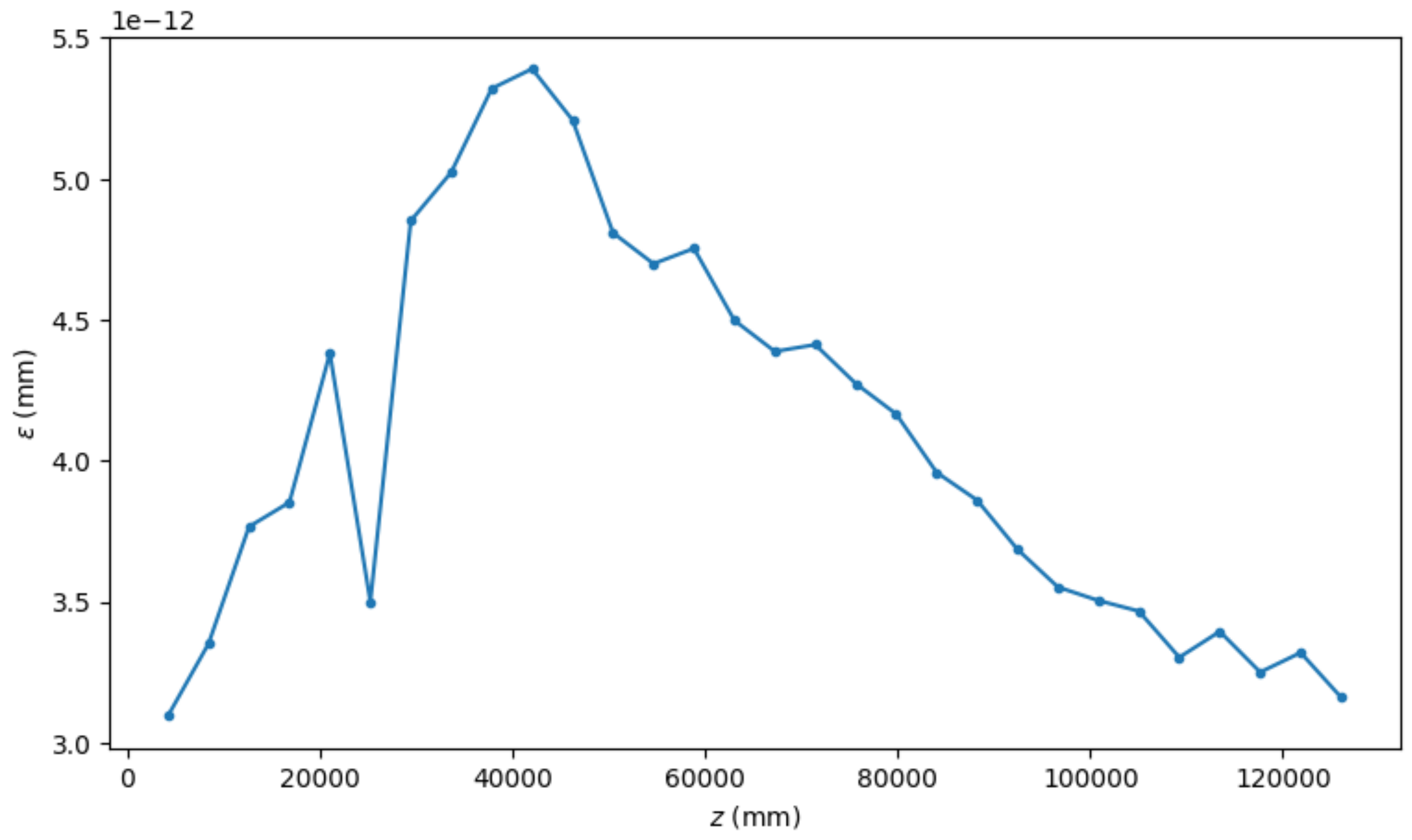


NORMALIZED EMITTANCES (MATHEMATICA)

*Three **normalized** emittance components, as calculated from the RMS covariance matrix:*



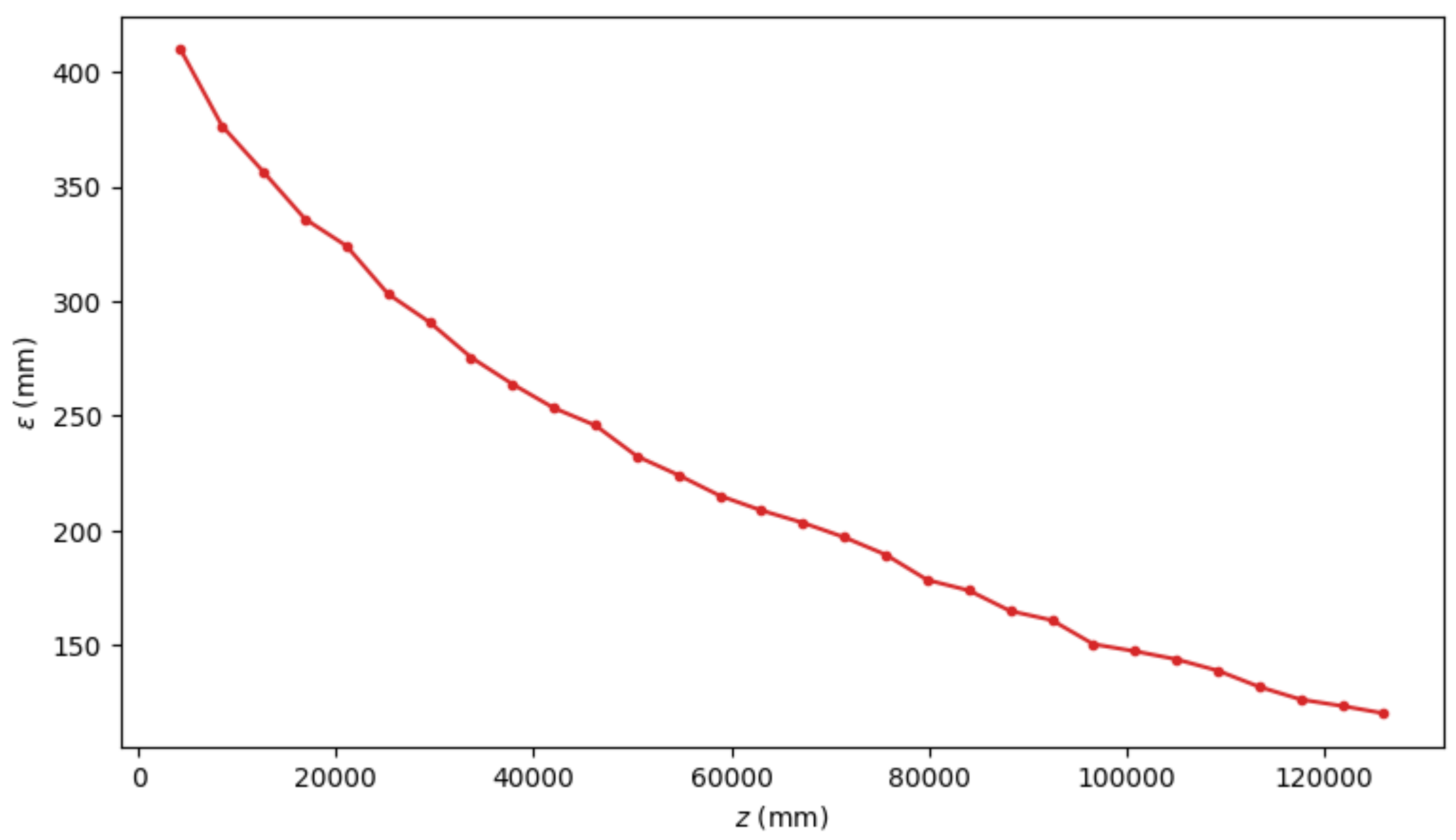


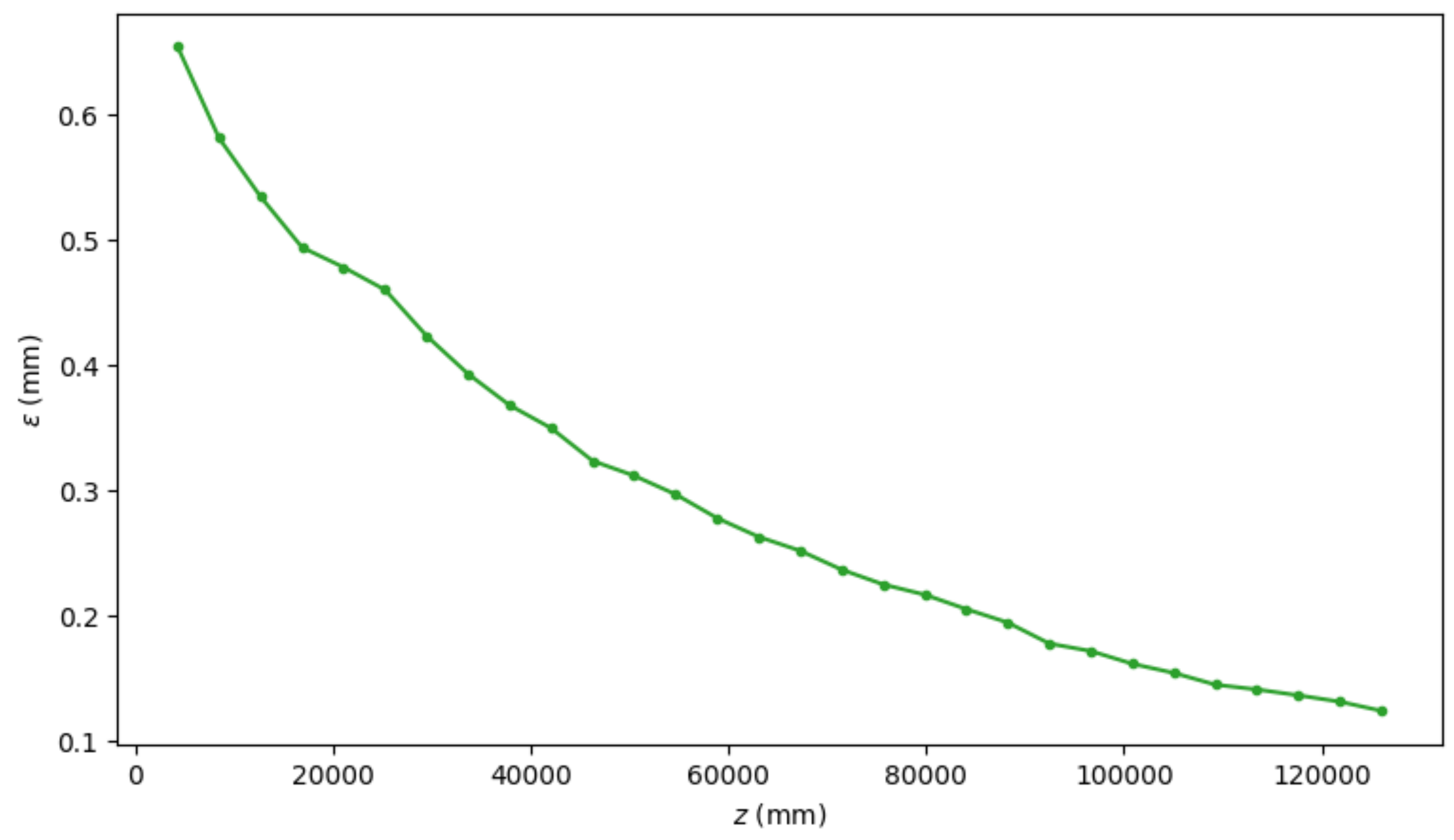


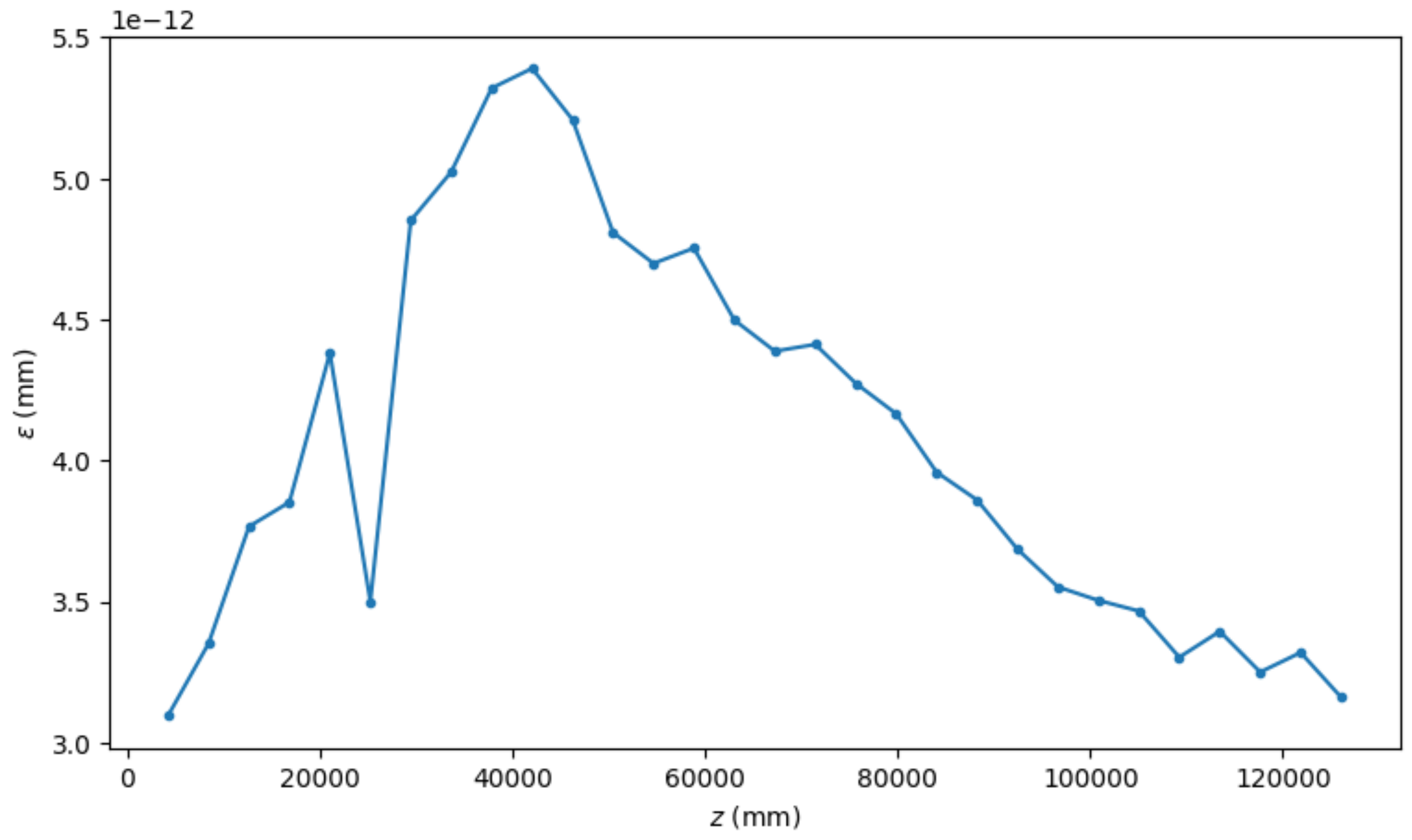
Assuming these are the transverse components, why is one so much smaller than the other upon normalization?

COMPARING DIRECTLY TO YURI'S NOTEBOOK

Values from Mathematica for first detector: {5.89331, 3.15642, 0.846919} cm





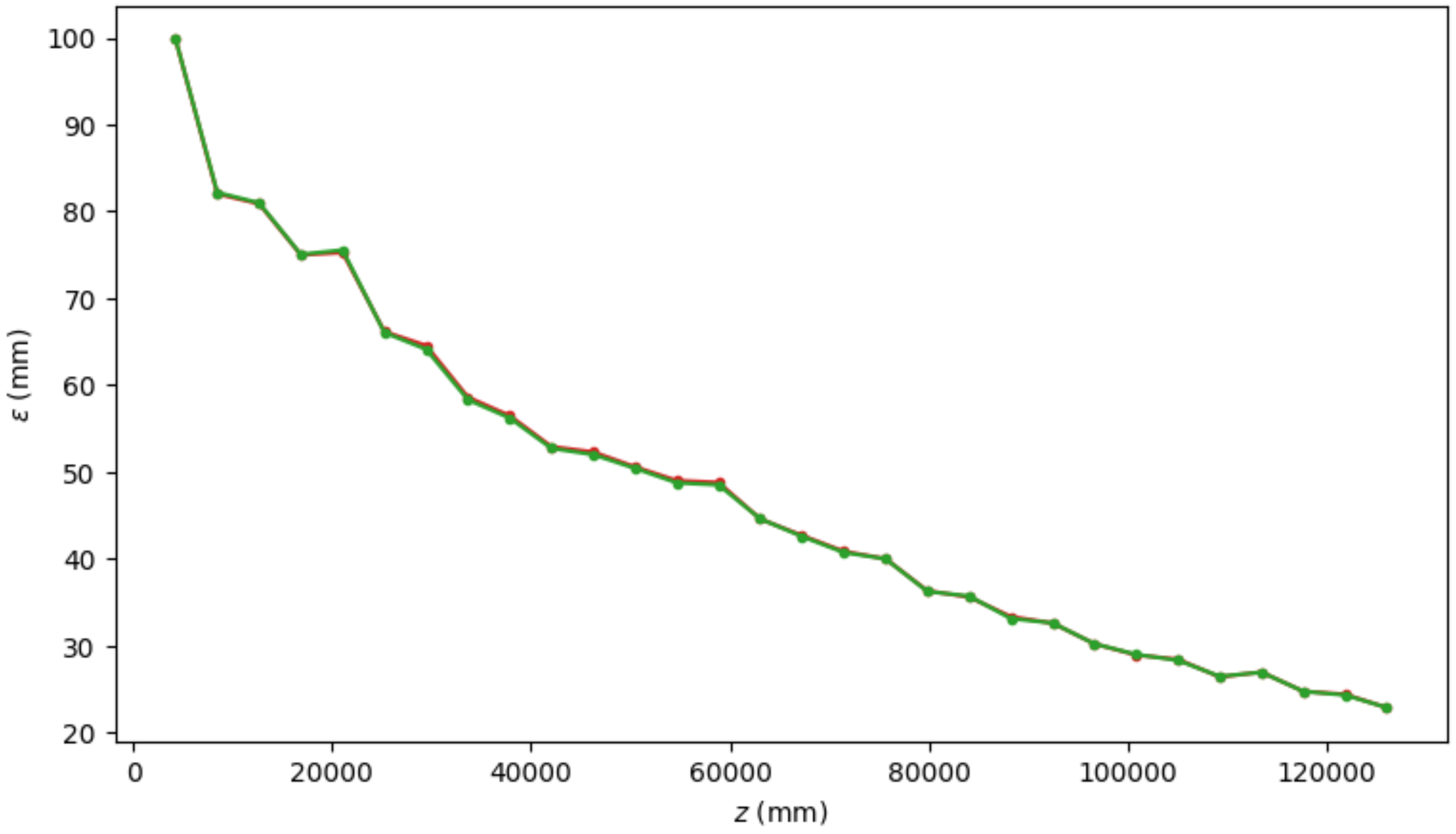


EMITTANCES FROM MATHEMATICA

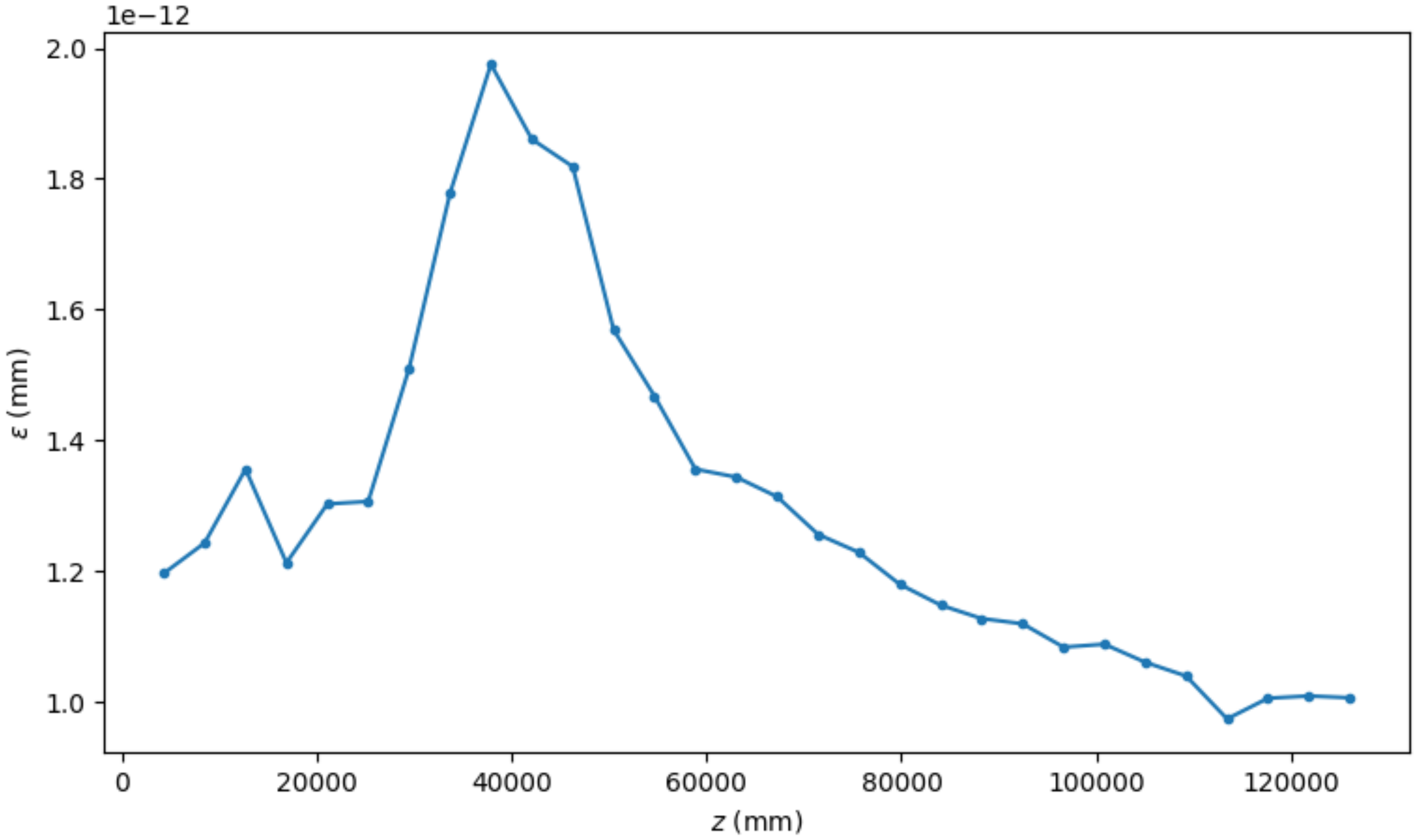
with Gaussian fit covariance matrix

EMITTANCES (MATHEMATICA)

Three emittance components, as calculated from the Gaussian fit covariance matrix:



