

HFOFO PROJECT UPDATES

Weeks of July 14-18 and 21-25, 2025

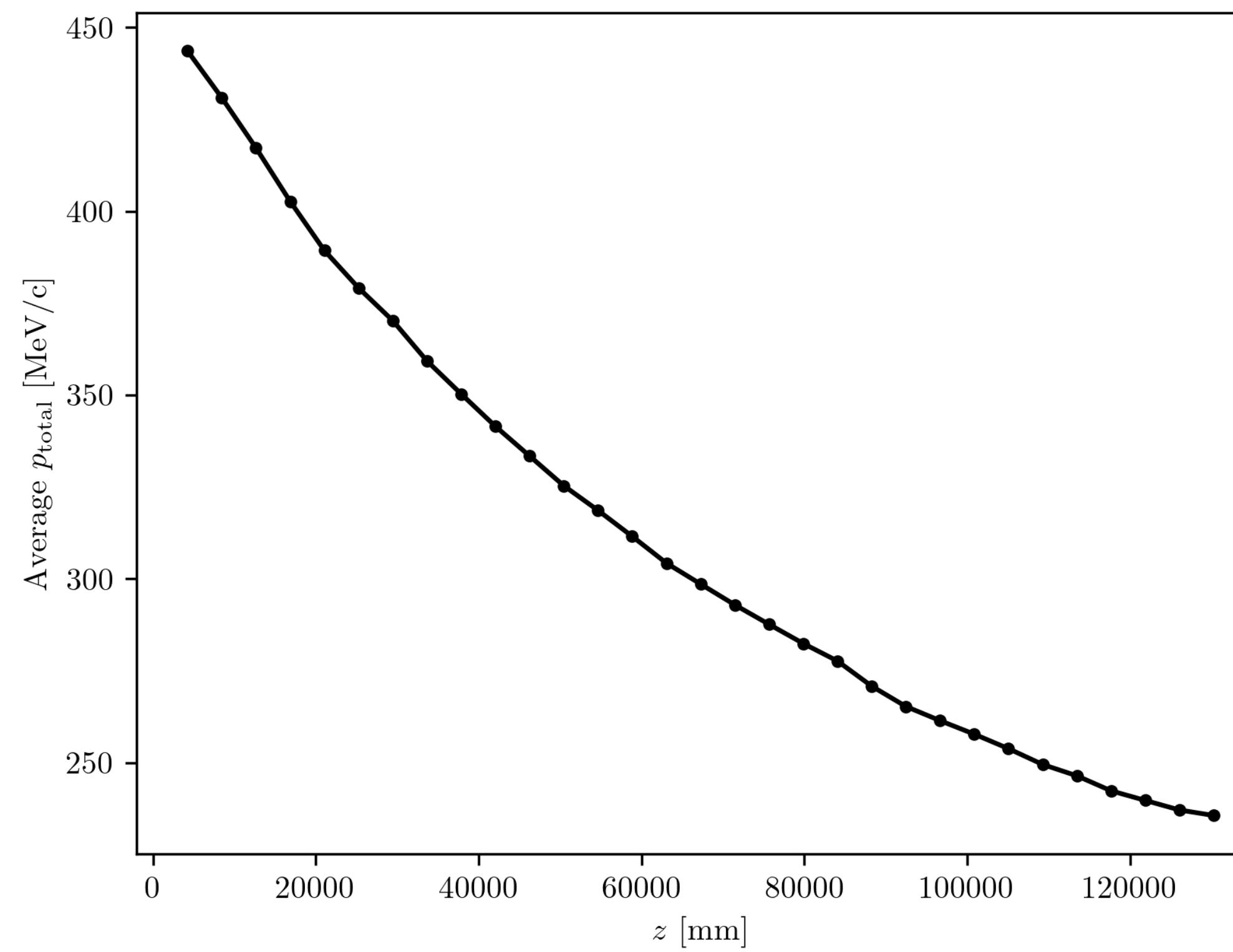
<https://github.com/criggall/muon-cooling/tree/main>