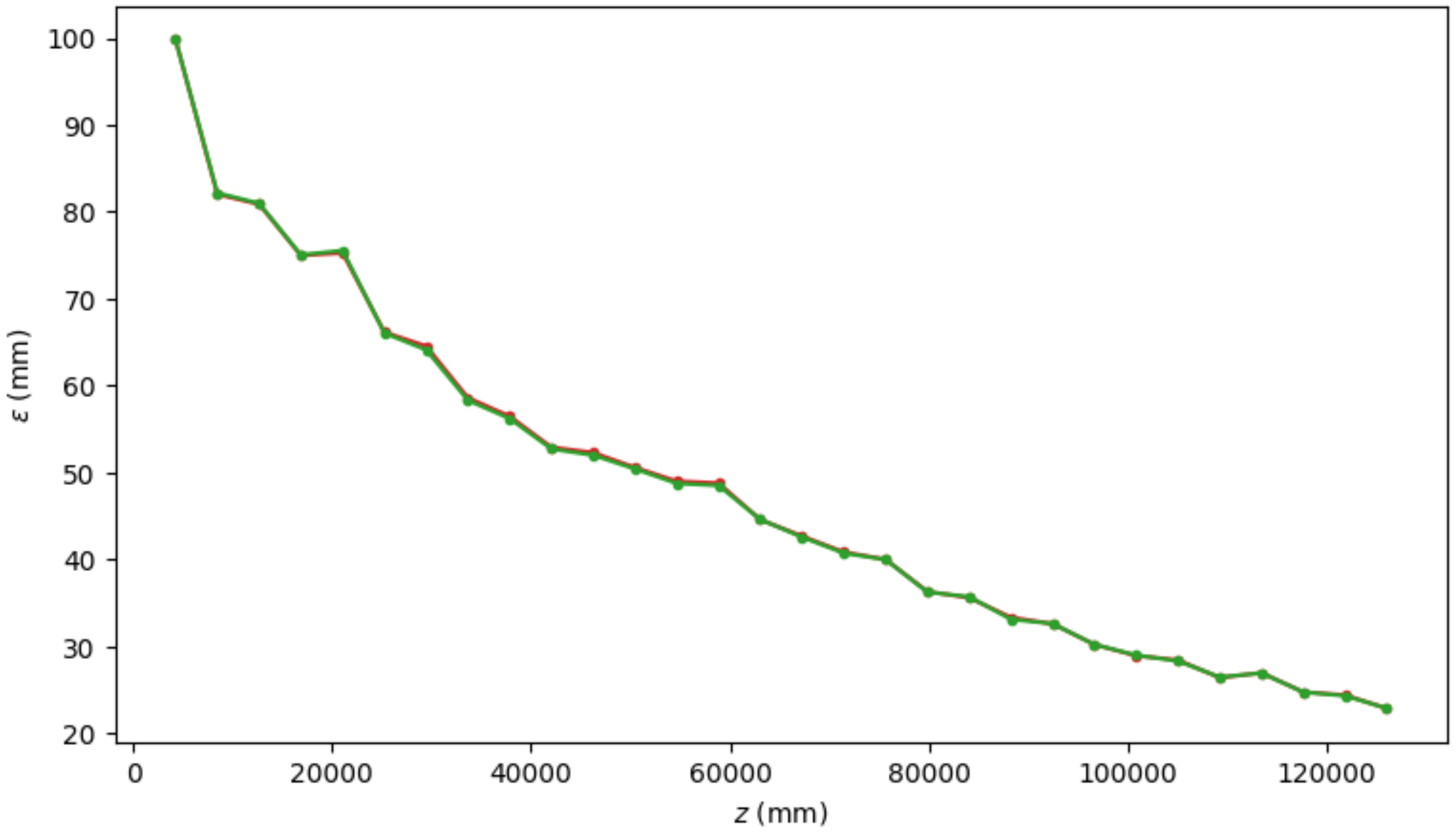
Downward trend = cooling ✓

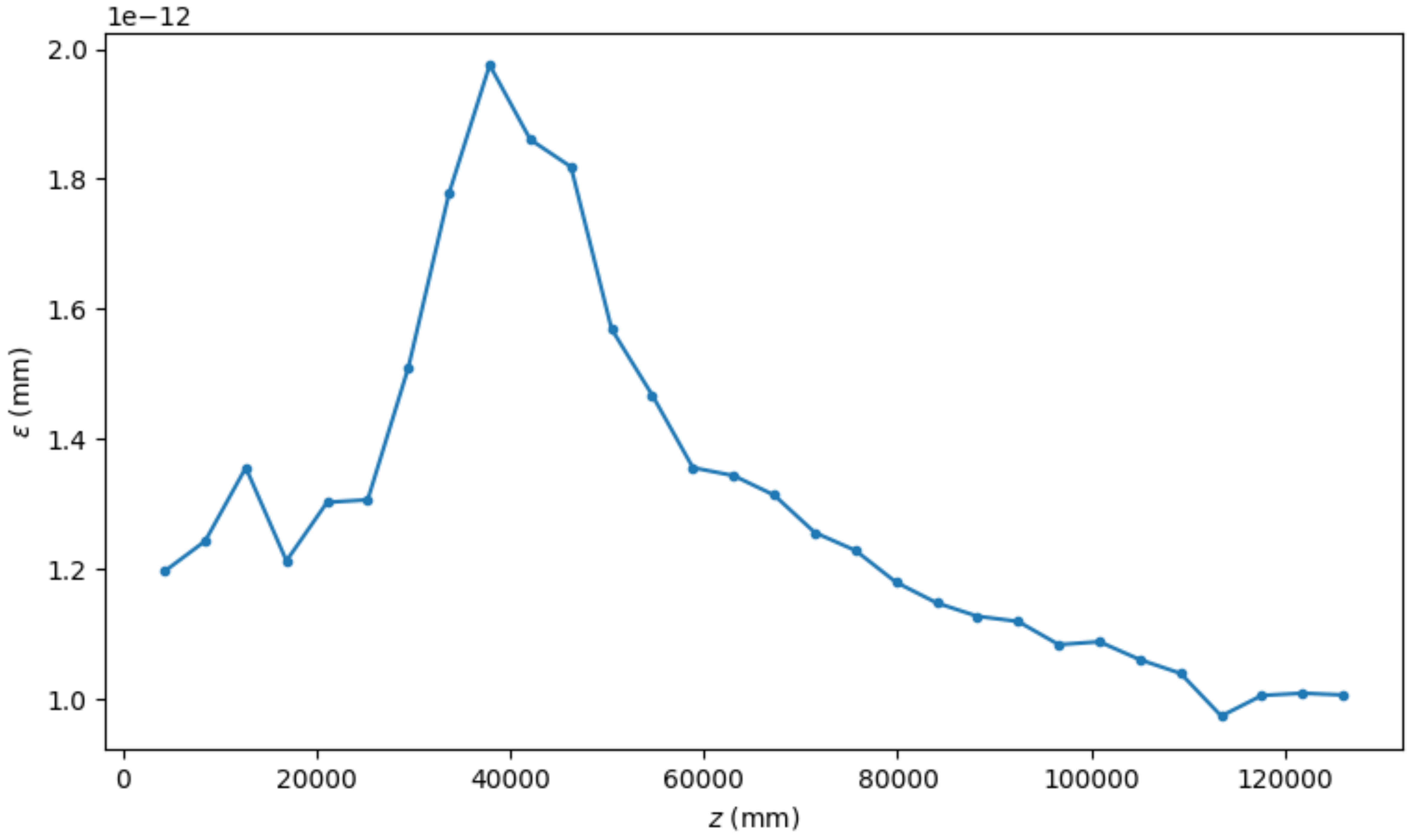


Again see maximum around 40m, but here the ~20m peak is much smaller

COMPARING DIRECTLY TO YURI'S NOTEBOOK

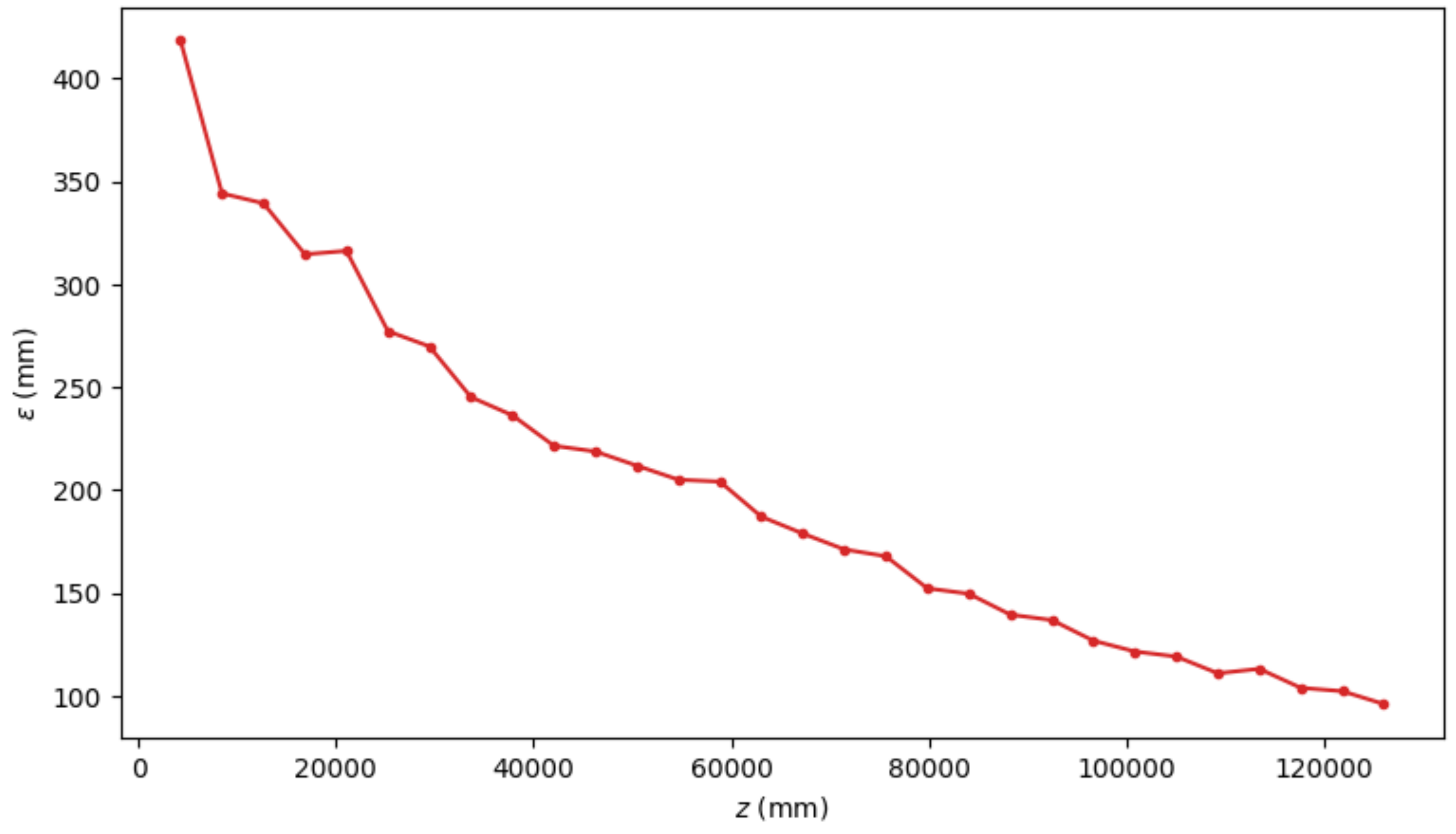
Values from Mathematica for first detector: {0.78215, 0.788655, 1.17555} cm

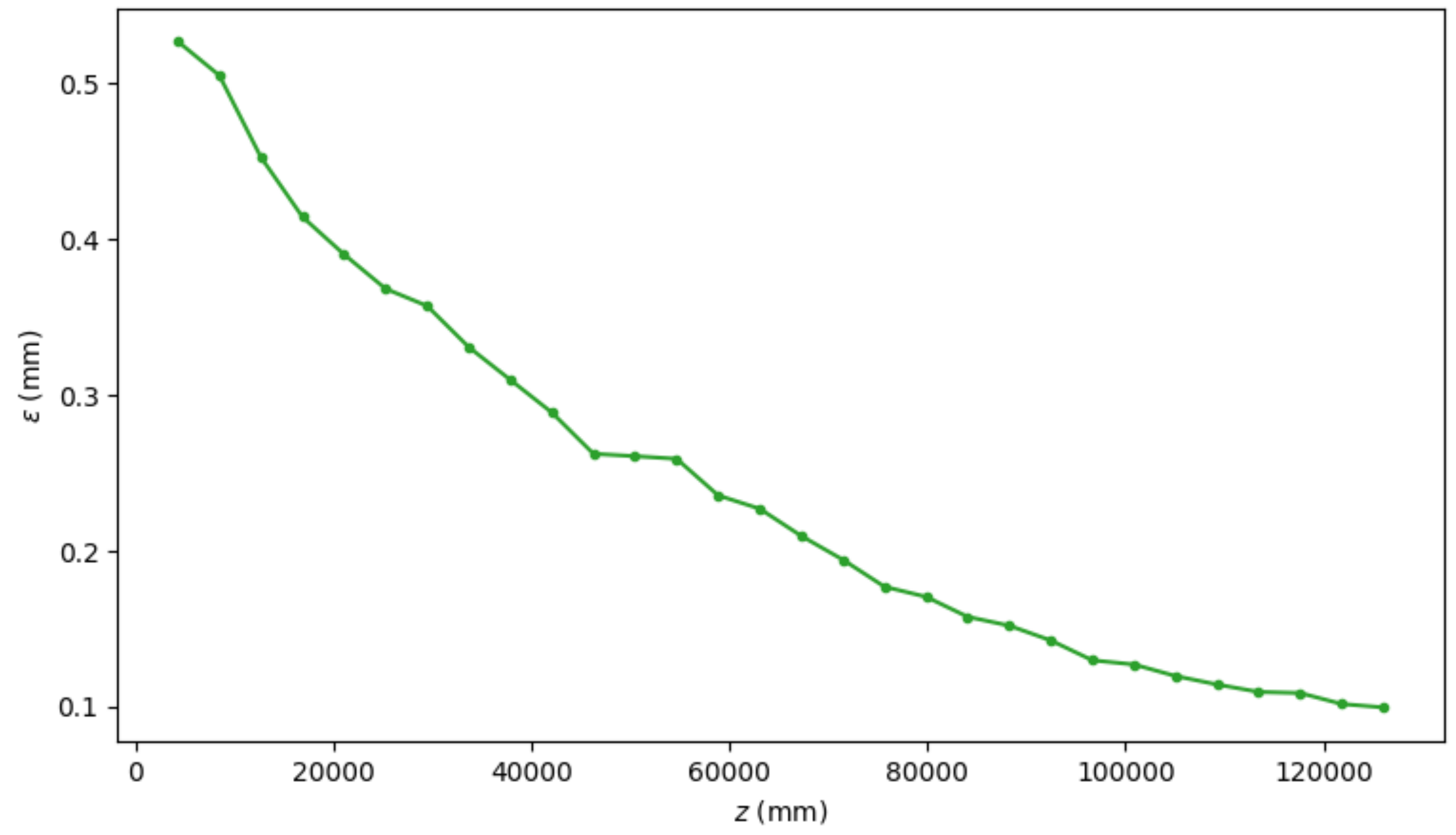


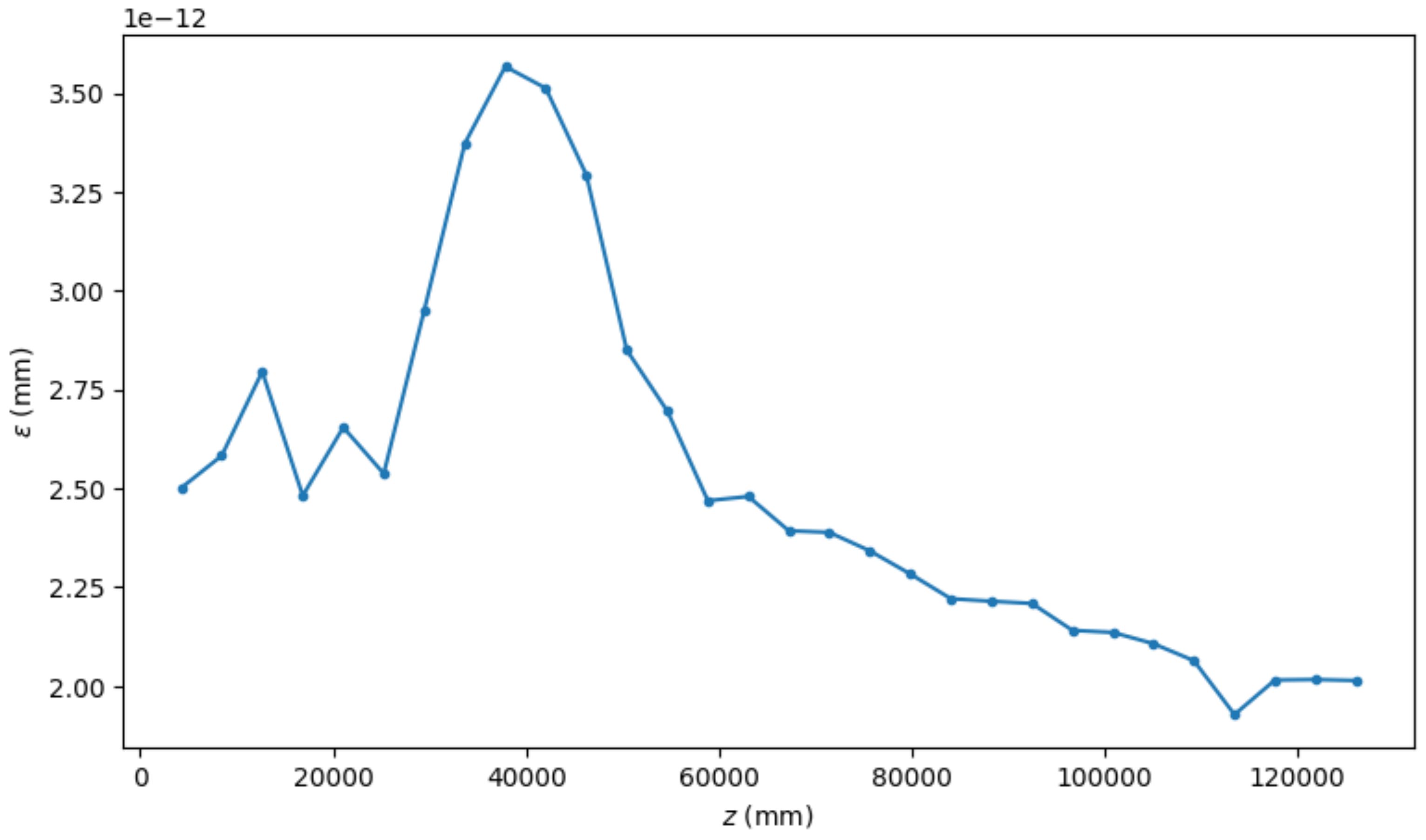


NORMALIZED EMITTANCES (MATHEMATICA)

*Three **normalized** emittance components, as calculated from the Gaussian fit covariance matrix:*



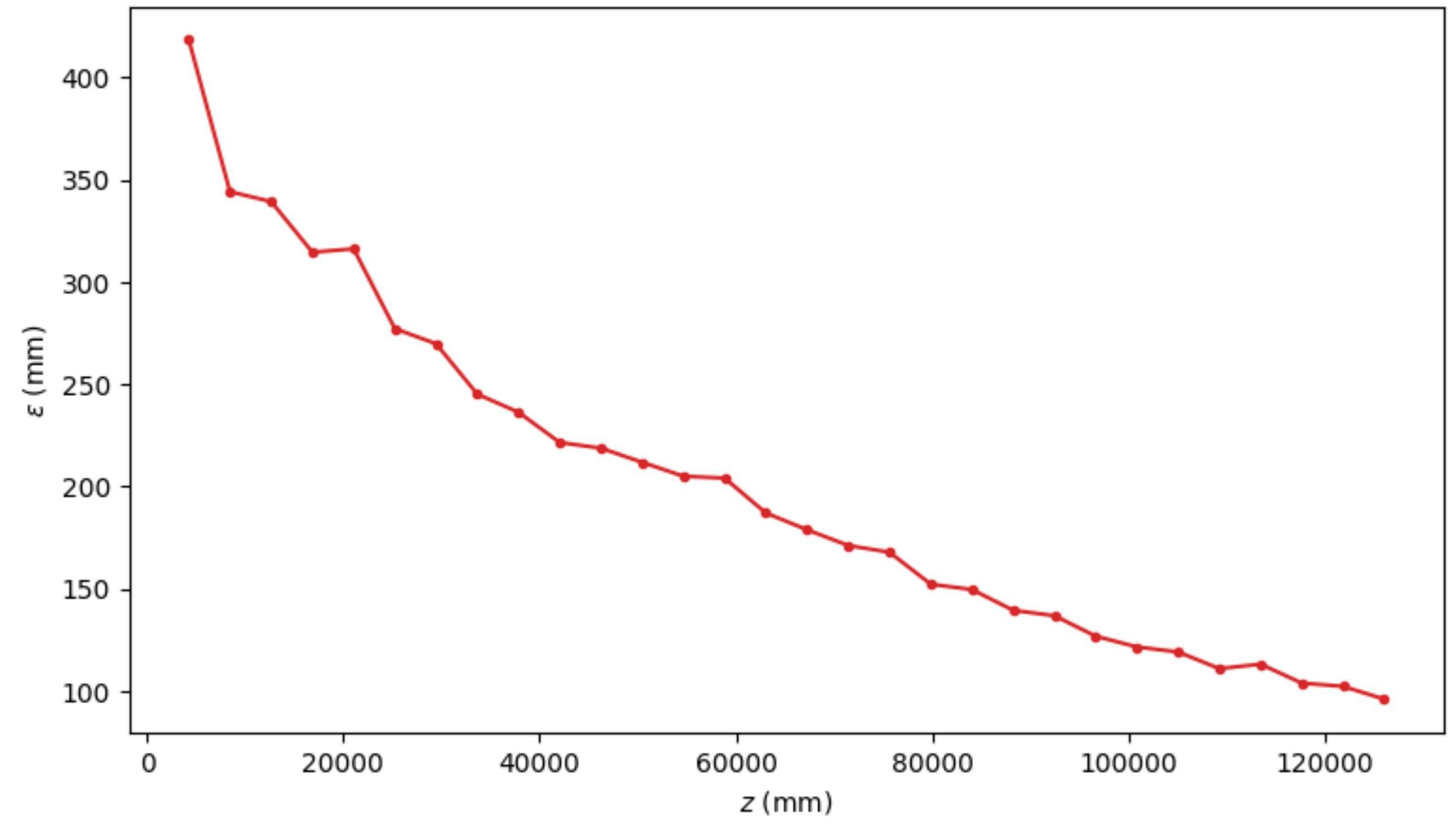


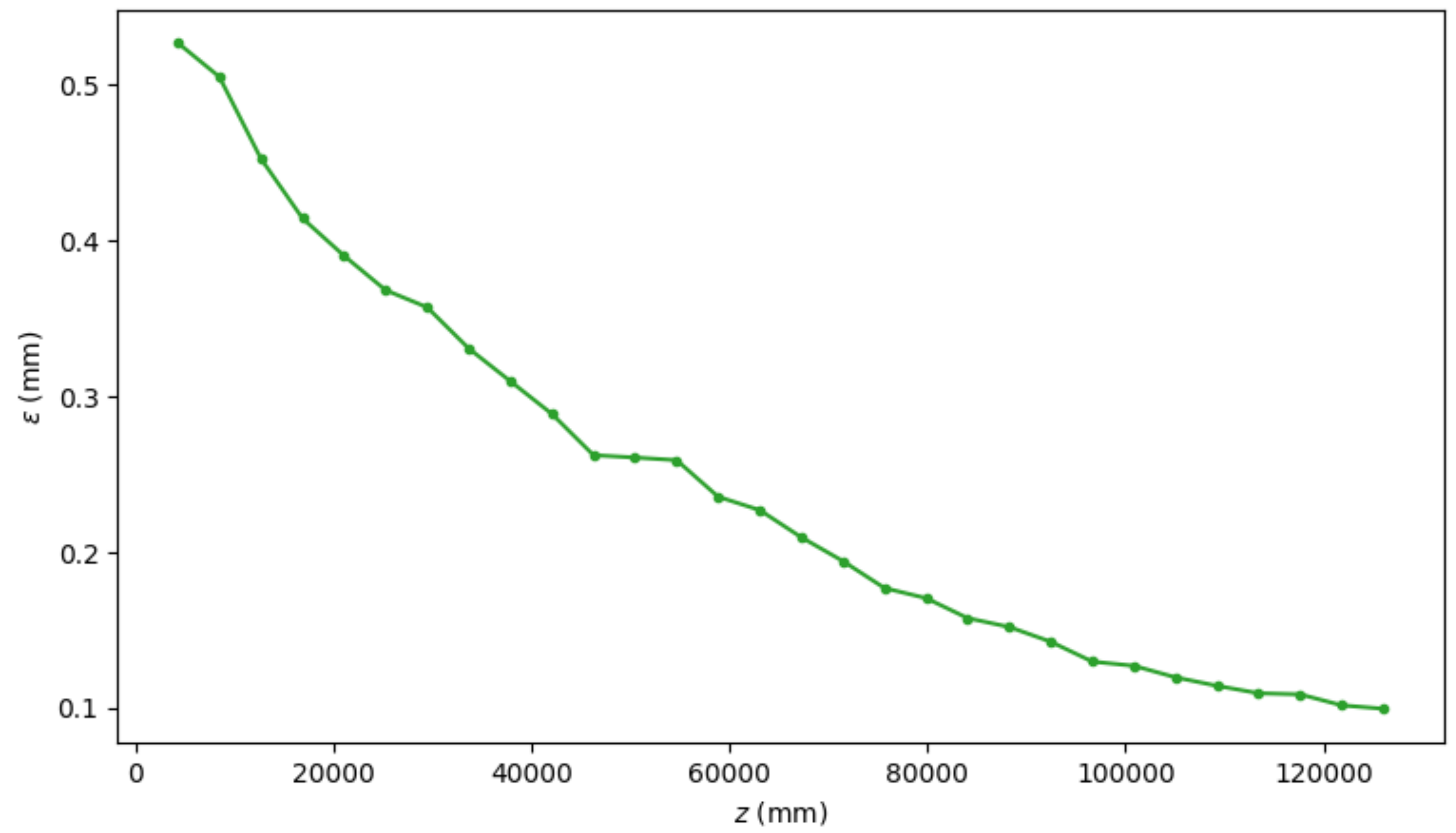


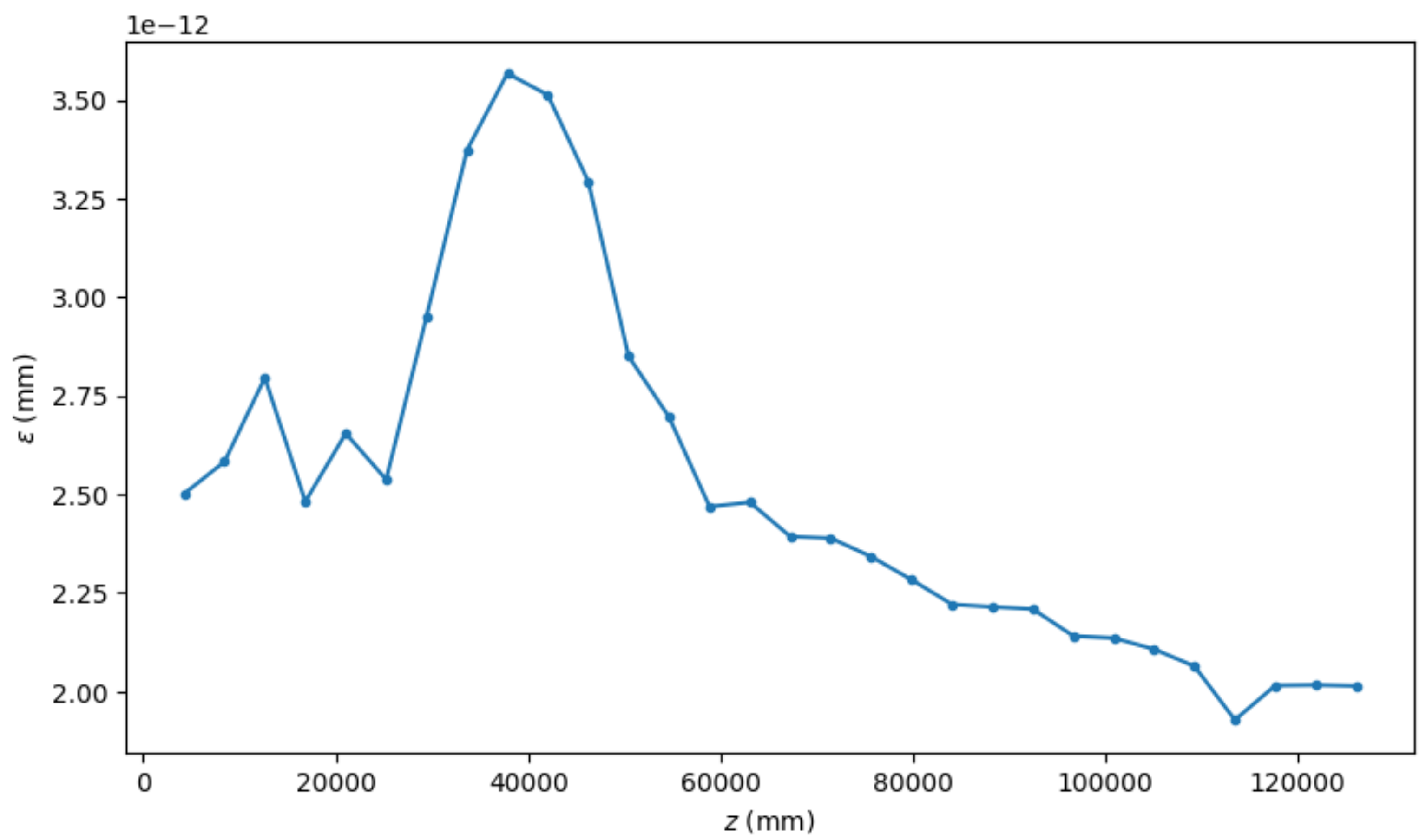
Again see one component being much smaller than the other

COMPARING DIRECTLY TO YURI'S NOTEBOOK

Values from Mathematica for first detector: {3.05319, 2.70211, 0.735605} cm

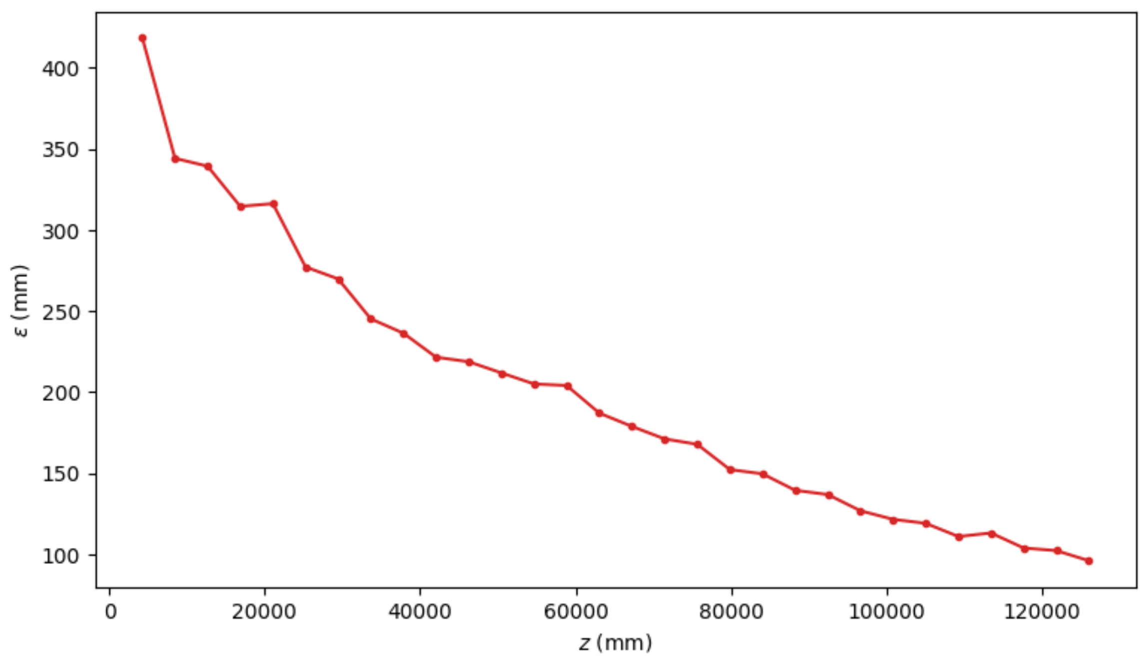


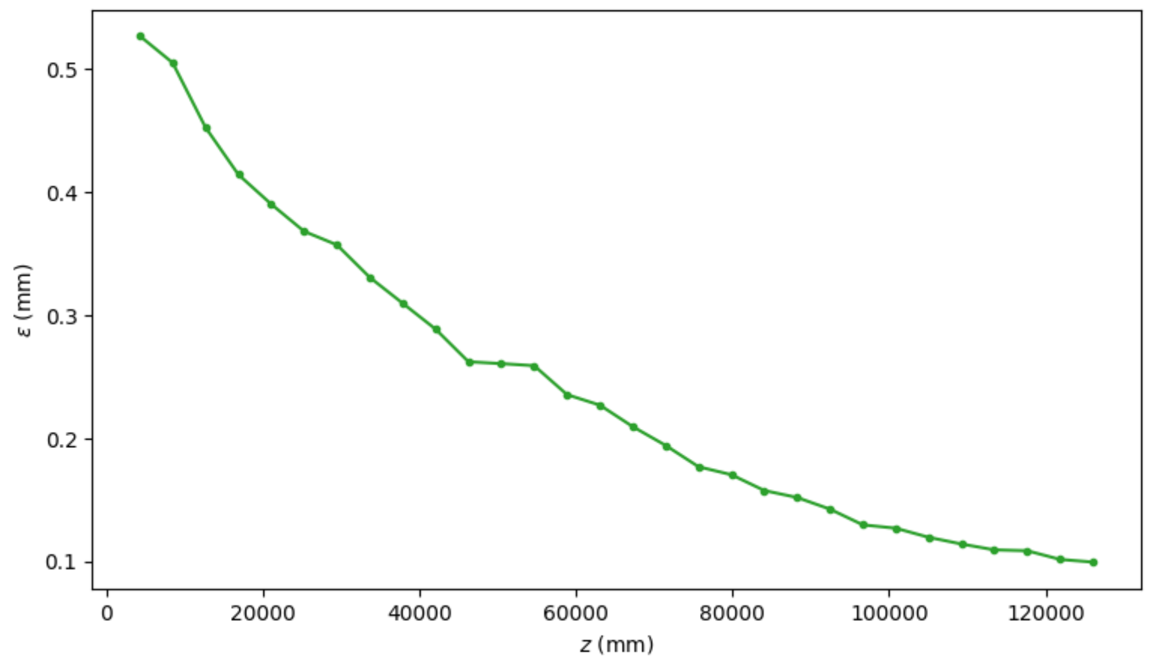


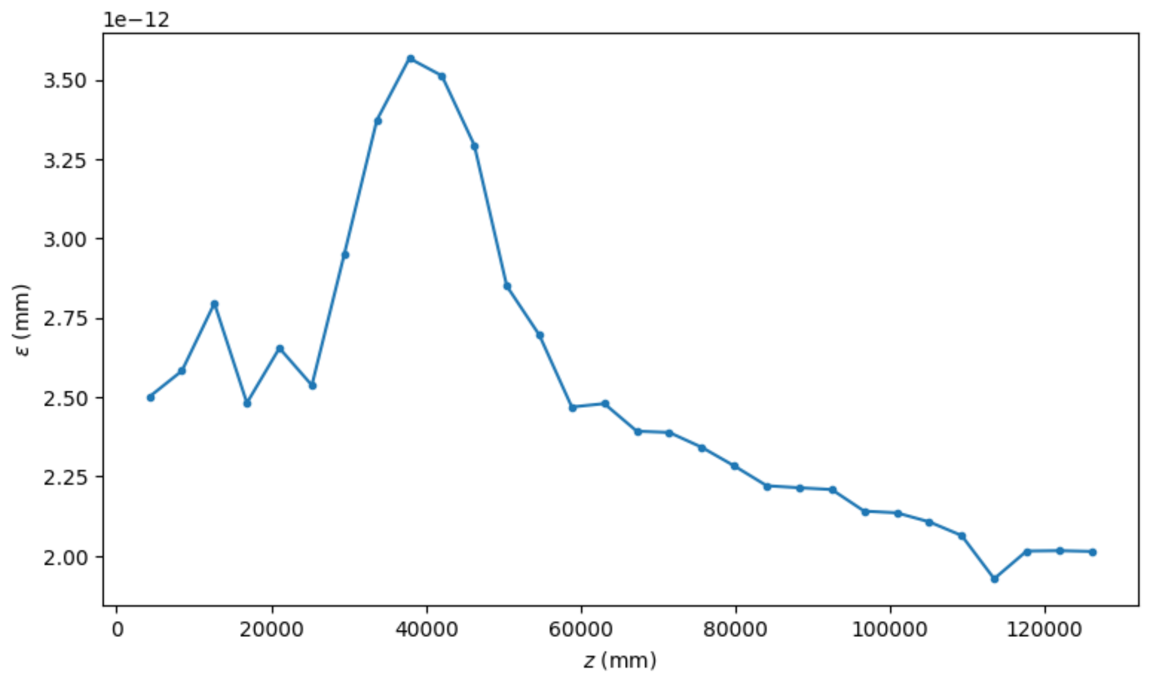


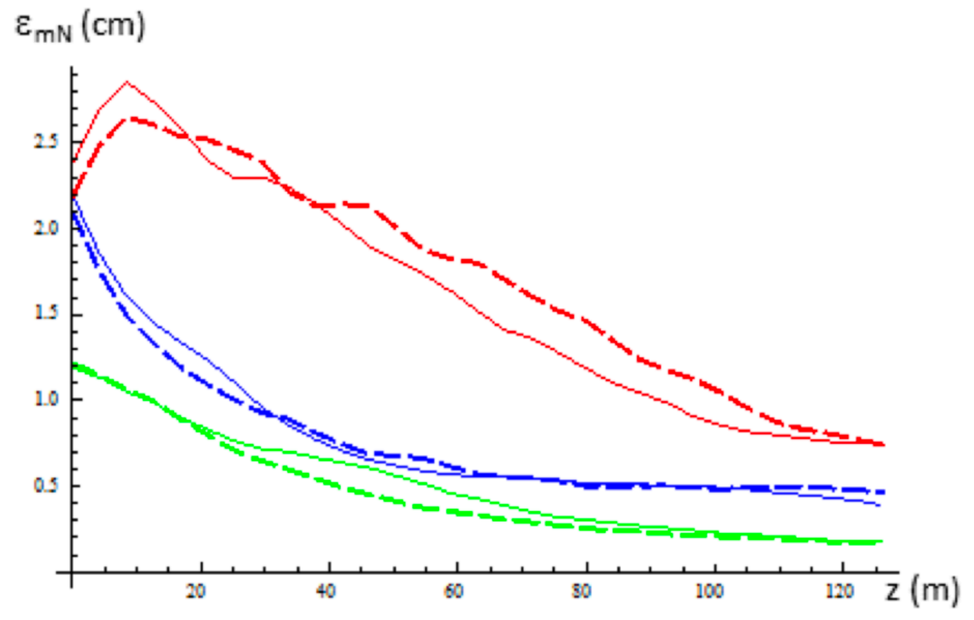
CONCLUSIONS

- Observe rather similar qualitative behavior across RMS vs. Gaussian fit approaches
- Some macroscopic resemblance to trends seen in Yuri's paper —
  - Two components decreasing with z and another peaking early in the channel
  - …But maximum of (presumably) longitudinal mode in Yuri's plot occurs around z = 10m, **not 40m, and is much less significant**
- None yielded three components of similar order of magnitude w.r.t. each other, and the values are **not the same order of magnitude** as those Yuri reports









NOTES ON OBSERVED DISAGREEMENT

- Observed significant disagreement with sampled points run through the Mathematica notebook directly $\implies$ **error(s) in python script**?
- Need to dissect python script further to ascertain the cause
- Could different fitting procedures be the cause?
- Do we even expect this notebook to offer a reasonable emittance calculation? Appears to be **for a 4D cooler so may not use appropriate approach**

NOTES ON OBSERVED DISAGREEMENT

Comparing the Mathematica-direct results at the first detector to Yuri's figure,

RMS Σ matrix:

{0.84044, 0.854529, 2.48799} cm

Normalized RMS Σ matrix:

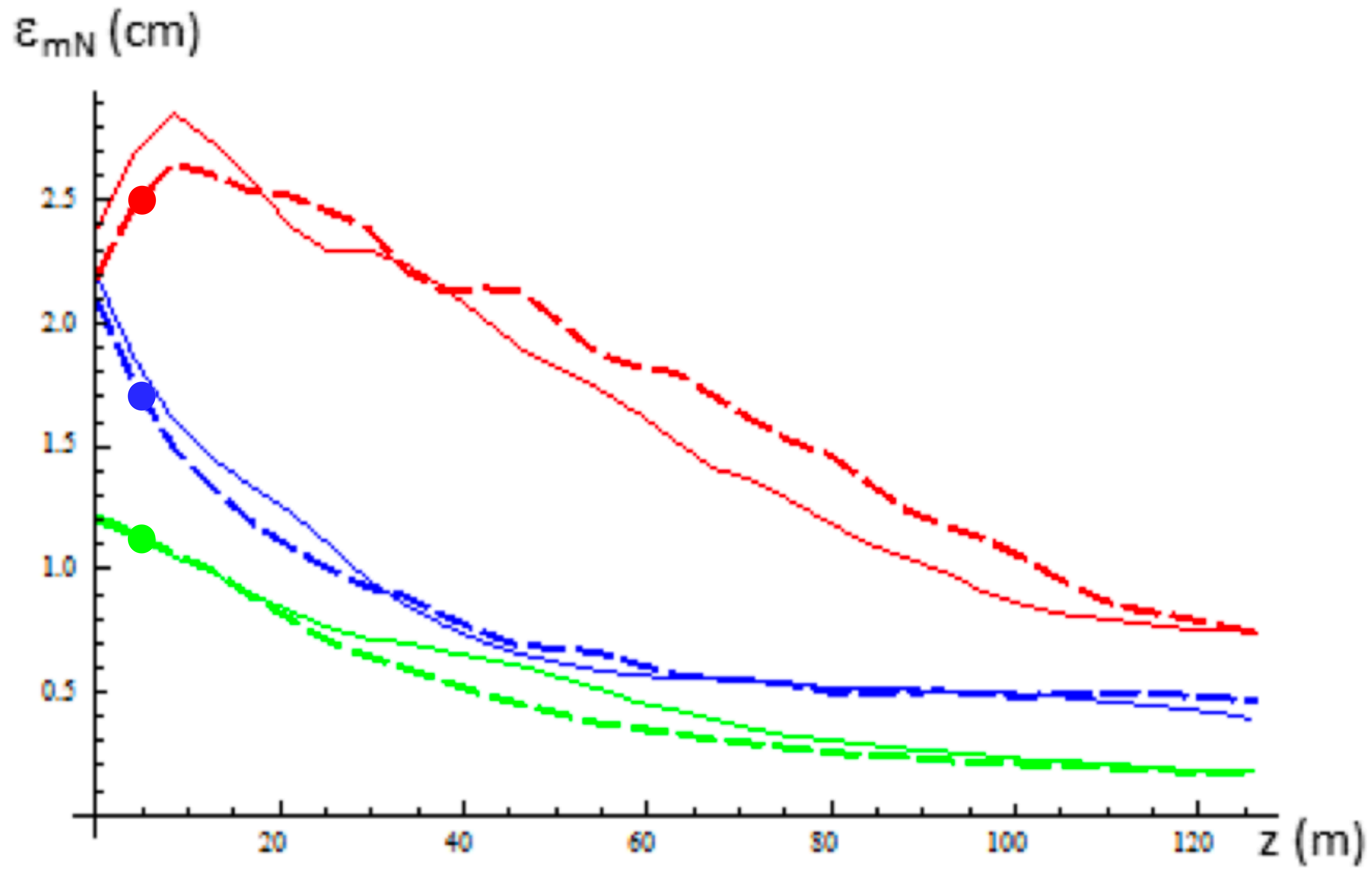
{5.89331, 3.15642, 0.846919} cm

Gaussian fit Σ matrix:

{0.78215, 0.788655, 1.17555} cm

Normalized Gaussian fit Σ matrix:

{3.05319, 2.70211, 0.735605} cm

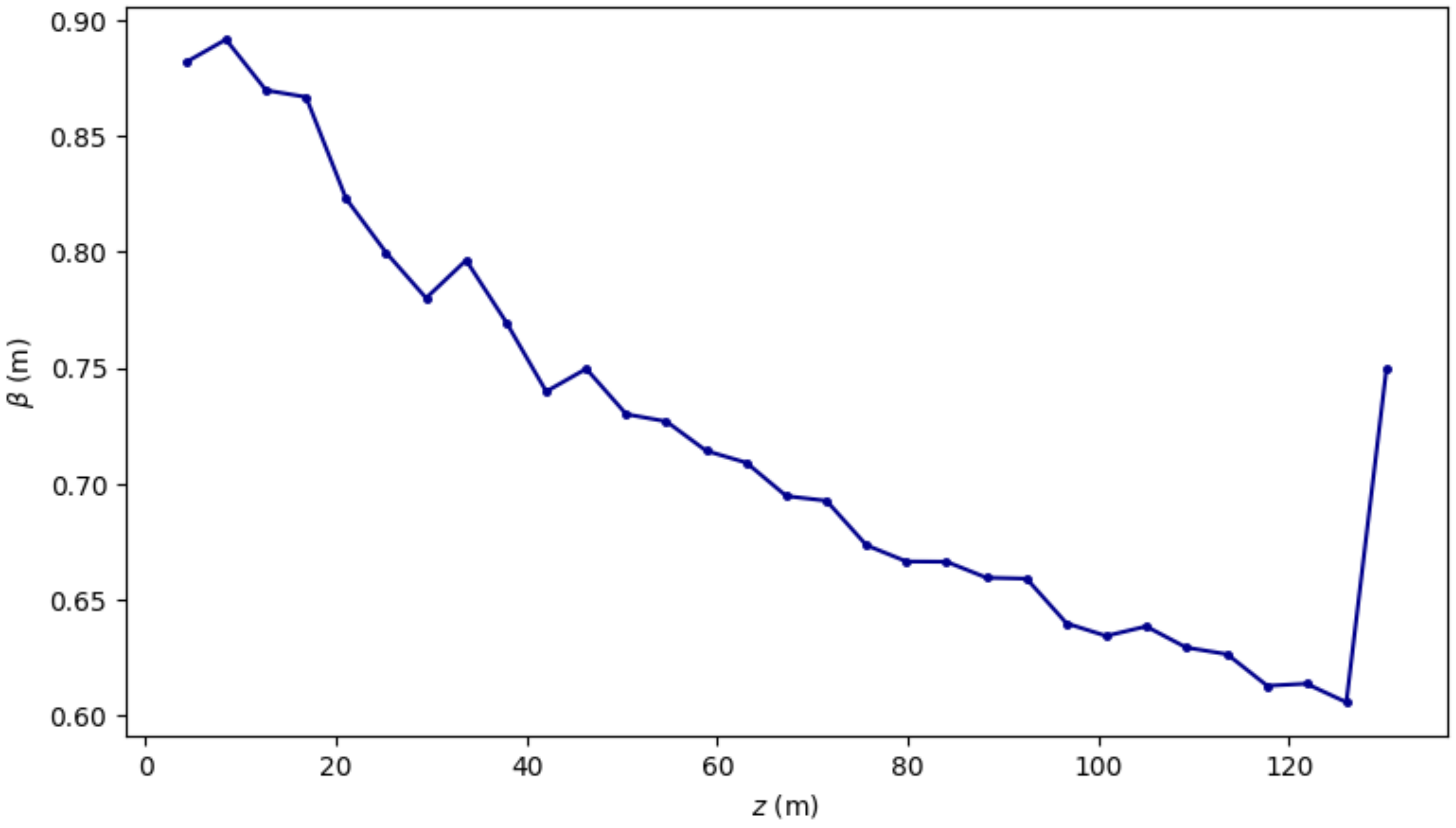


∴ This is **not** the approach Yuri used for HFOFO.

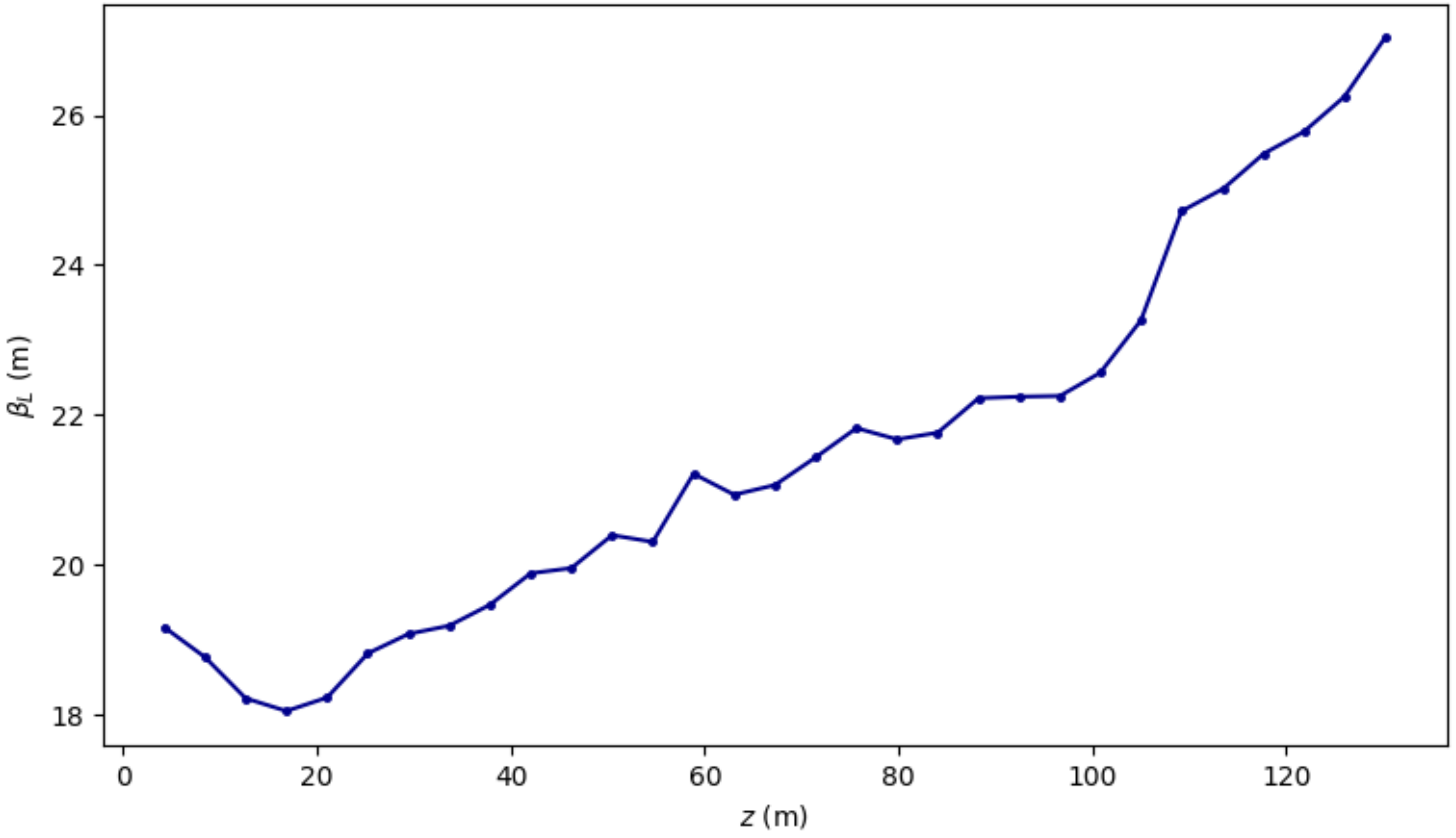
EMITTANCES FROM ECALC9

BETA FUNCTIONS (ECALC9)

β reduced along channel
→ evidence of cooling?



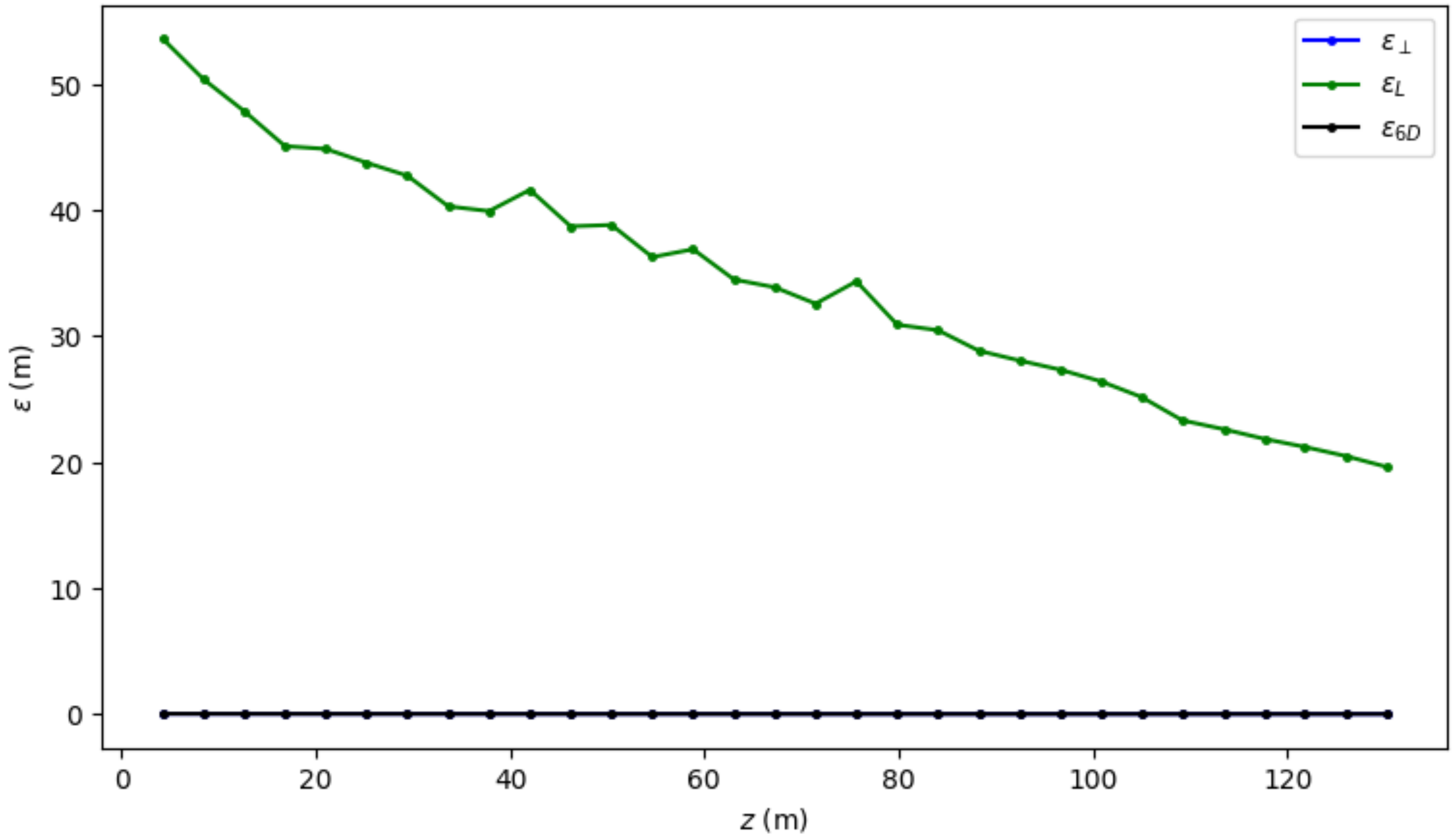
β_L increasing along channel → expect to see longitudinal emittance increasing?



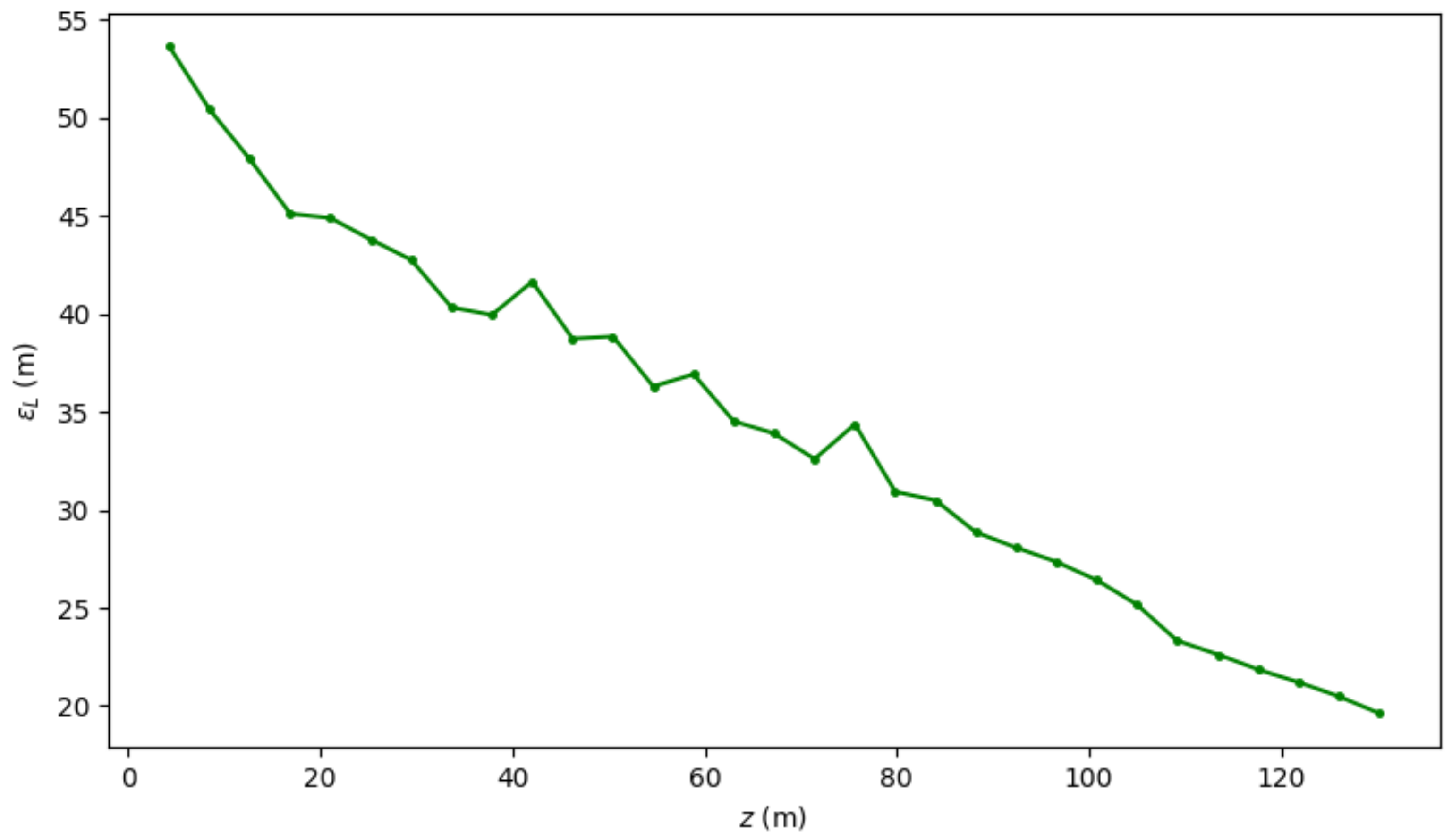
What order of magnitude makes sense for these values?

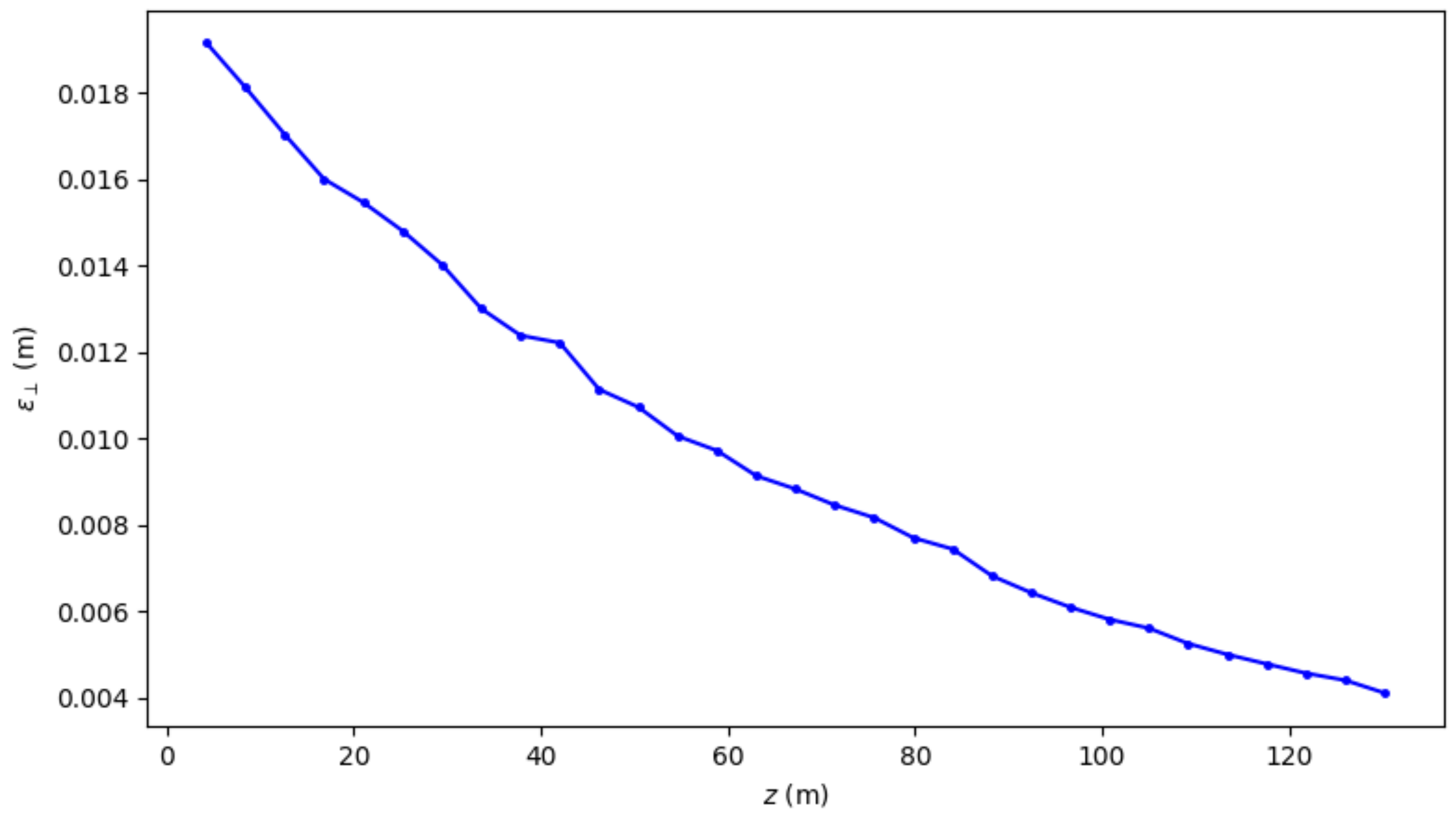
EMITTANCES (ECALC9)

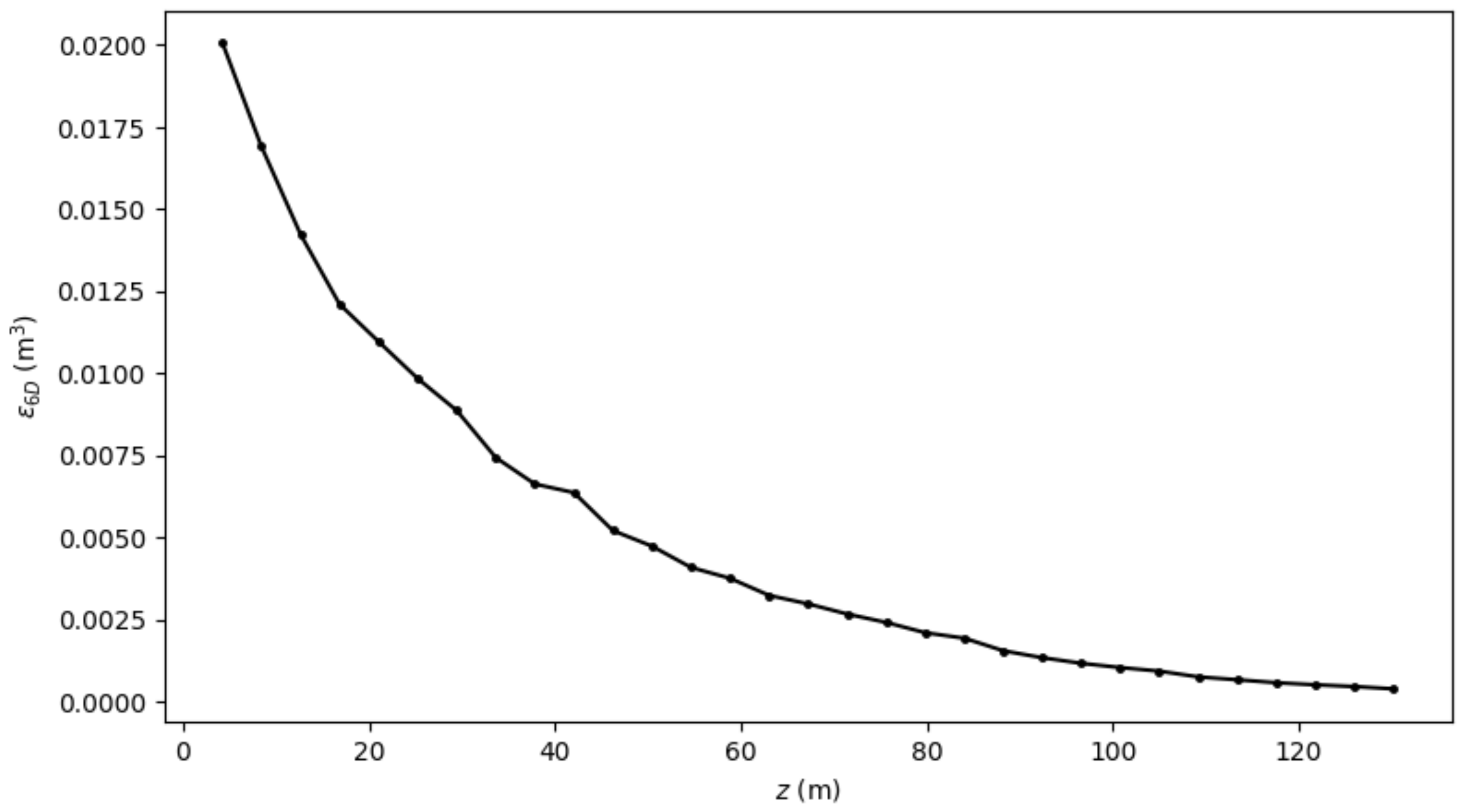
Longitudinal emittance is much larger than transverse ✓



EMITTANCES (ECALC9)



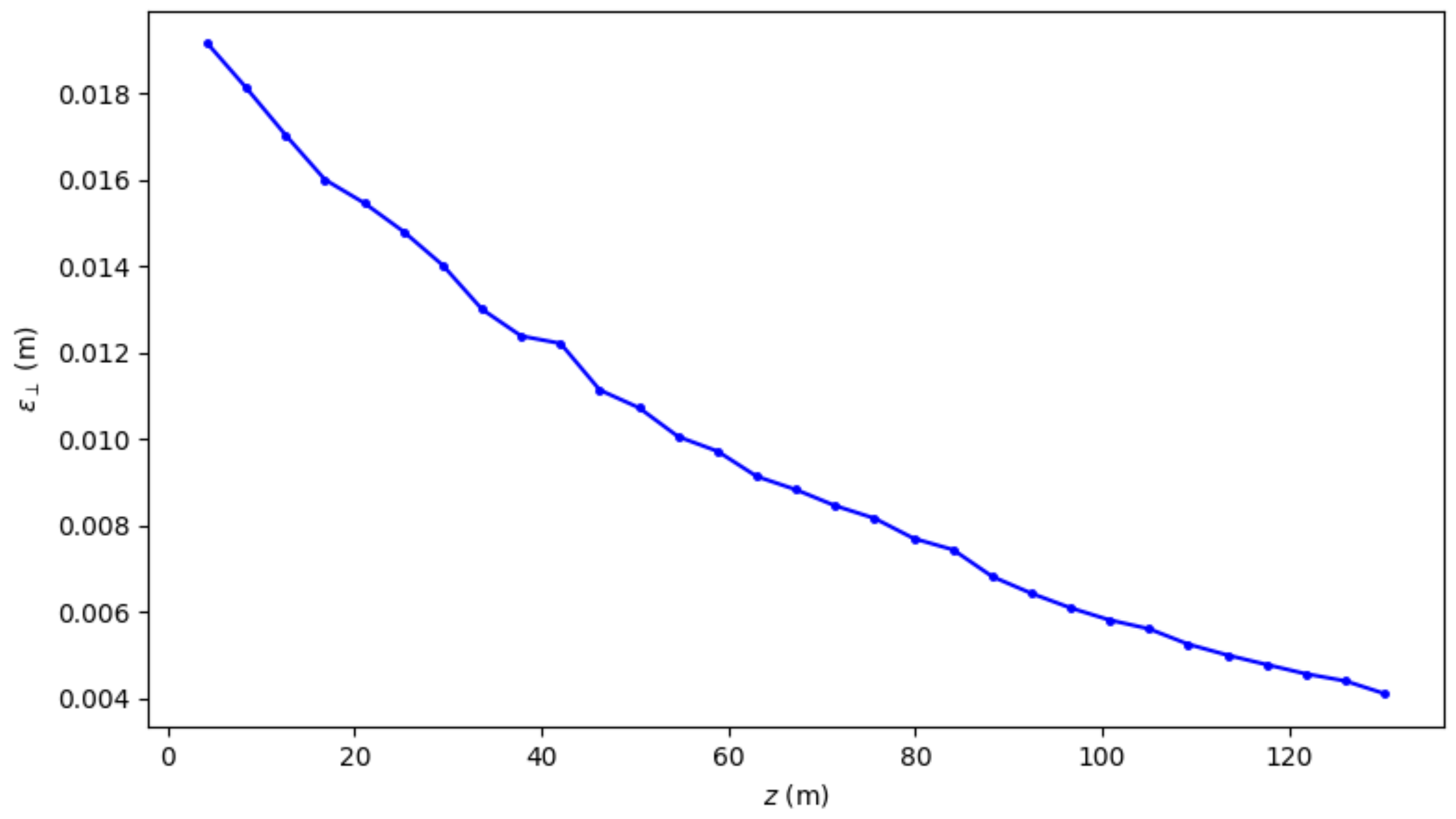


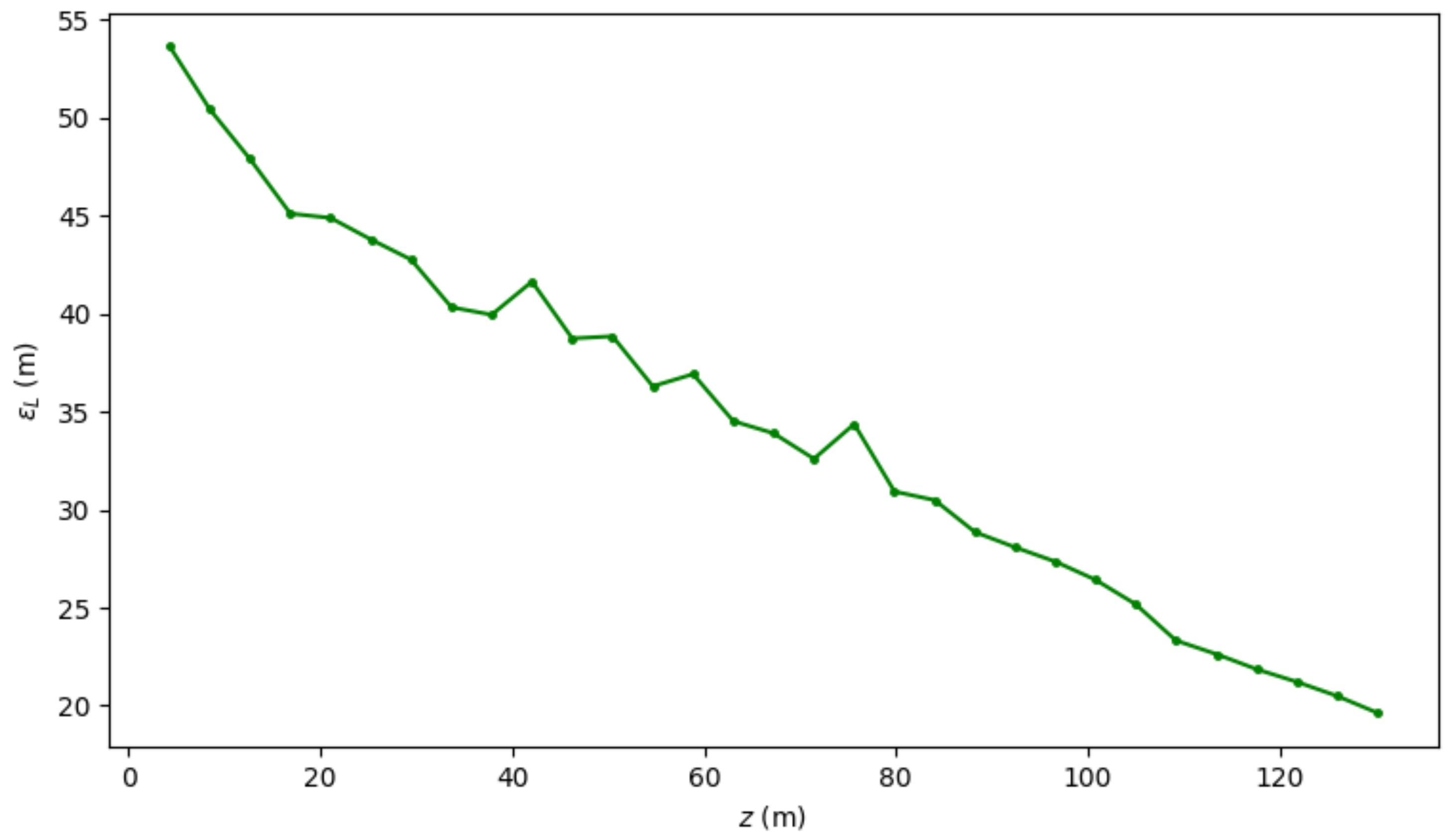


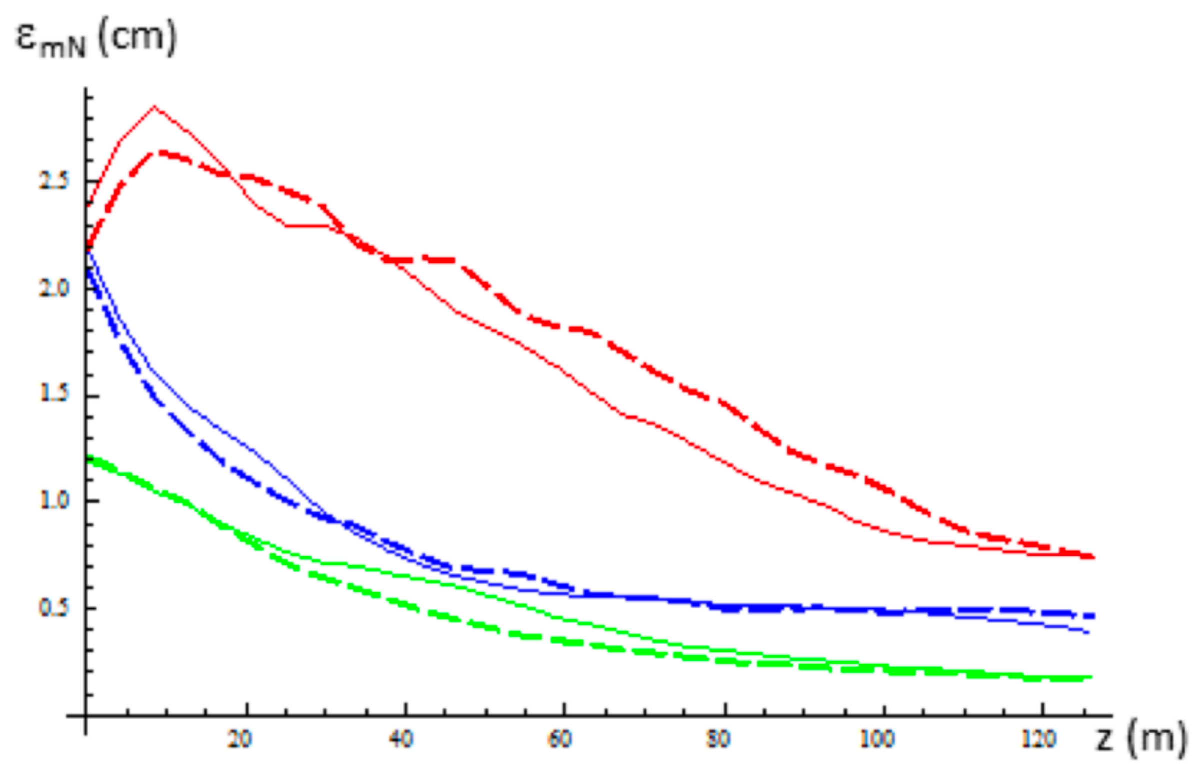
Unlike the Mathematica results, here we see a downward trend in both the longitudinal and the transverse emittances → 6D cooling ✓

CONCLUSIONS

- No initial increase in longitudinal emittance observed with this approach
- Again see values **orders of magnitude off** from each other and from Yuri's results
- That said, we do not necessarily expect great agreement as ecalc9 makes the assumption of azimuthal symmetry and does not apply corrections for timing, momentum, etc.

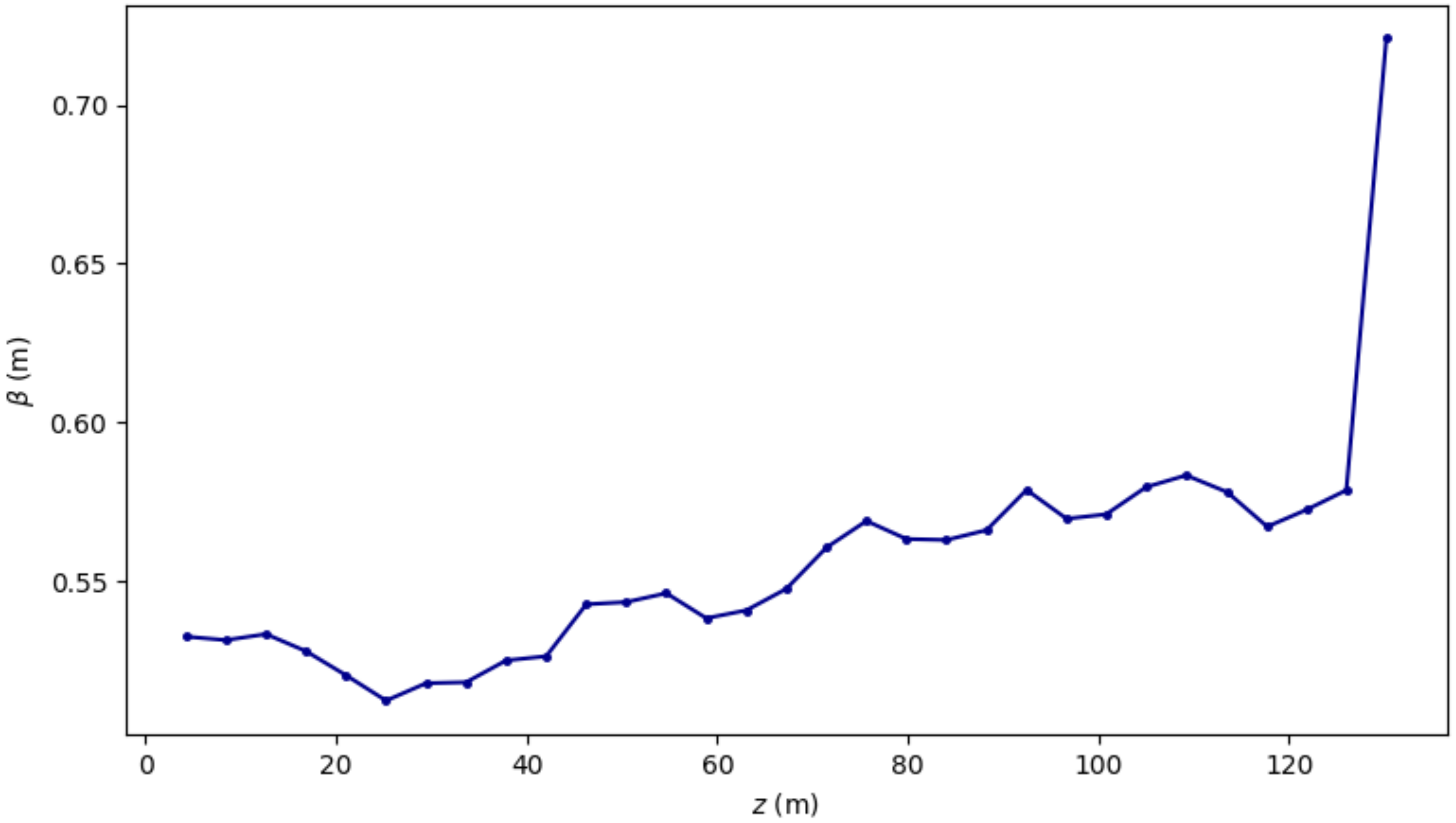




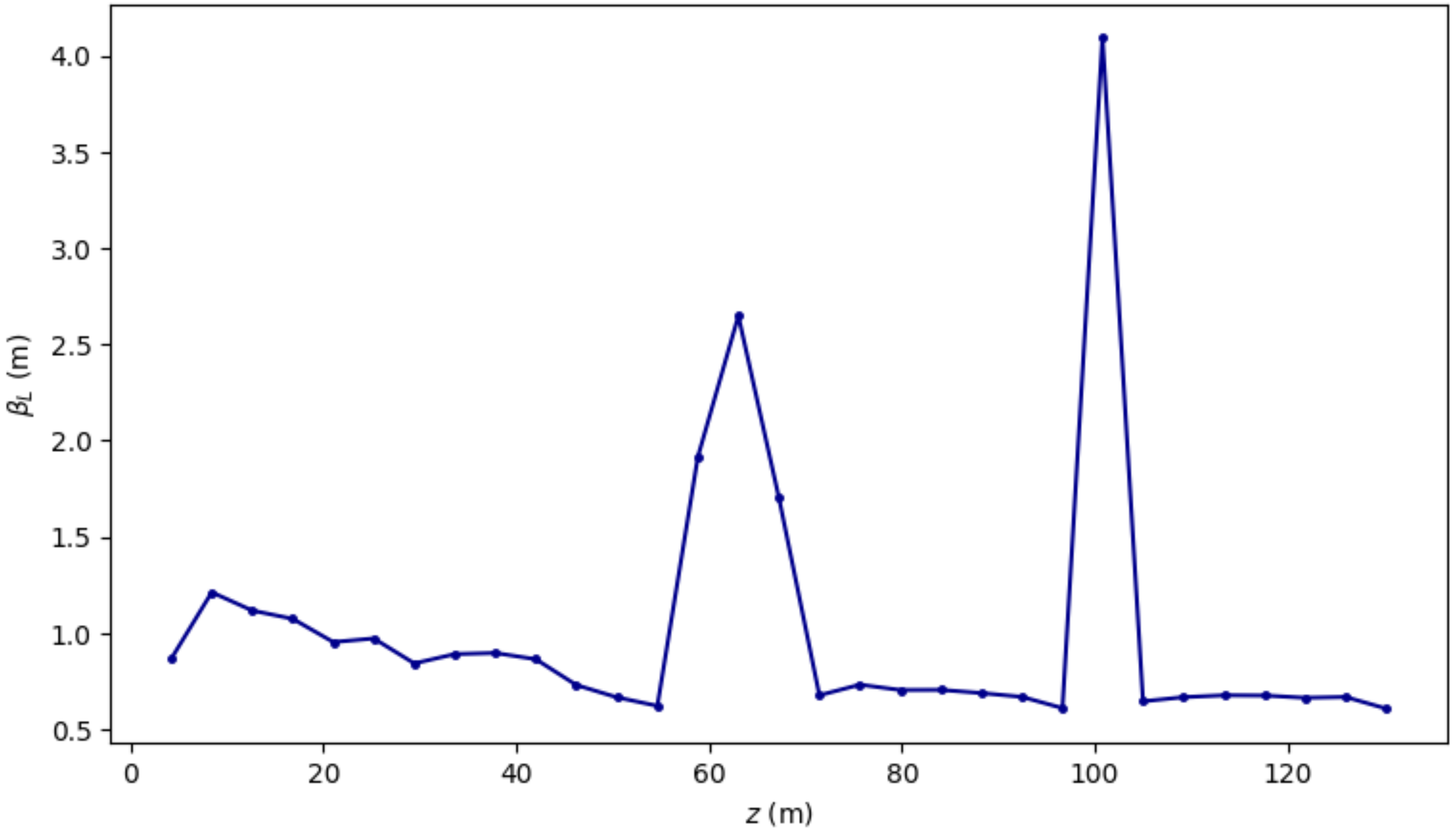


EMITTANCES FROM ECALC9F

BETA FUNCTIONS (ECALC9F)



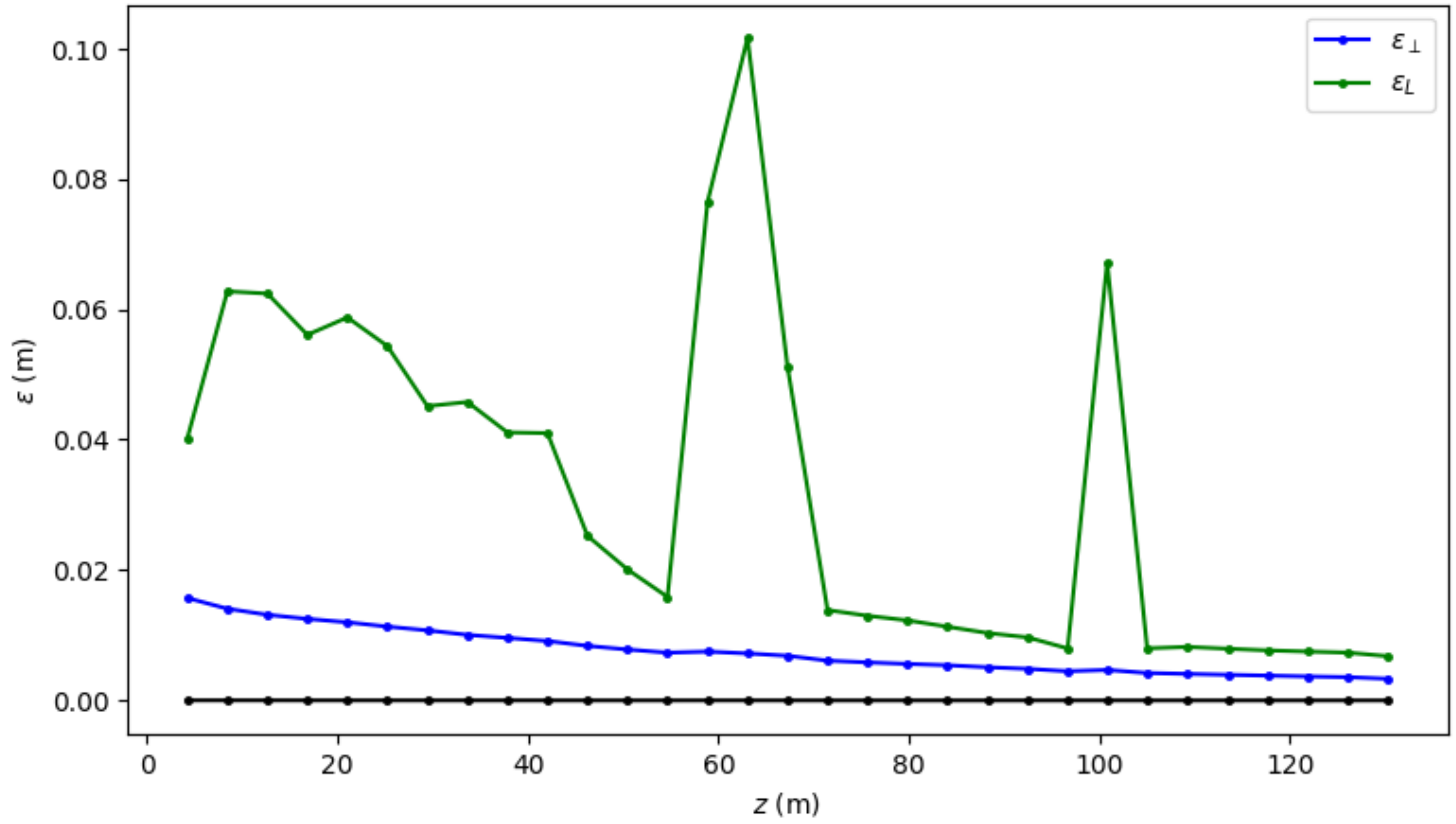
Rather large jump in β at the last detector, but this was seen in the earlier plots as well — perhaps explainable by fringe fields?



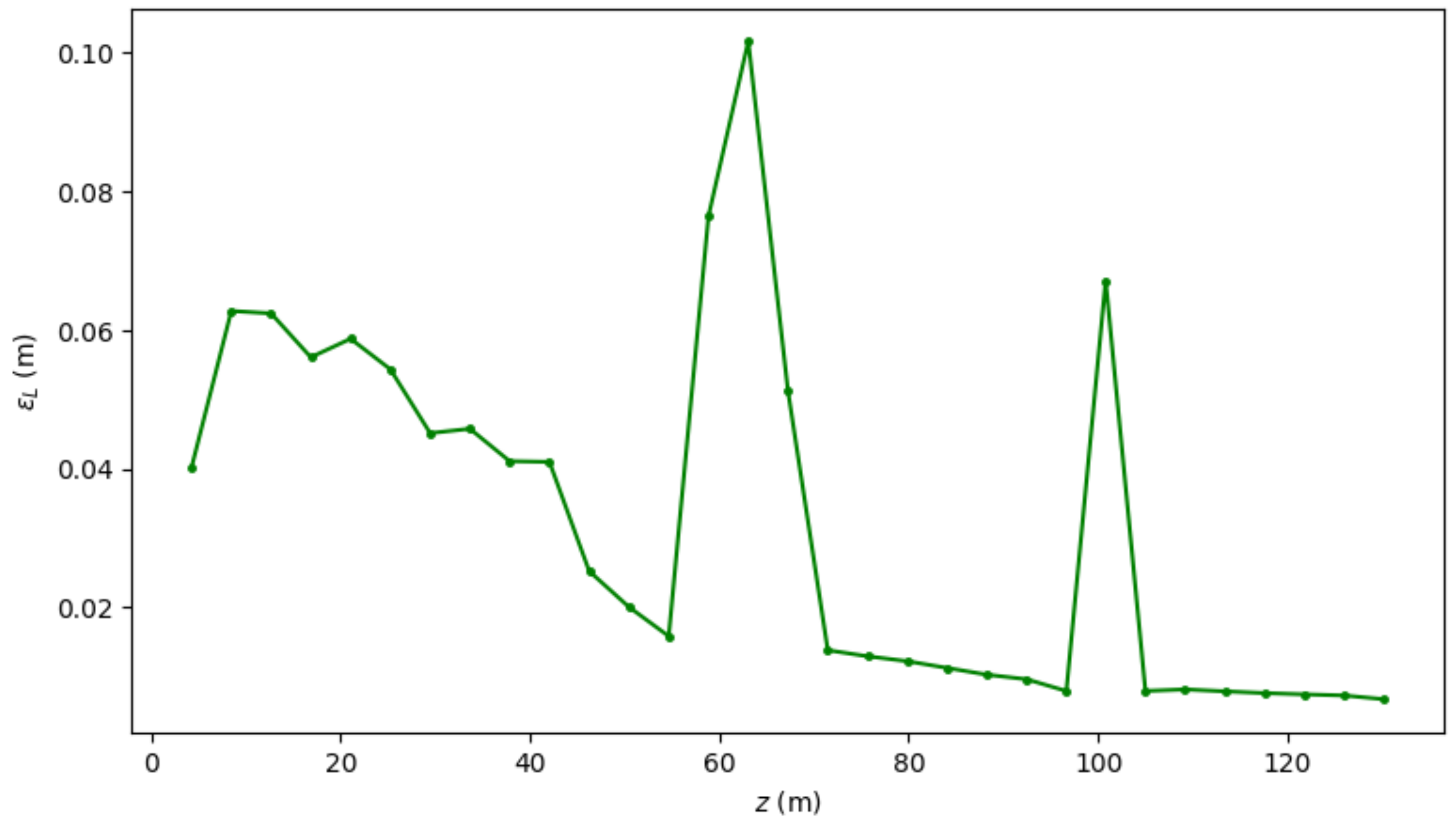
Observe significant peaks in β_L around 60m and 100m — with no apparent motivation from channel geometry

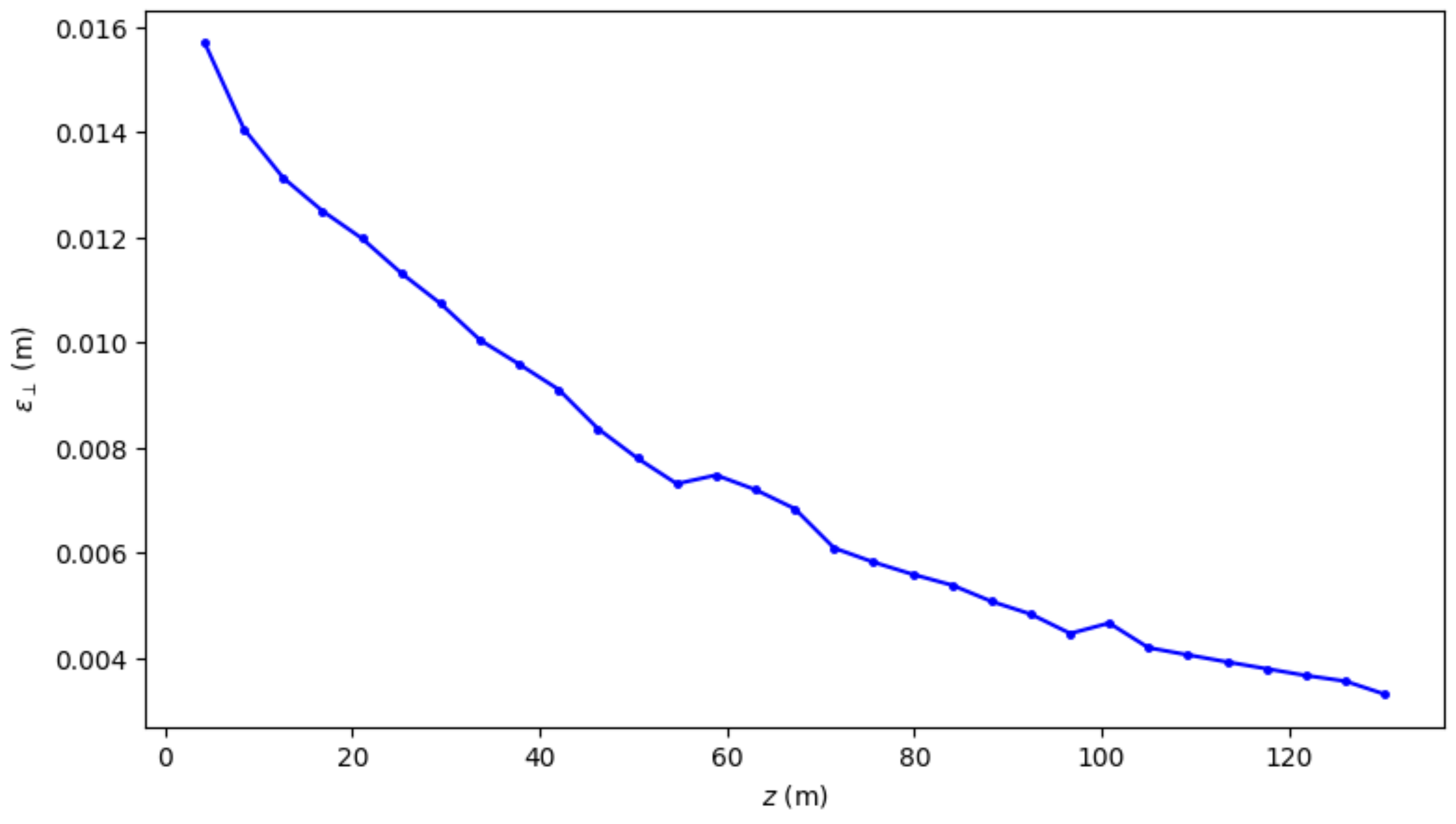
EMITTANCES (ECALC9F)

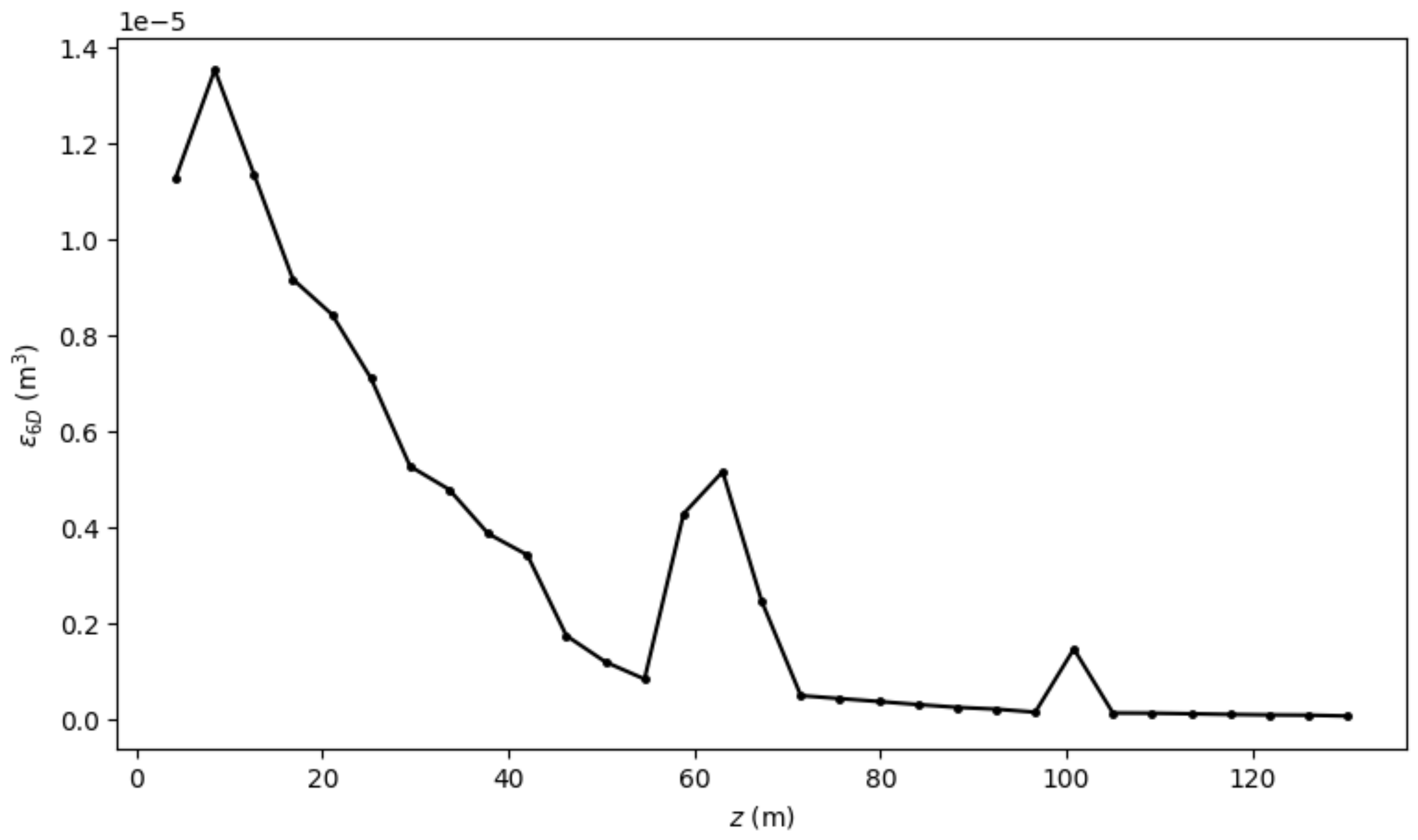
Observe peaks in longitudinal emittance at same locations



EMITTANCES (ECALC9F)



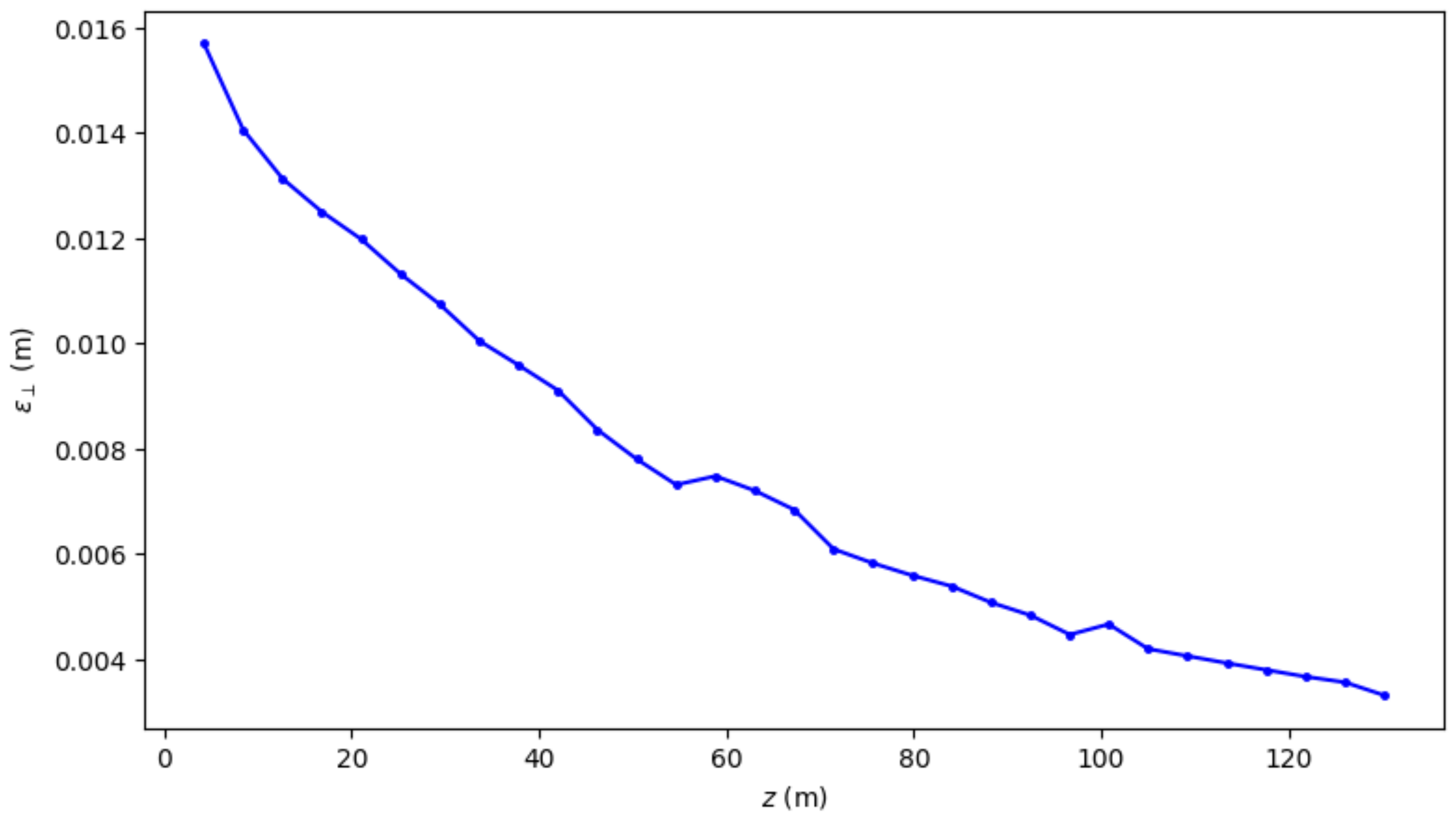


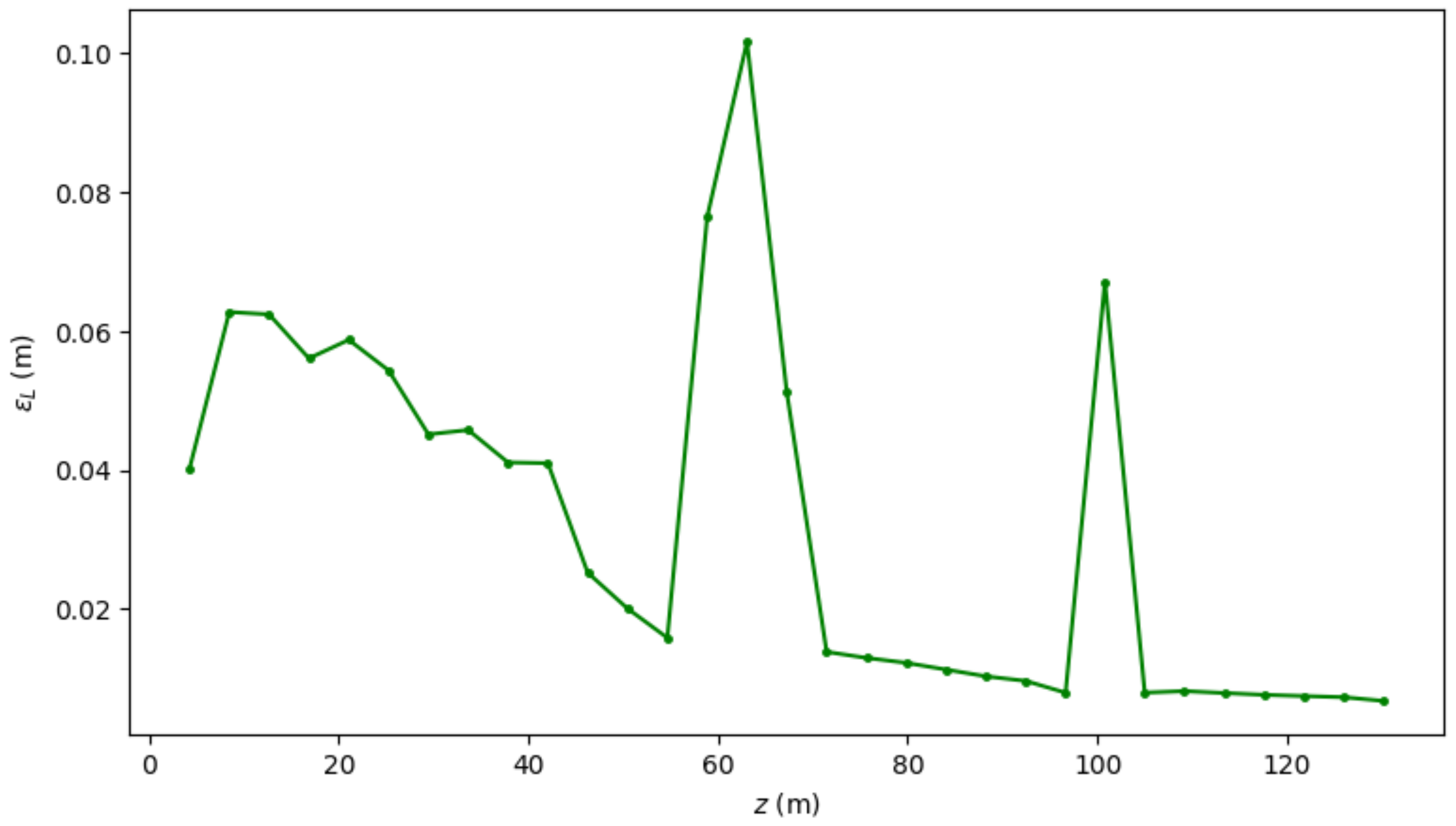


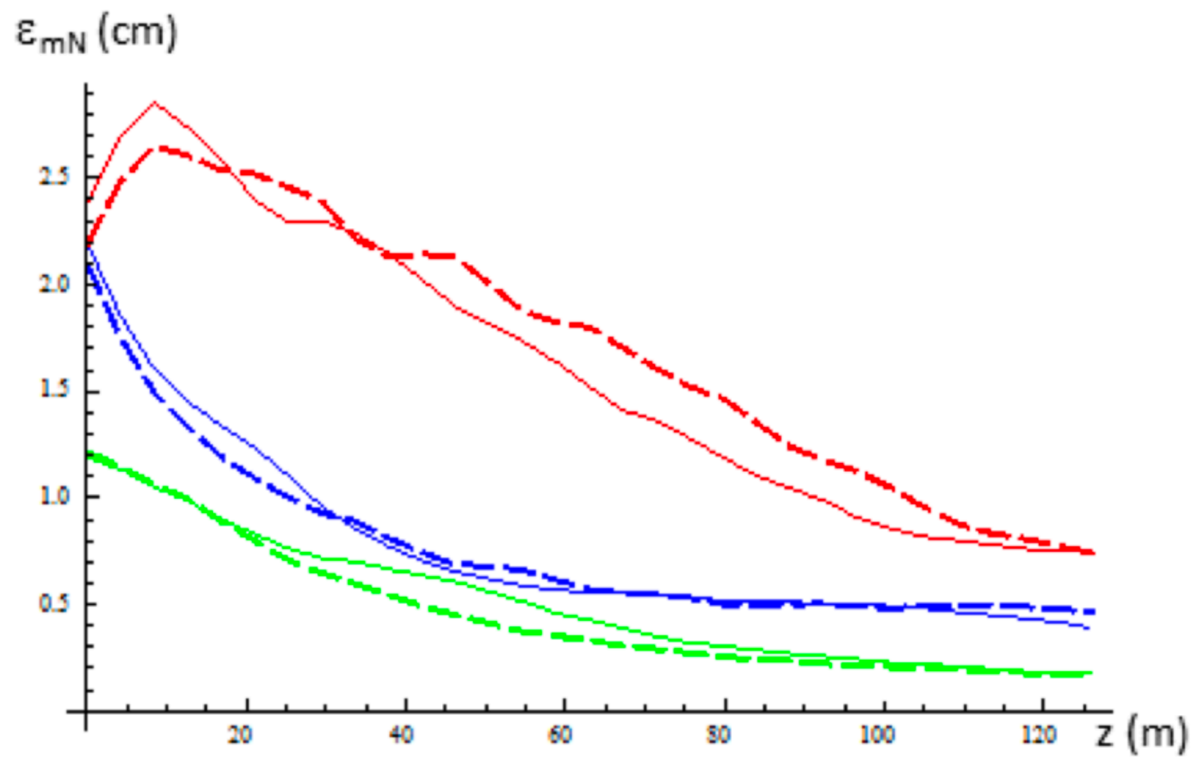
Zooming in on the transverse and 6D emittances, we see again features occur at these z

CONCLUSIONS

- Observe **wildly different behavior in longitudinal emittance**
- Similar features appear in transverse emittance — again not seen in Yuri's plot
- Perhaps need to study the influence of the **choice of corrections**? Currently using those specified in the provided .inp file



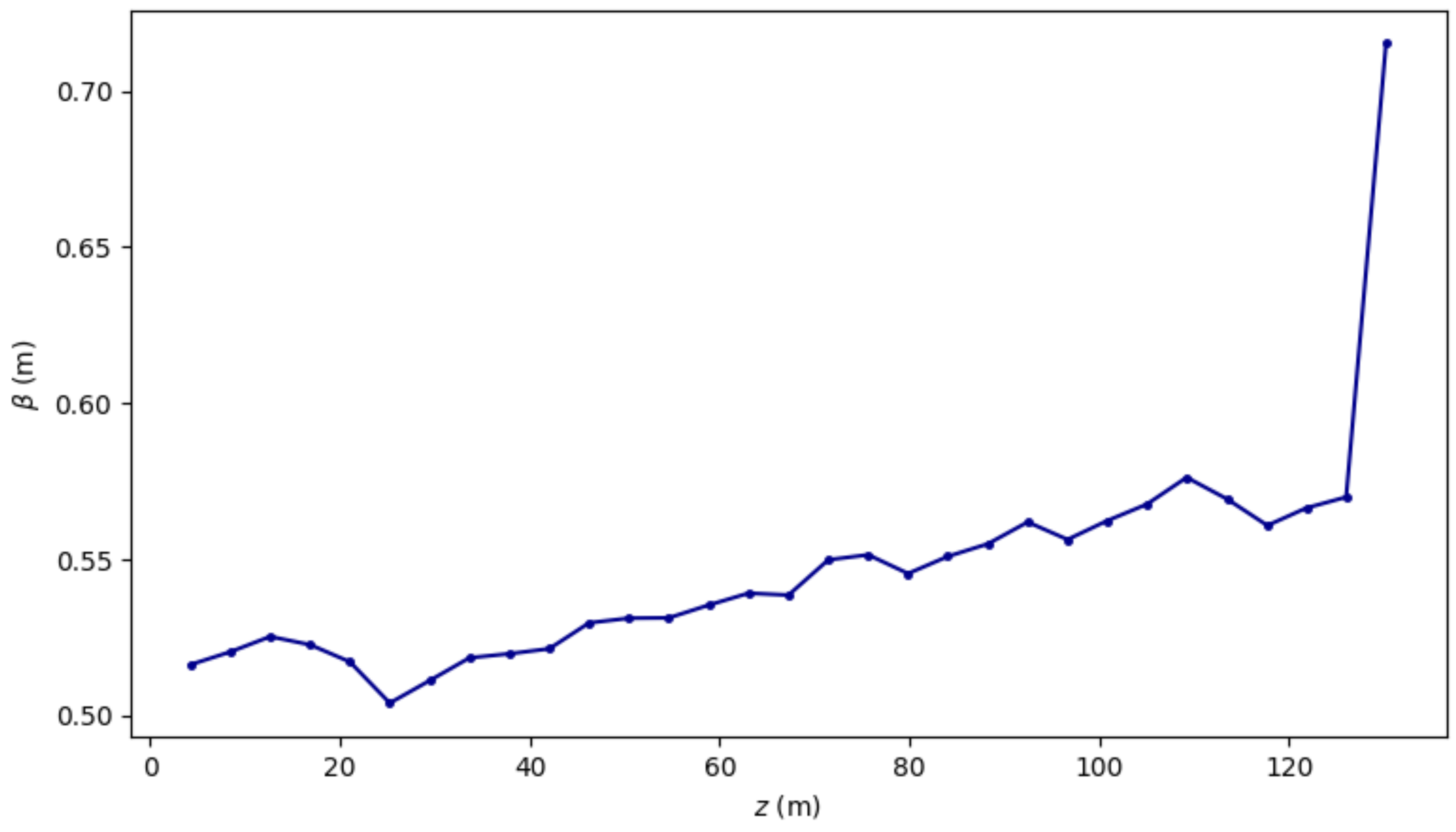




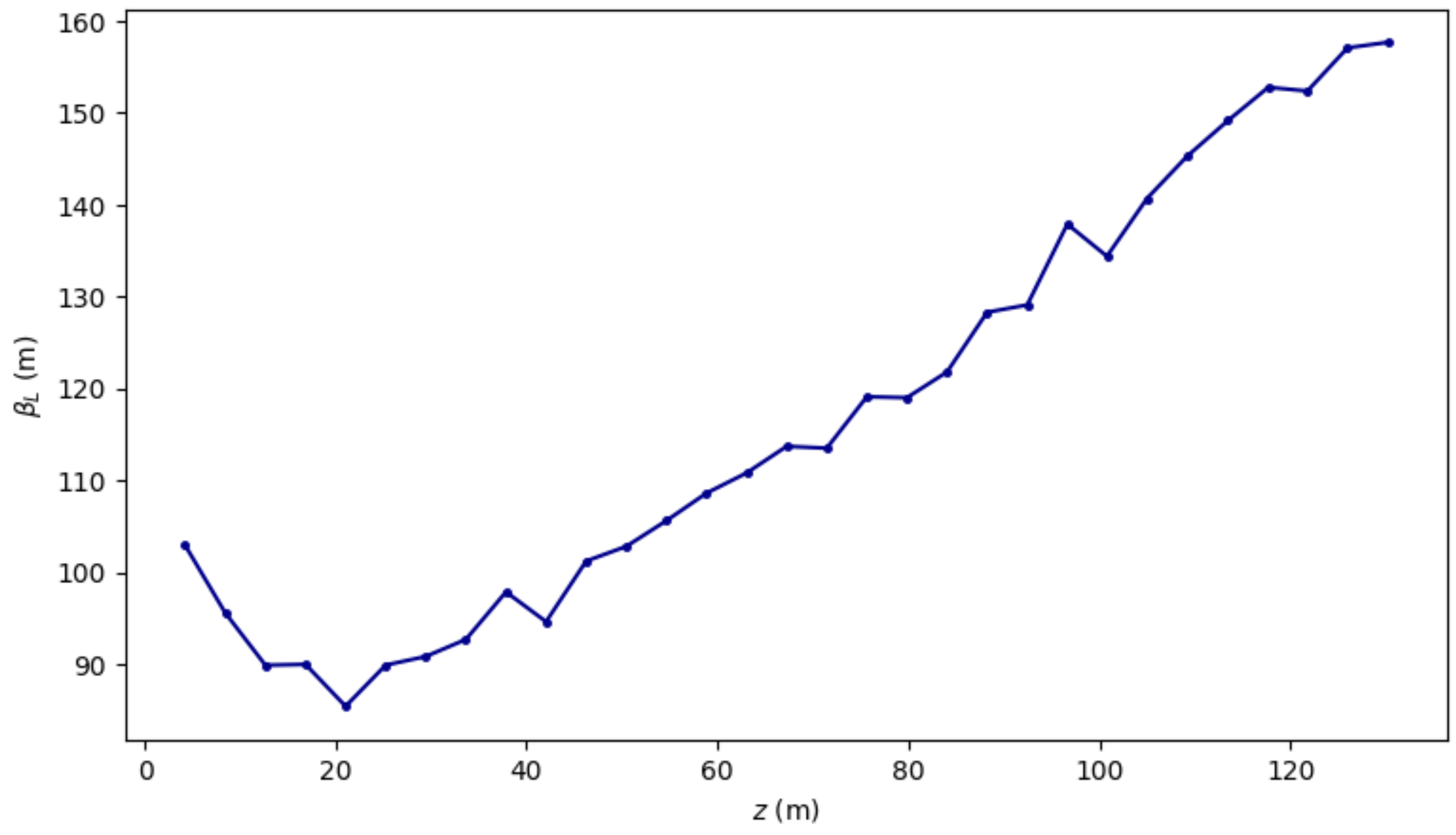
EMITTANCES FROM EMITCALC

BETA FUNCTIONS (EMITCALC)

Note large value at last detector again

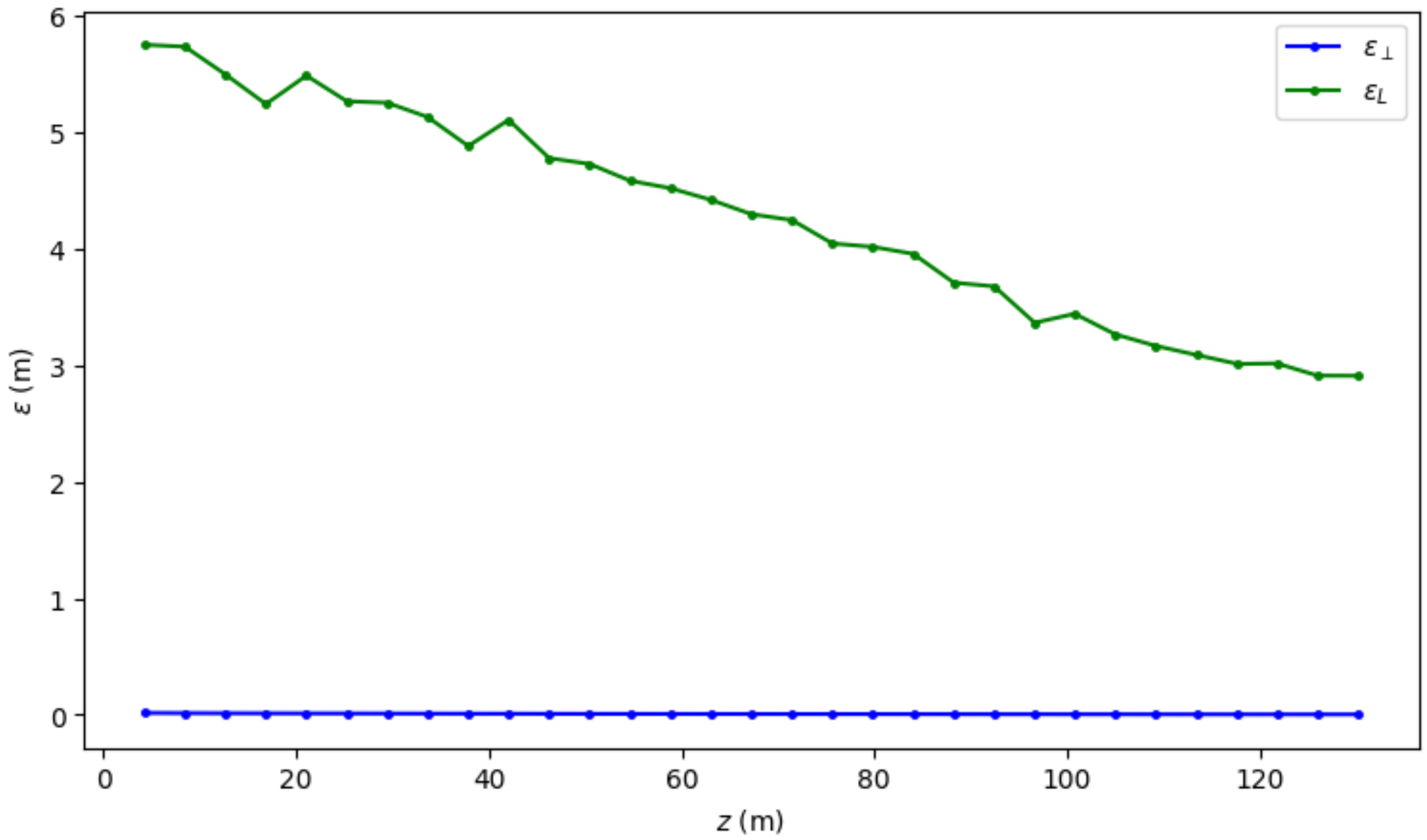


Growth along channel — more similar to ecalc9 result

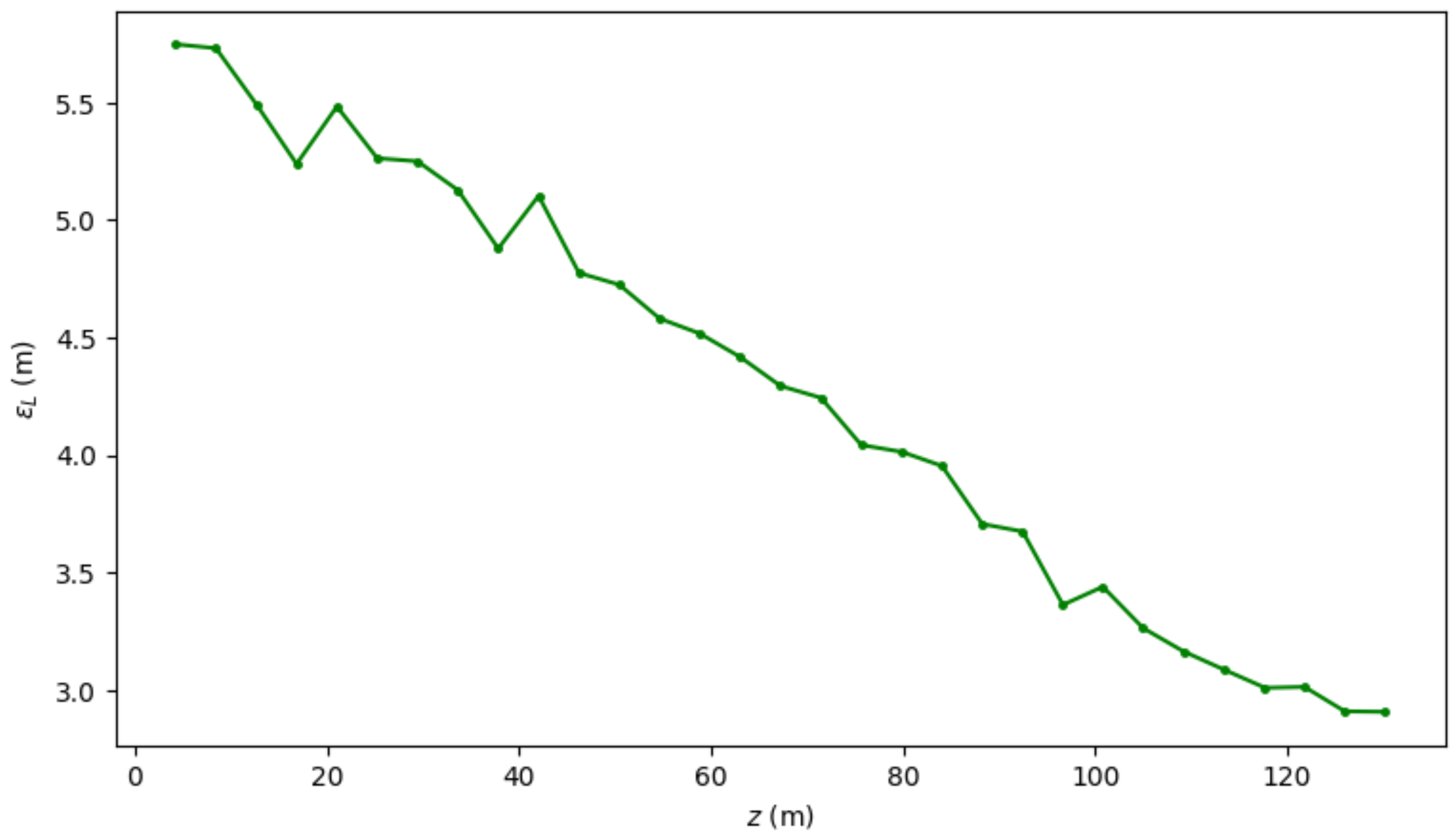


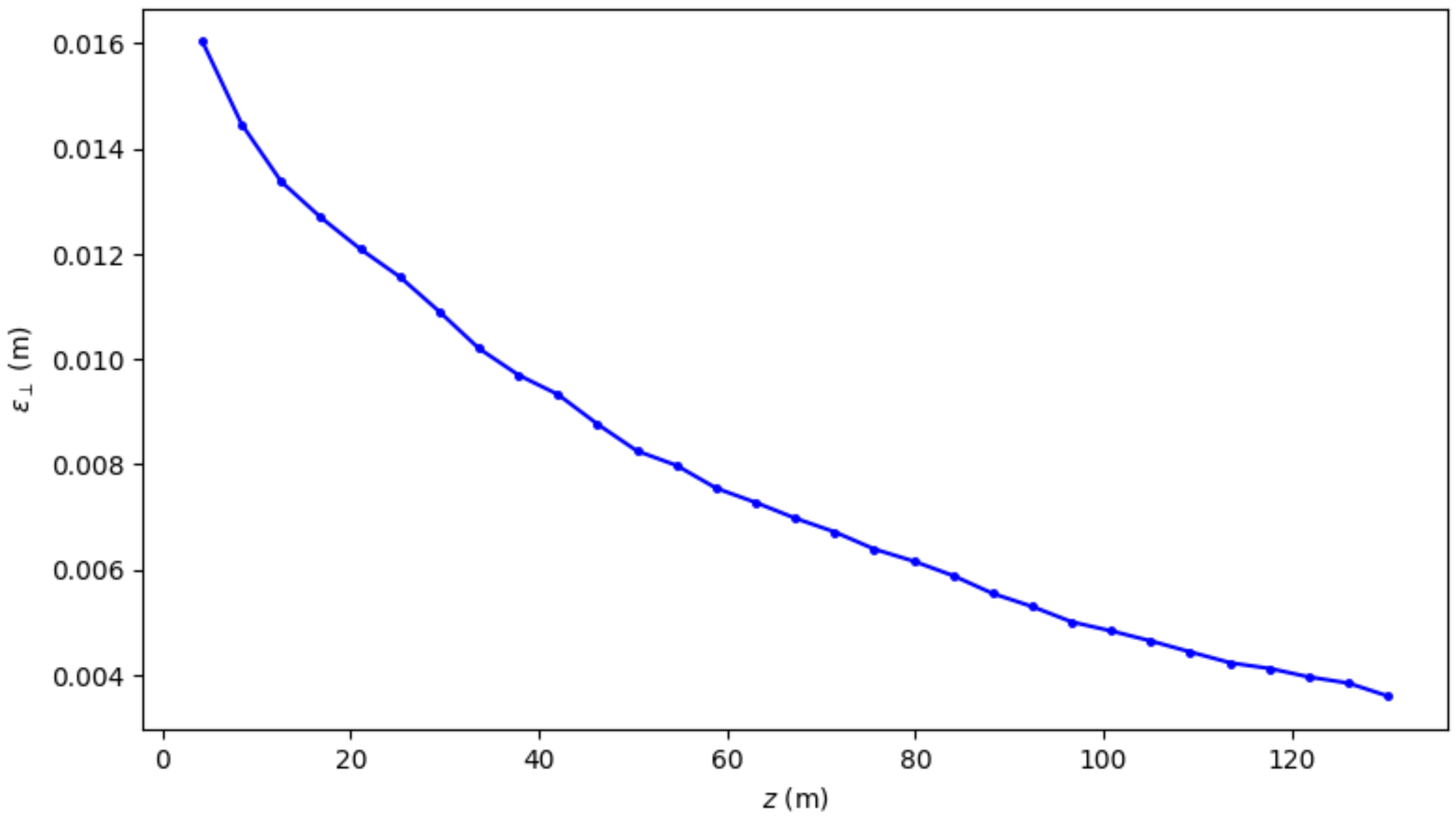
EMITTANCES (EMITCALC)

Longitudinal emittance much larger than others ✓



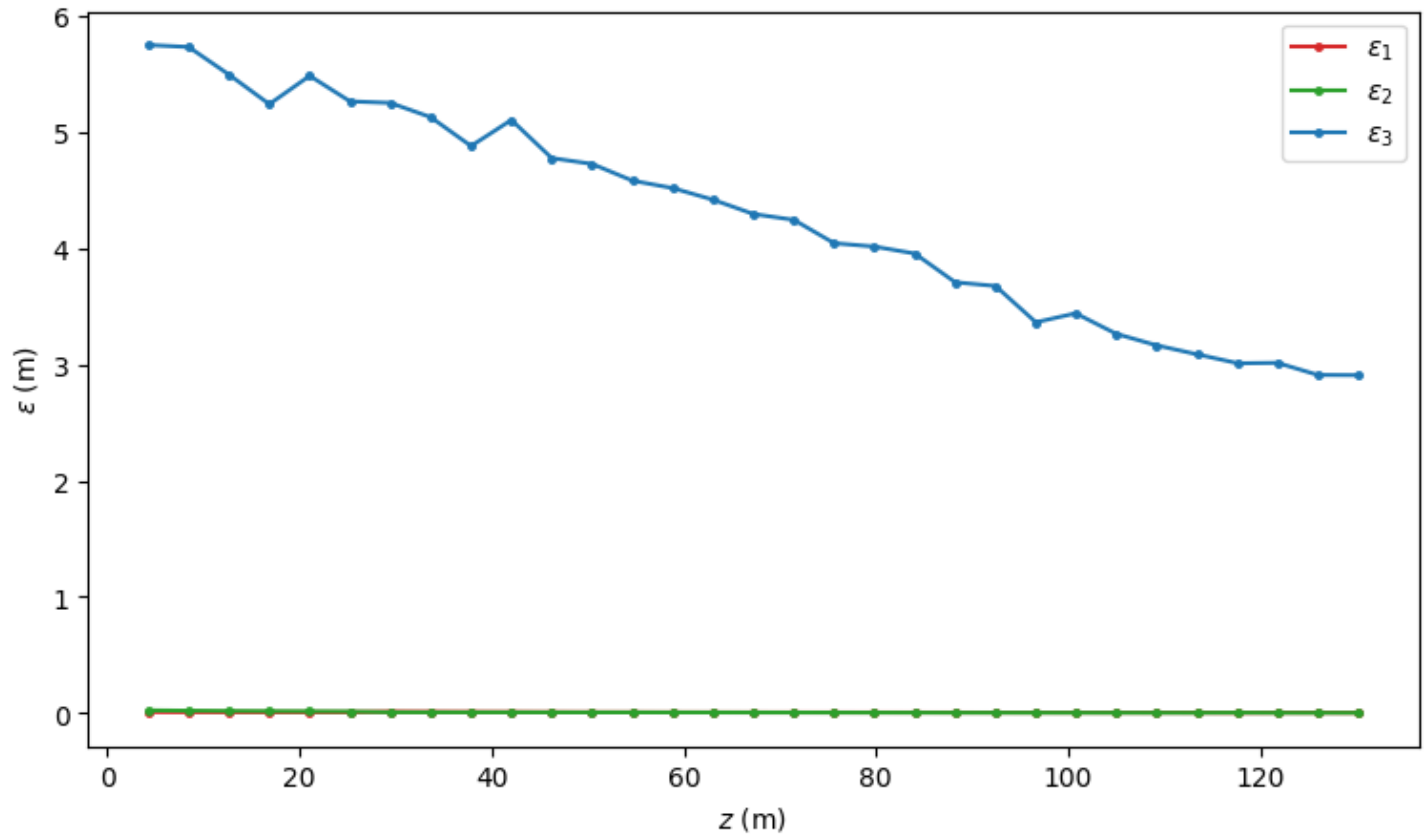
EMITTANCES (EMITCALC)

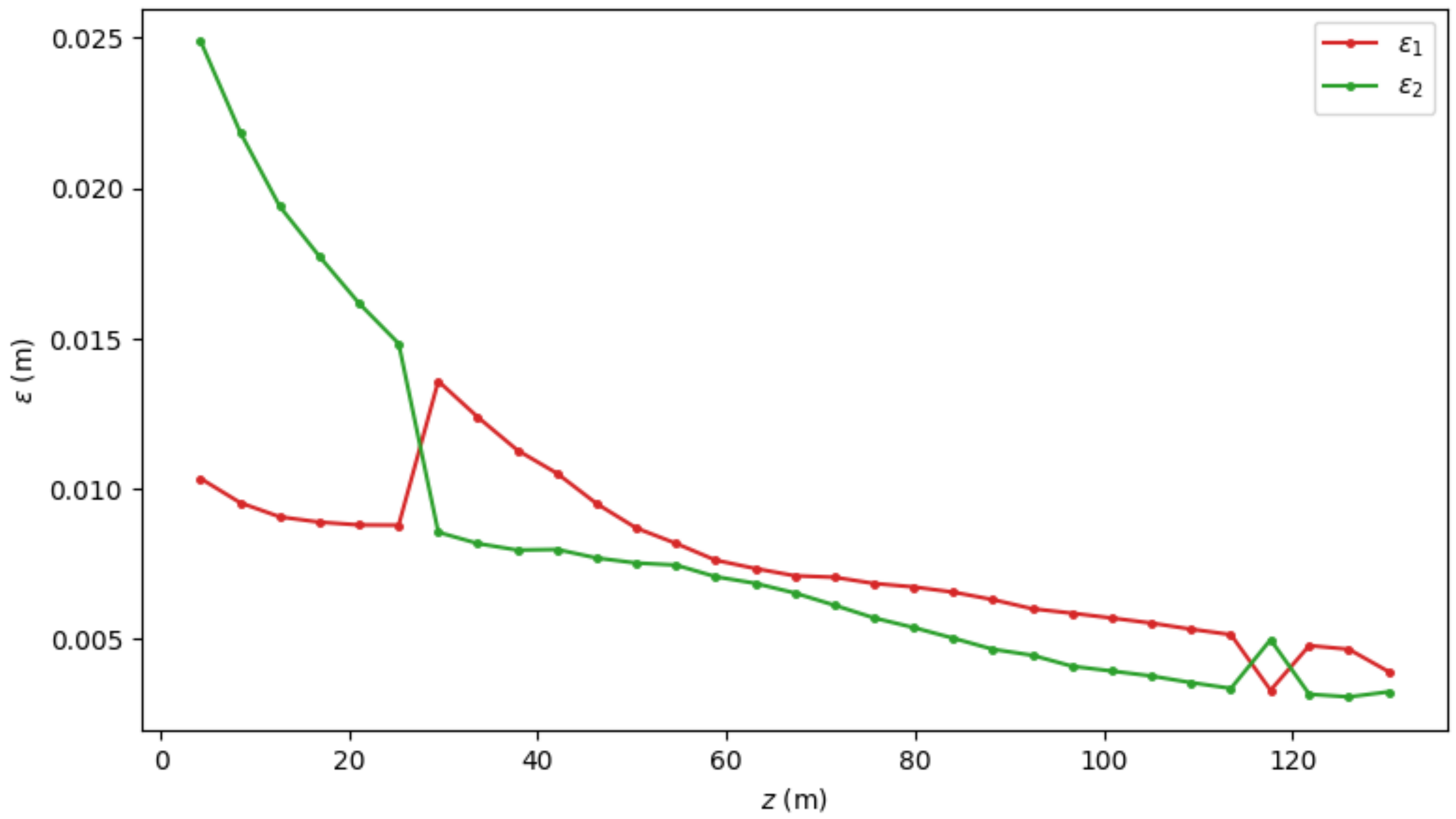




Decrease in both longitudinal and transverse emittances → 6D cooling ✓

EMITTANCES (EMITCALC)



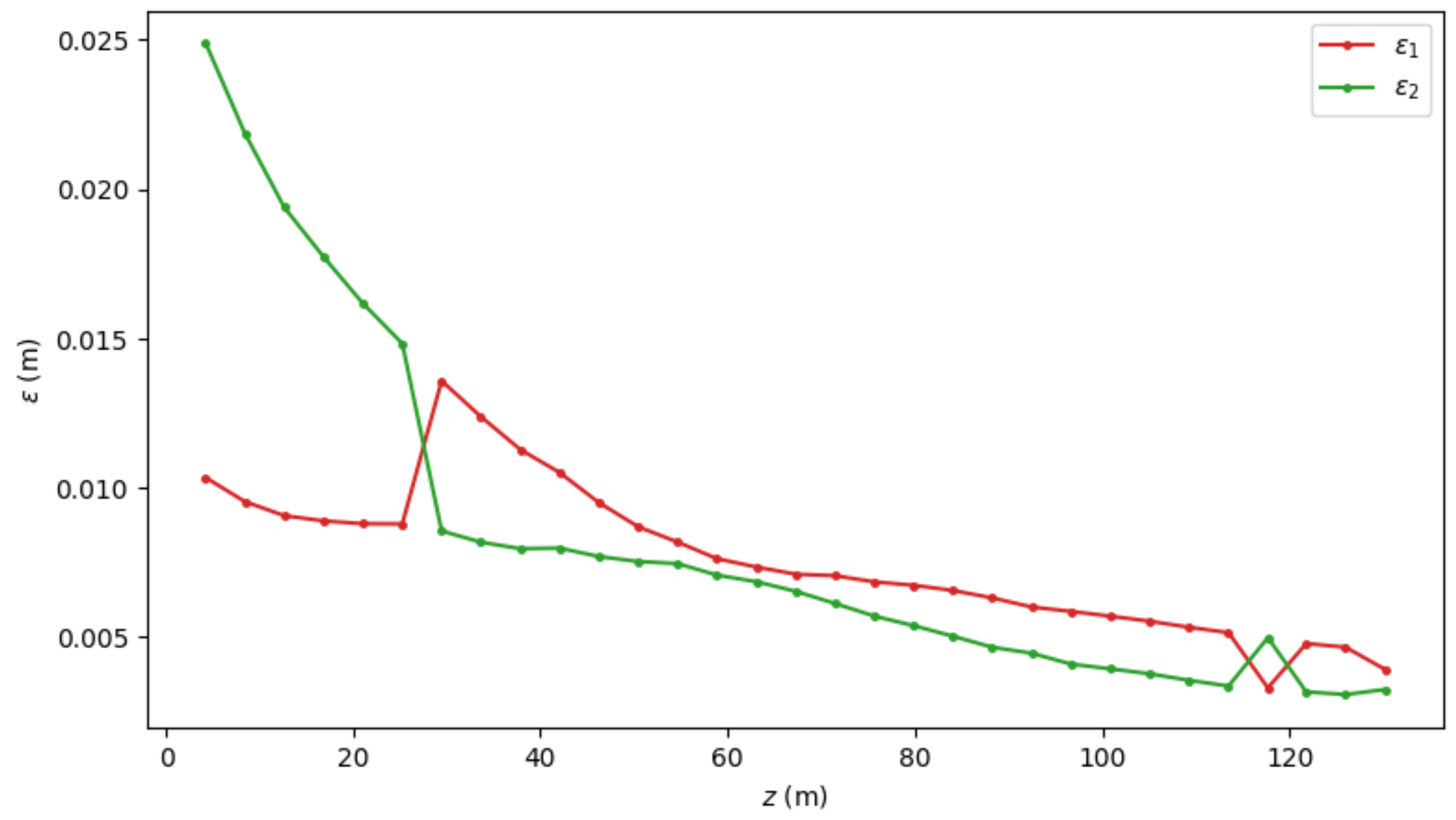


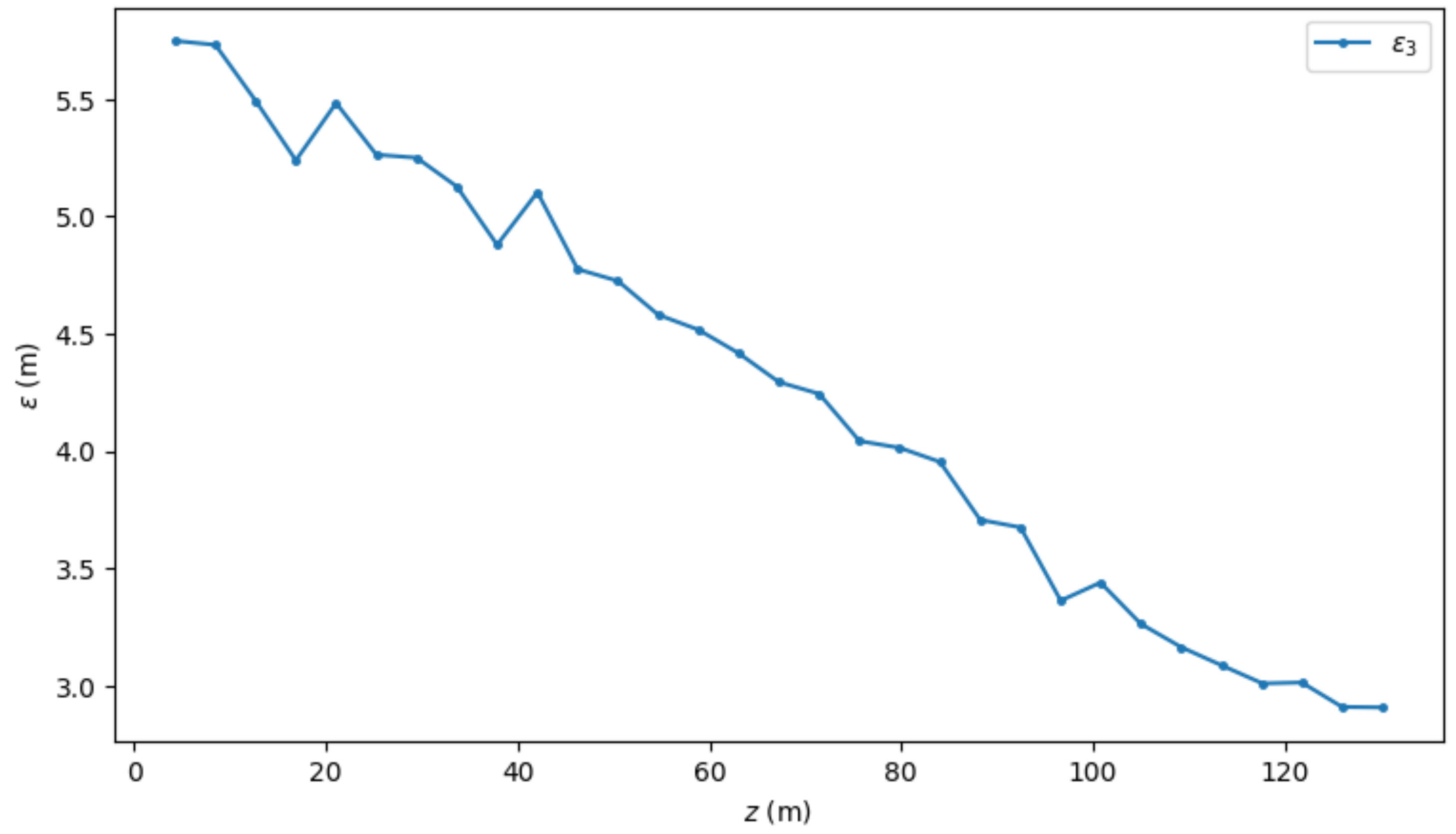
$$\varepsilon_3 \iff \varepsilon_L$$

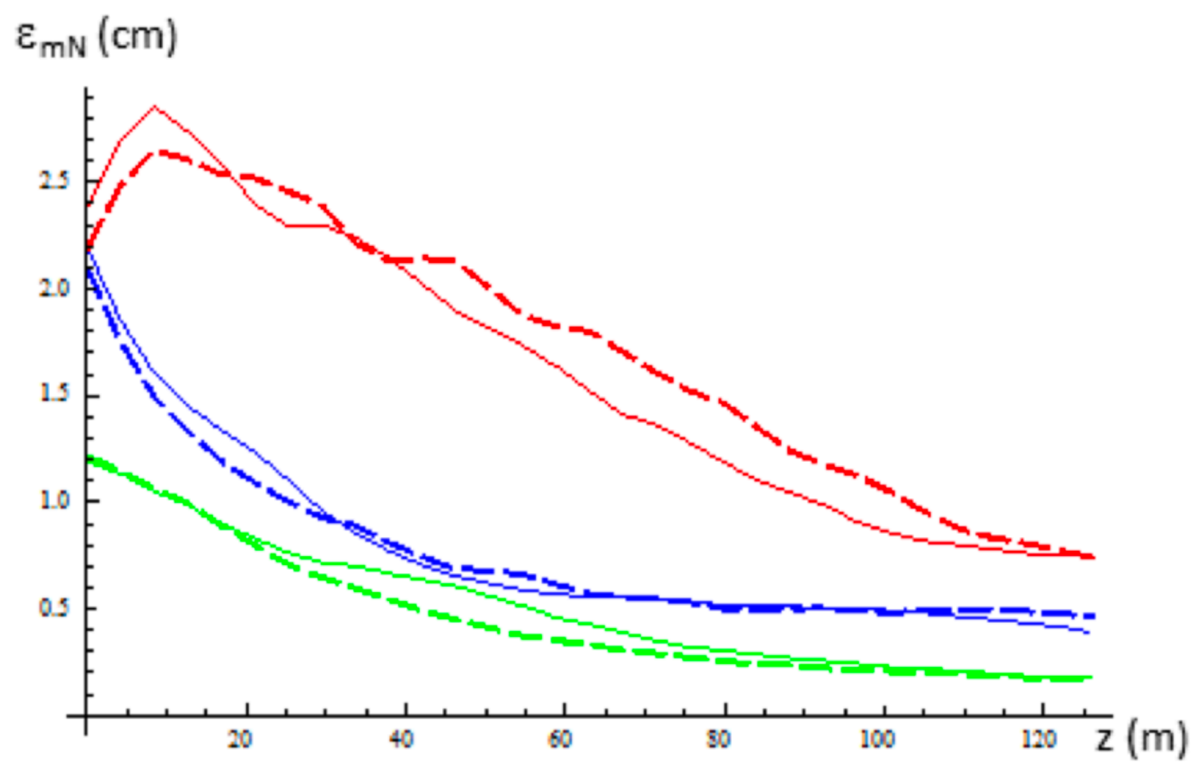
Should really be two downward-sloping curves?
Are jumps explainable by script assigning
arbitrary labels for modes?

CONCLUSIONS

- This approach appears to be the **closest to Yuri's results** both qualitatively and quantitatively
- Initial and final values for transverse modes (ε_2 and ε_3) are in very close agreement
- Longitudinal emittance (ε_1) values are larger than Yuri reported and do not show an initial increase — at least at this sampling







SUMMARIZING RESULTS

CONSIDERING EMITTANCE REDUCTION - EMITCALC

	$\frac{\varepsilon_{1,initial}}{\varepsilon_{1,final}}$	$\frac{\varepsilon_{2,initial}}{\varepsilon_{2,final}}$	$\frac{\varepsilon_{L,initial}}{\varepsilon_{L,final}}$
emitcalc	2.64	7.70	1.98
Yuri's paper	5.75	7.40	1.16

Note: Yuri's values were obtained by digitizing the plot in his paper — so the values should not be trusted to the level of precision given

CONSIDERING EMITTANCE REDUCTION - ECALC9(F)

	$\frac{\varepsilon_{6D,initial}}{\varepsilon_{6D,final}}$
ecalc9	51.86
ecalc9f	151.28
Yuri's paper	109.80

Note: This value came from Yuri's reported values for initial and final 6D emittances

∴ Rather expectedly, neither ecalc9 nor ecalc9f appears to be a viable candidate for HFOFO

CONSIDERING EMITTANCE REDUCTION - MATHEMATICA

	$\frac{\varepsilon_{1,initial}}{\varepsilon_{1,final}}$	$\frac{\varepsilon_{2,initial}}{\varepsilon_{2,final}}$	$\frac{\varepsilon_{L,initial}}{\varepsilon_{L,final}}$
RMS	3.41	3.42	0.95
Normalized RMS	3.41	5.26	0.98
Gaussian	4.37	4.37	1.19
→ Normalized Gaussian	4.36	5.31	1.24
Yuri's paper	5.75	7.40	1.16

NEXT STEPS

- While there remains the possibility of errors in my translation of Yuri's Mathematica notebook to python, given the poor agreement and evidence to suggest that this notebook is not optimized for HFOFO, I would like to suggest that for now we **move forward with using emitcalc**

- This would allow our focus to instead be on **understanding the magnetic field** in HFOFO

  → Implementing dipoles instead of solenoids

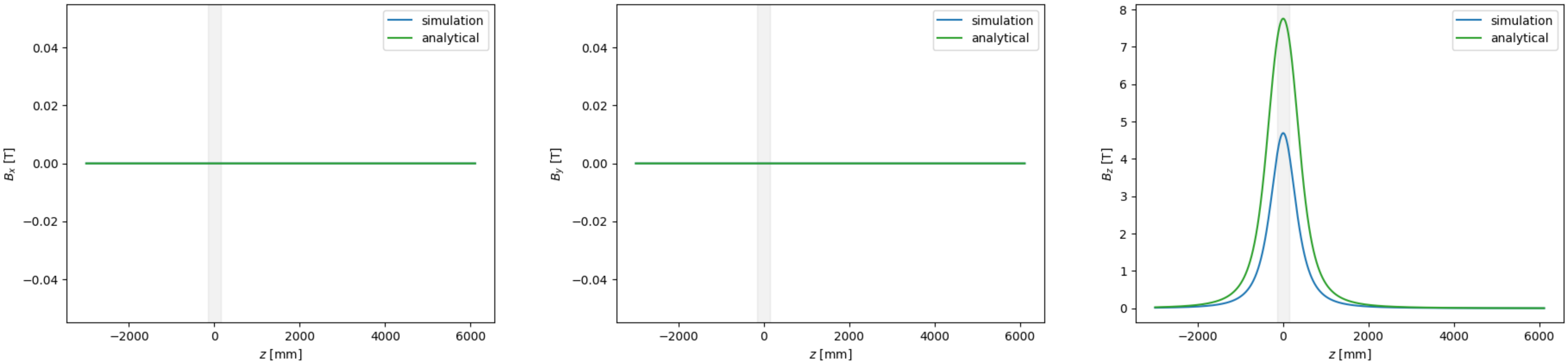
  → Optimization based on beam dynamics

- Thinking about what emittance means — What is the emittance we measure in a physical experiment, and should we be more concerned with this? **What is the most useful way to define emittance?** How much should we be concerned with outliers in the distribution, etc.? Is there a standardized approach to adequately compare across channel designs?

- How can I intuitively verify the results numerically — i.e., what values do we expect?

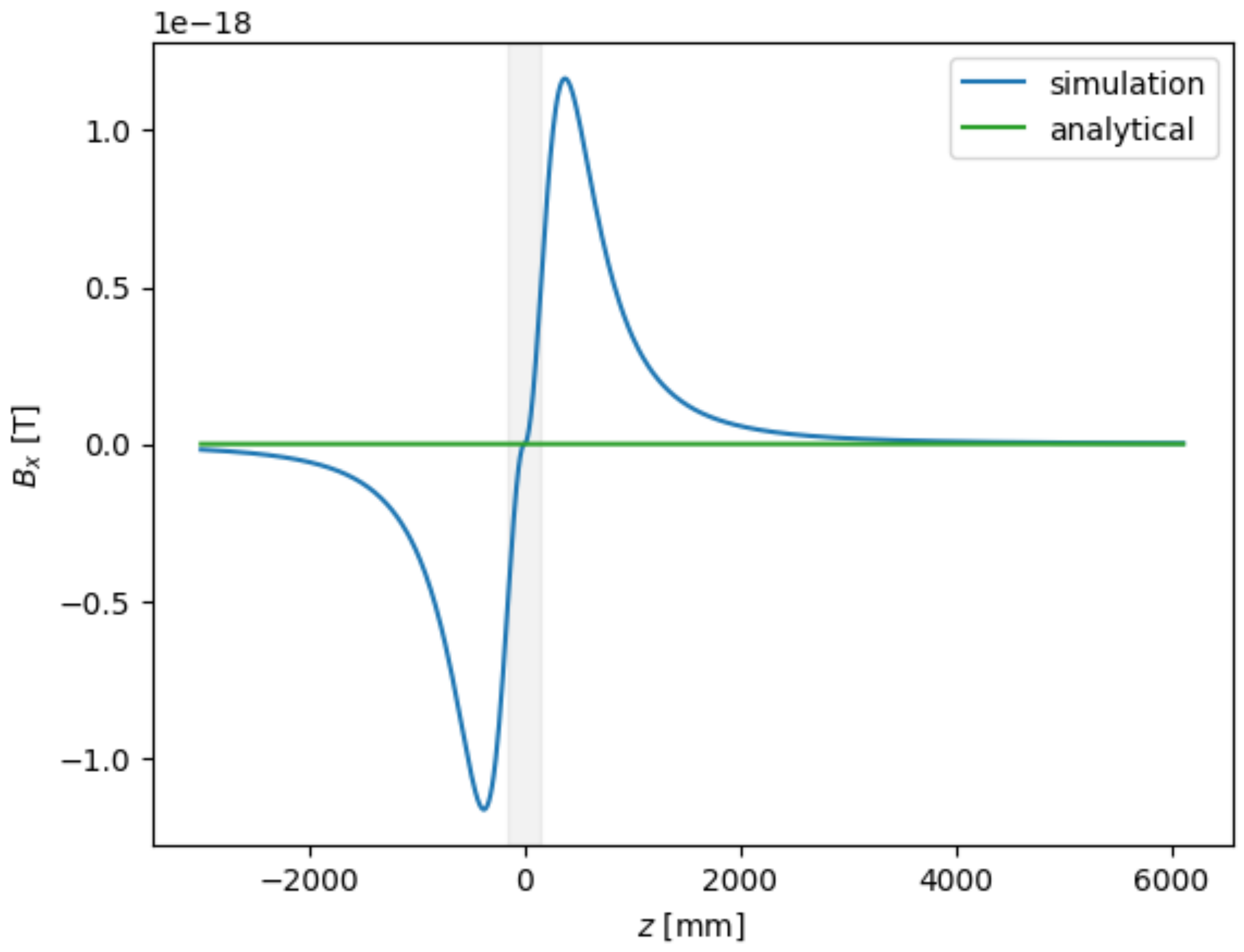
STUDYING MAGNETIC FIELD COMPONENTS IN HFOFO

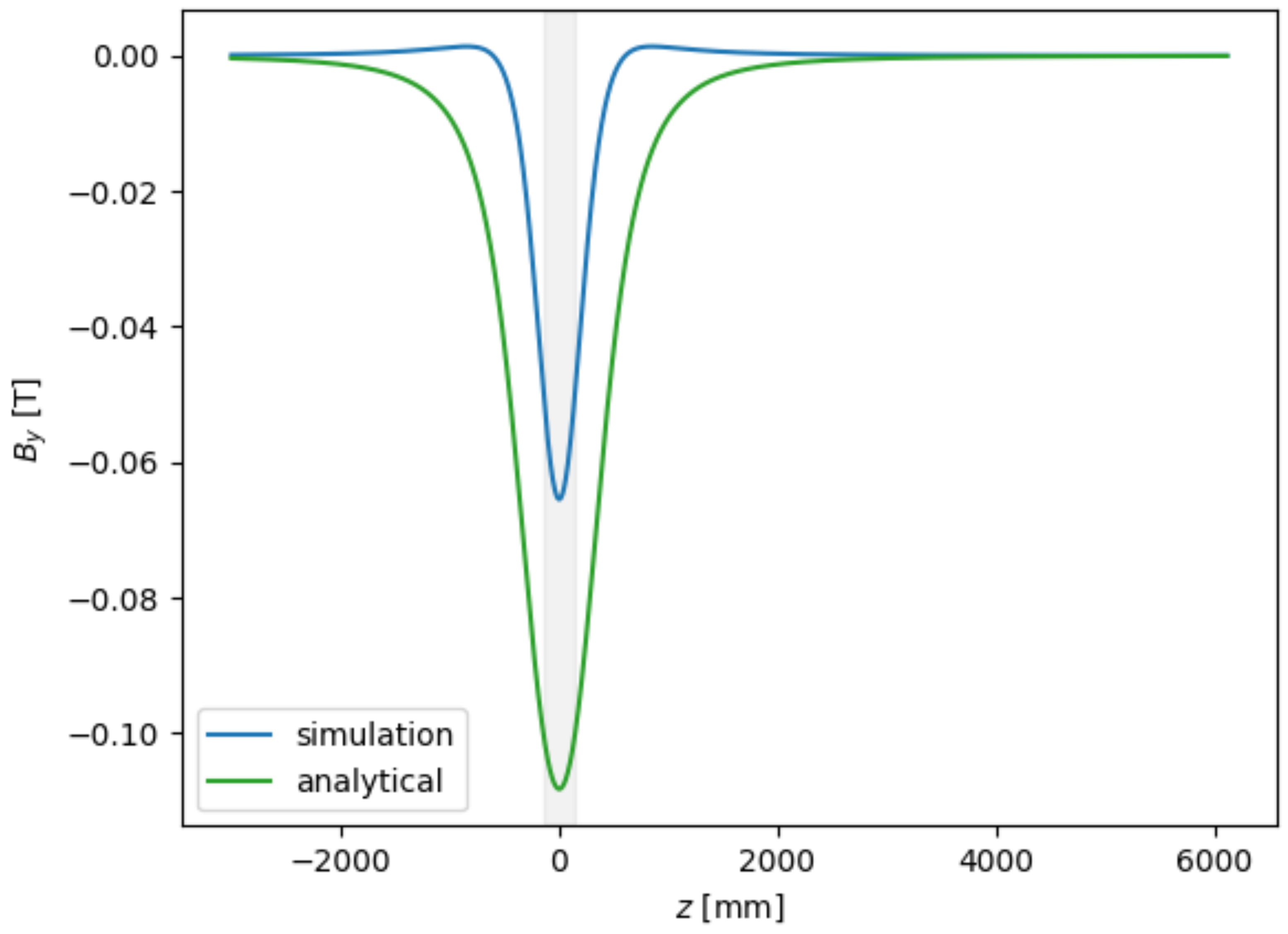
SINGLE HFOFO COIL - NO ROTATIONS

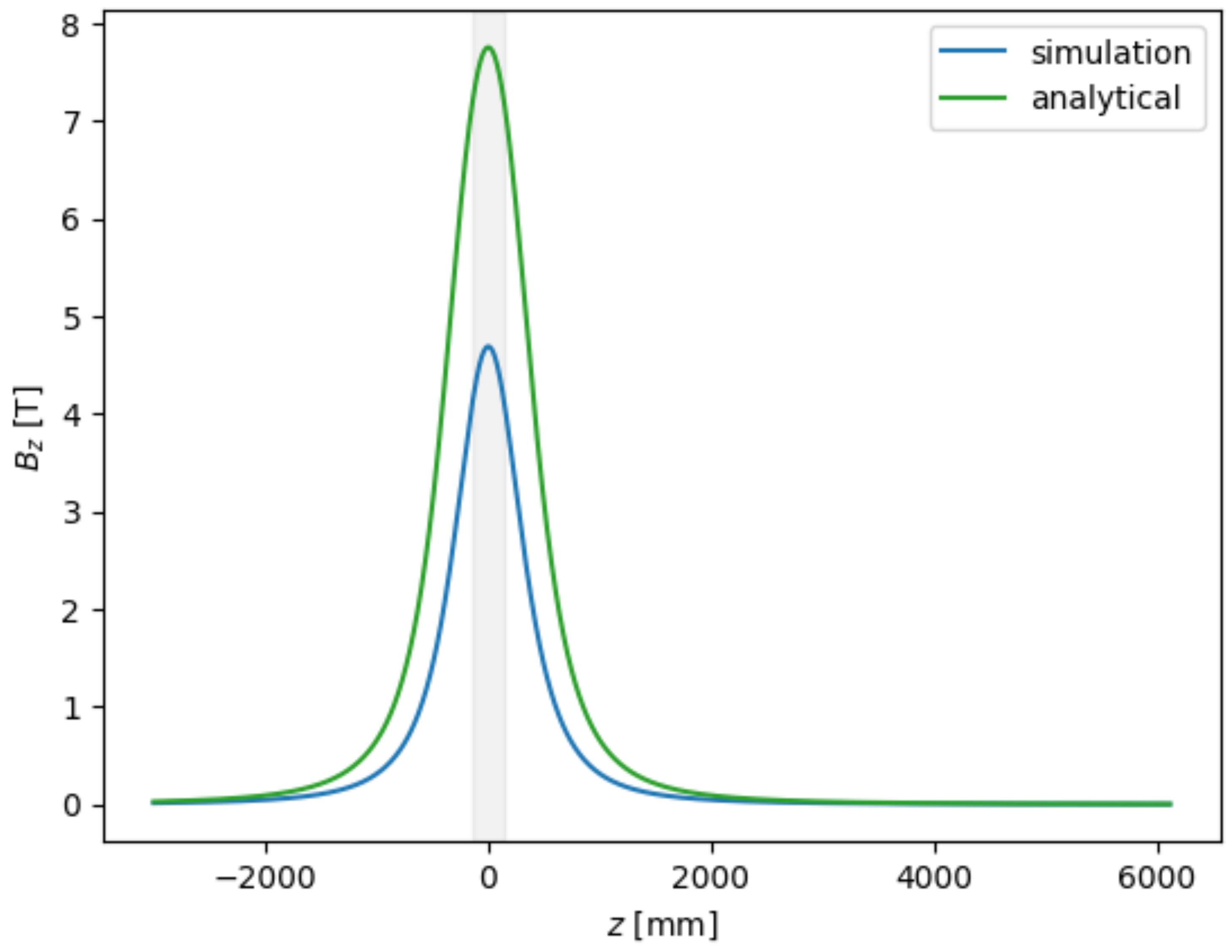


Analytical values appear to underestimate compared to the simulation, but general shape appears (at visual inspection) the same

SINGLE HFOFO COIL - WITH 0.8° PITCH





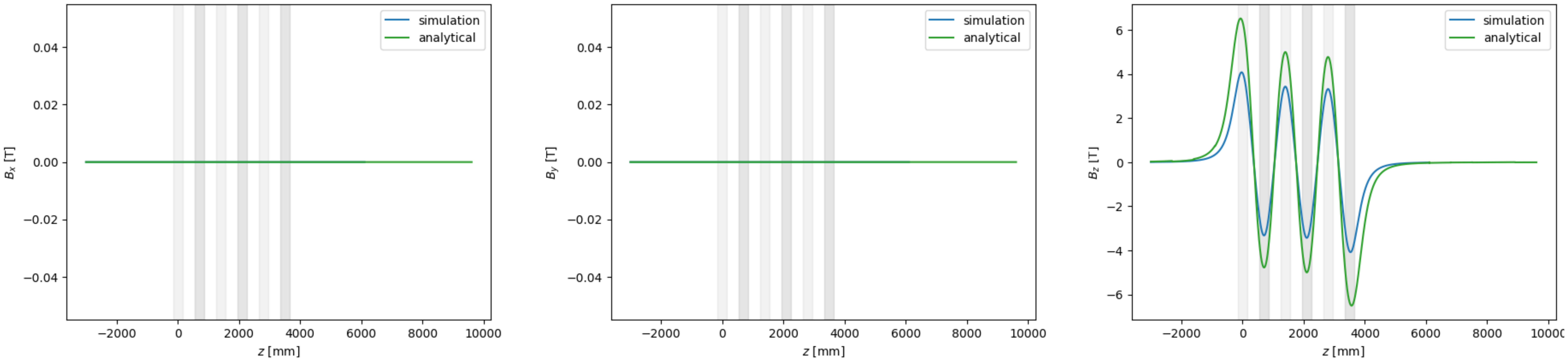


Again see analytical approximation overestimating

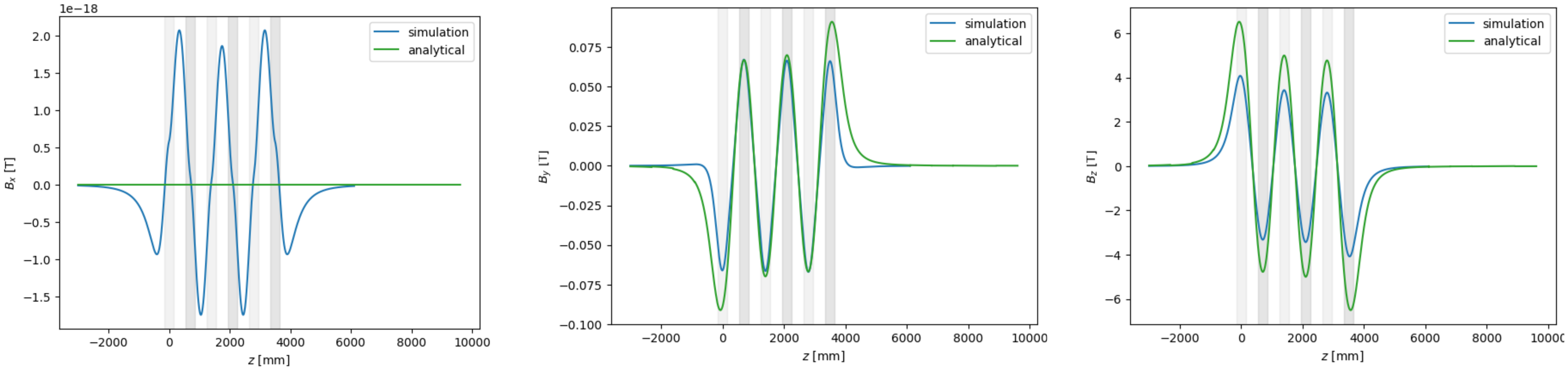
Very small nonzero x-component in simulation which is unexpected

Some deviation in y-component behavior at edges of nonzero region

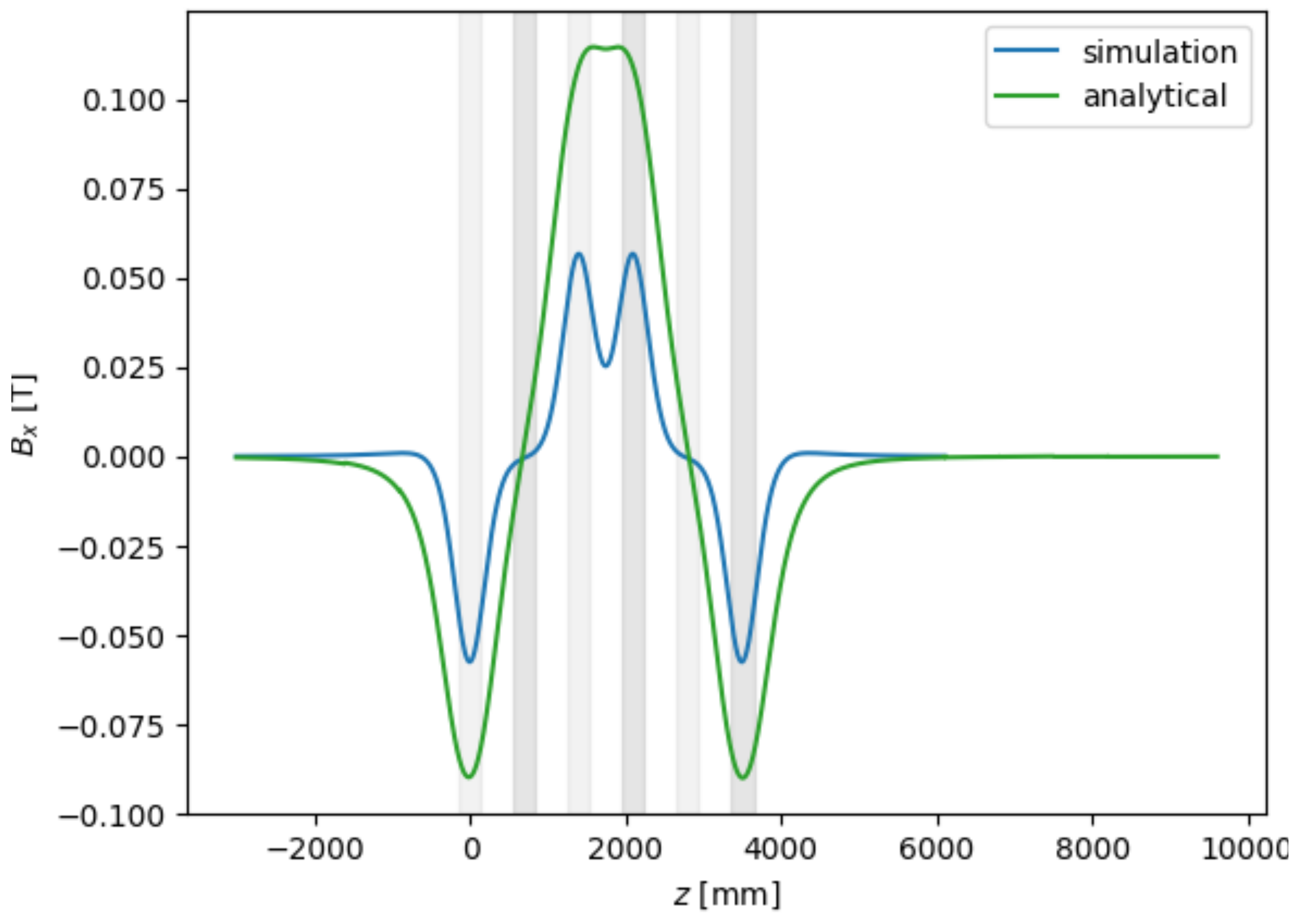
HFOFO PERIOD - NO ROTATIONS

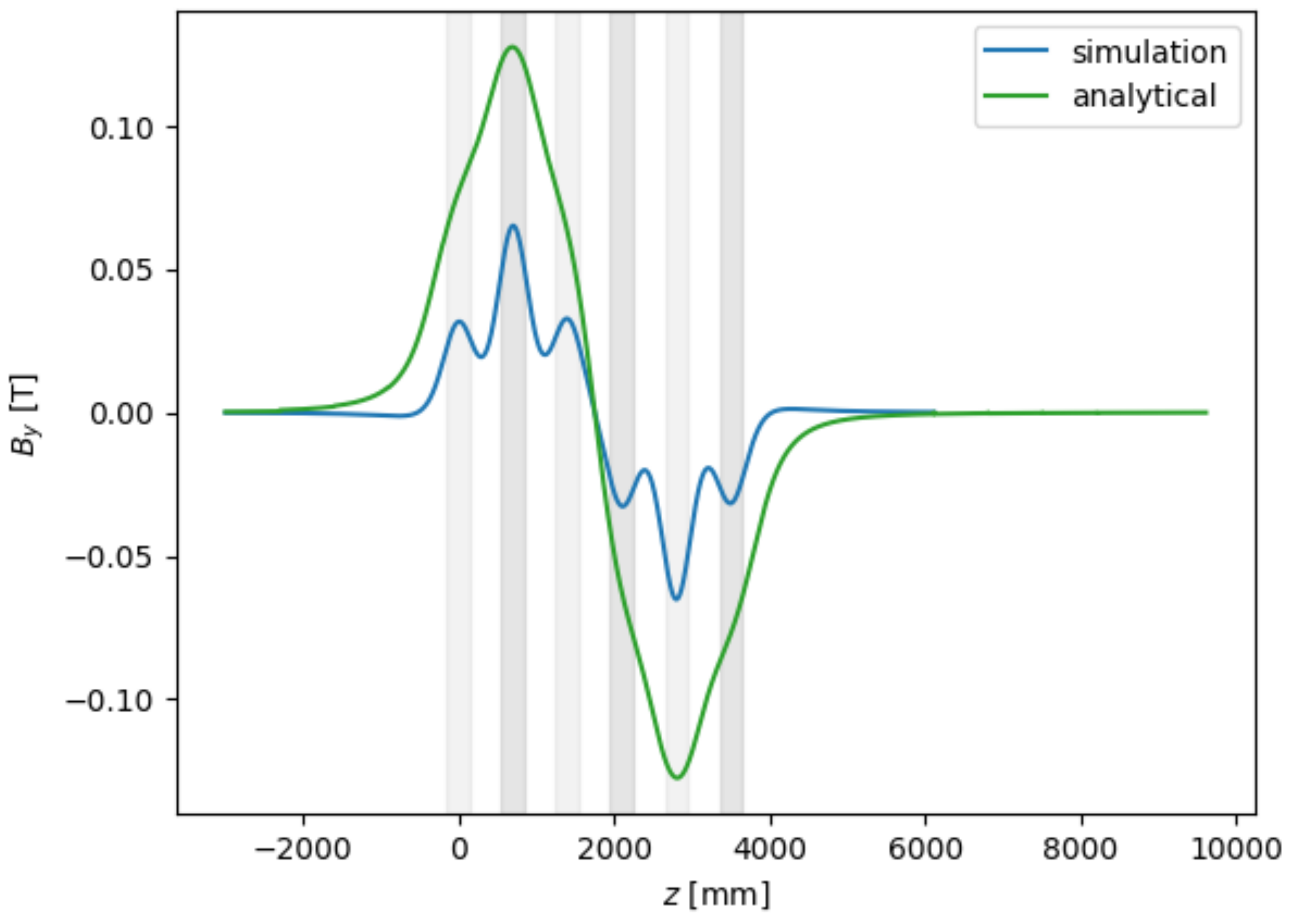


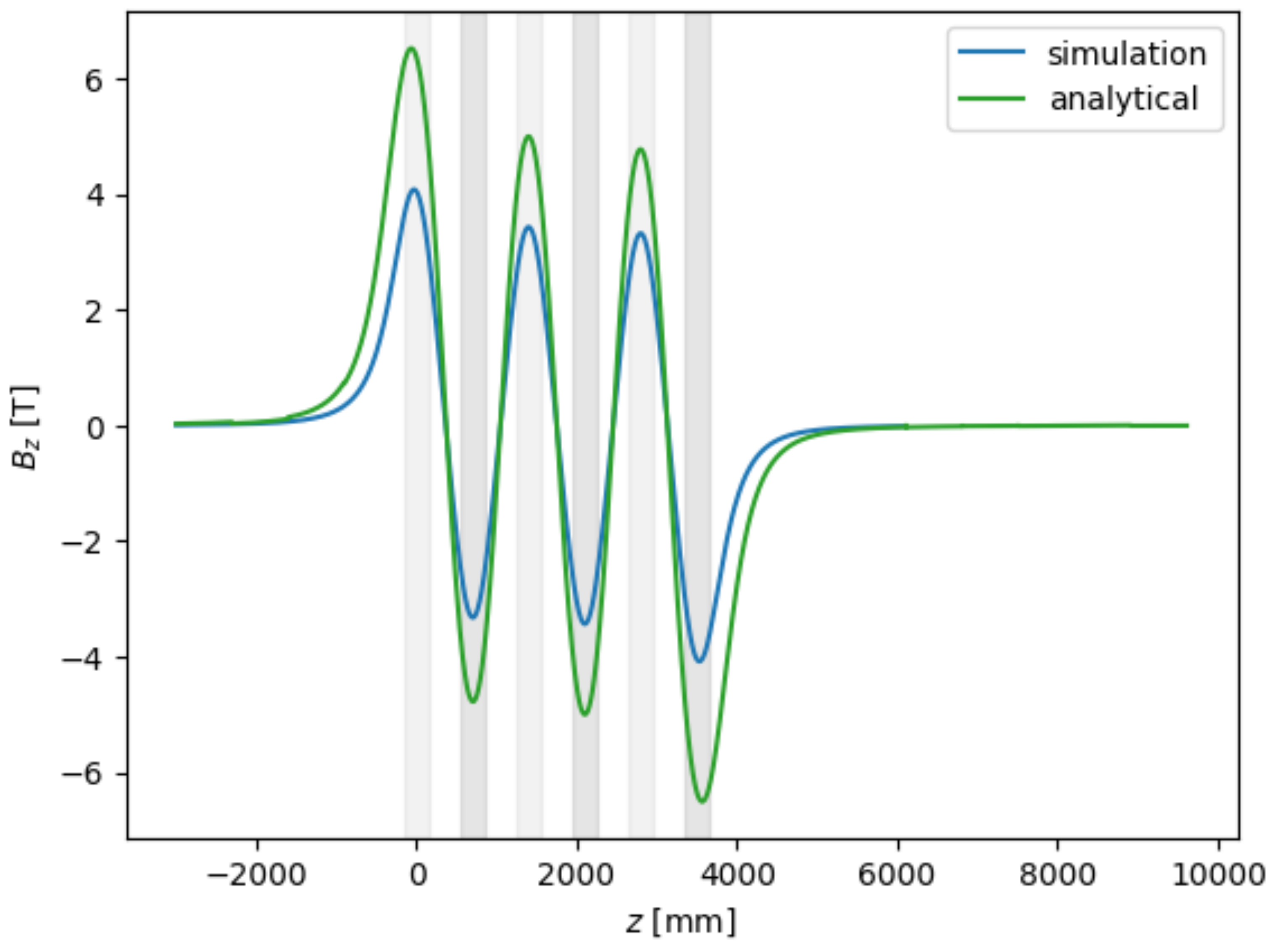
HFOFO PERIOD - WITH 0.8° PITCH



HFOFO PERIOD - WITH 0.8° PITCH + PERIODIC ROLLS

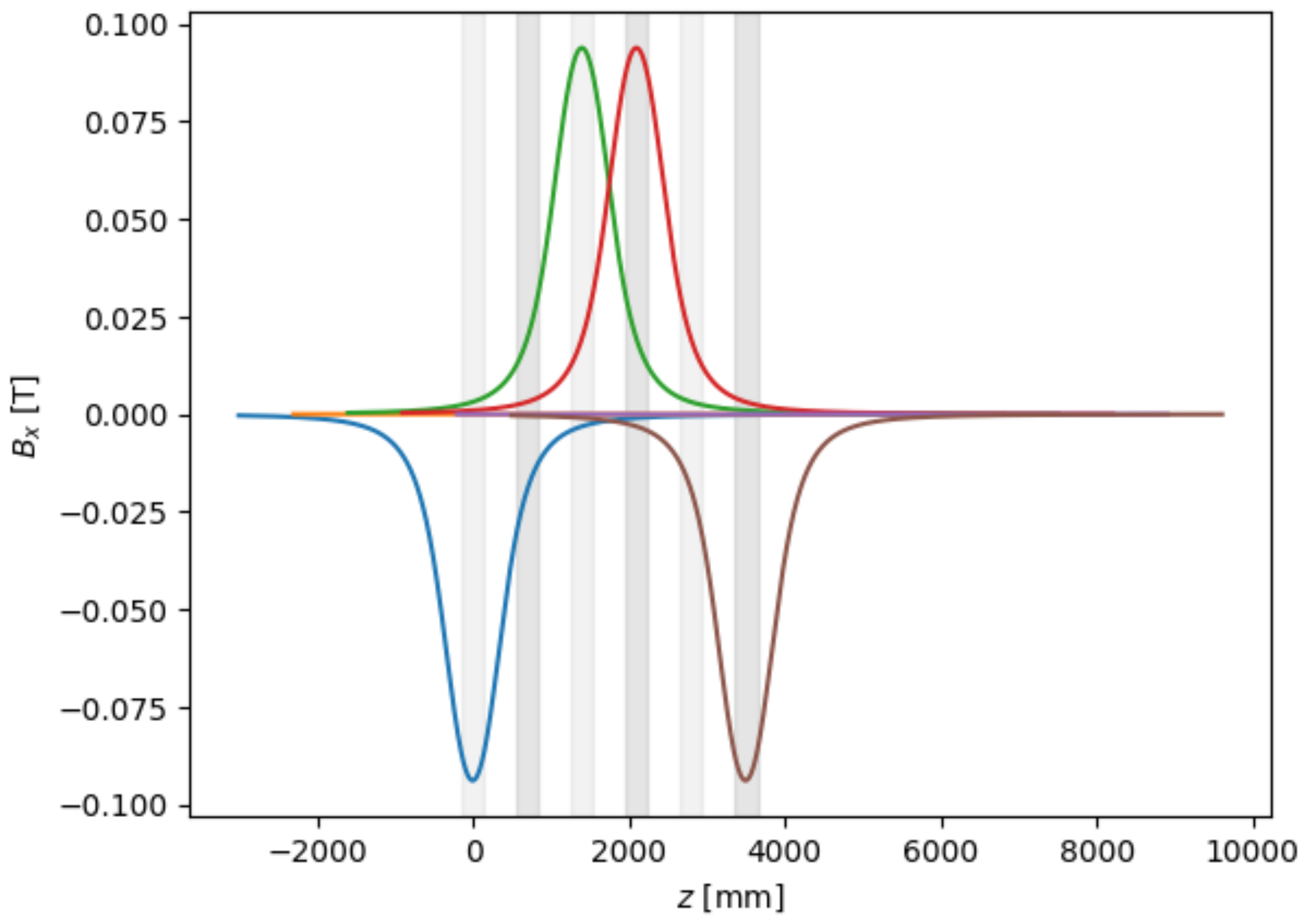


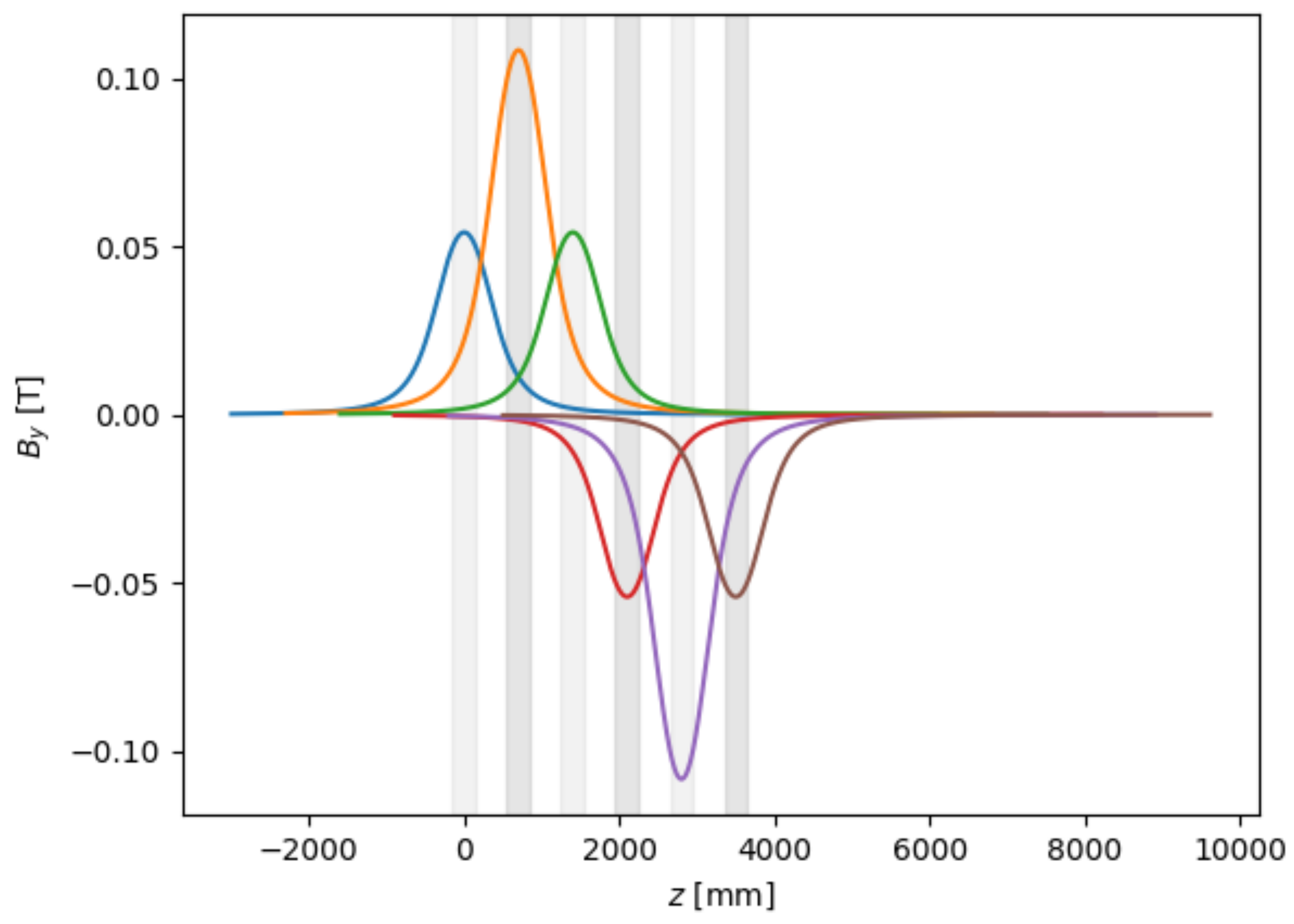




$$\theta_z = 240°, 0°, 120°, 240°, 0°, 120°$$

HFOFO PERIOD - WITH 0.8° PITCH + PERIODIC ROLLS





$$\theta_z = 240^\circ, 0^\circ, 120^\circ, 240^\circ, 0^\circ, 120^\circ$$