STUDYING MOMENTUM EVOLUTION

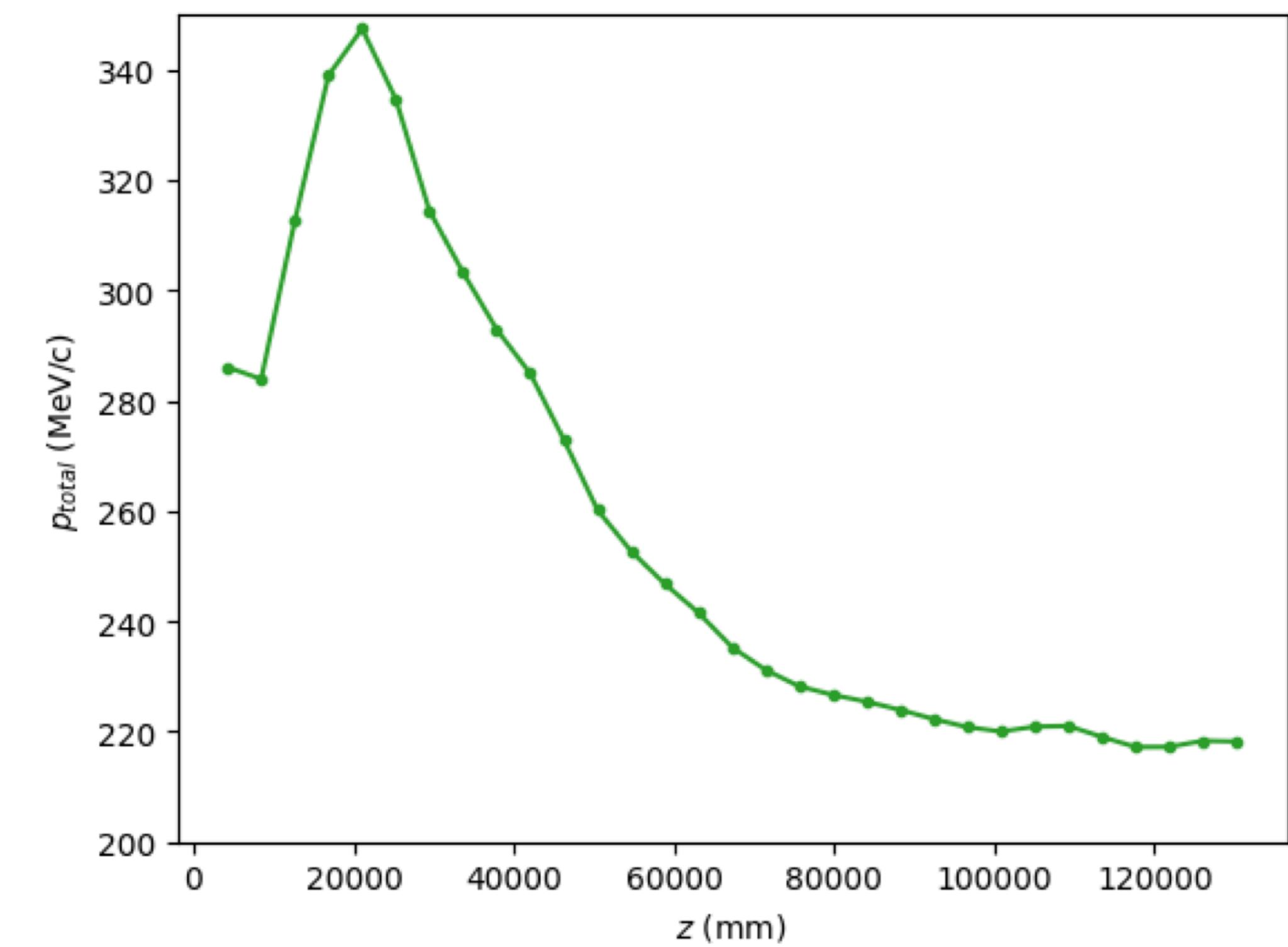
MOMENTUM EVOLUTION

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With the beam used by Yuri:



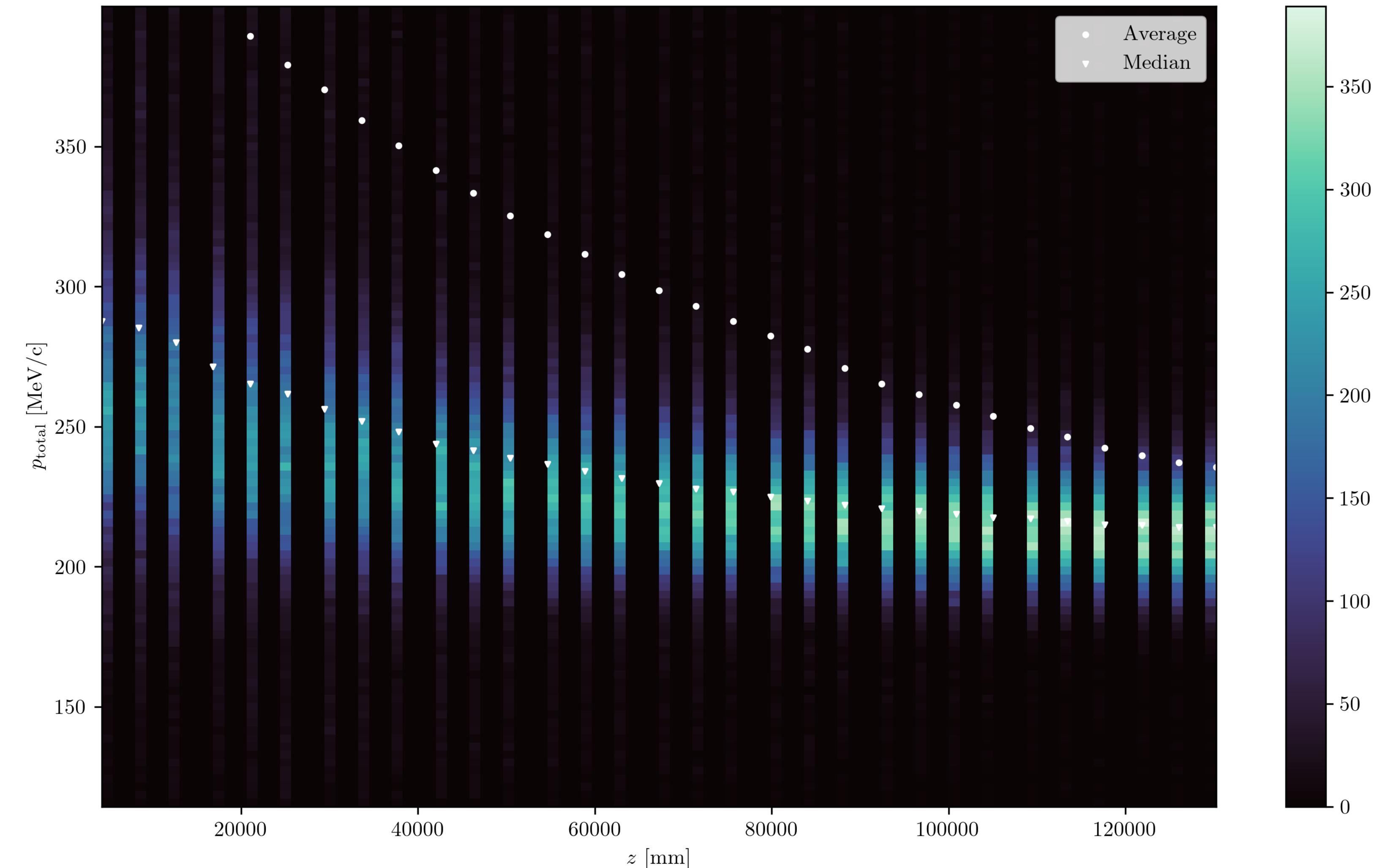
From before, with the file Diktys provided:



Note: It would be interesting to compare initial and final phase space distributions as well

MOMENTUM EVOLUTION

4

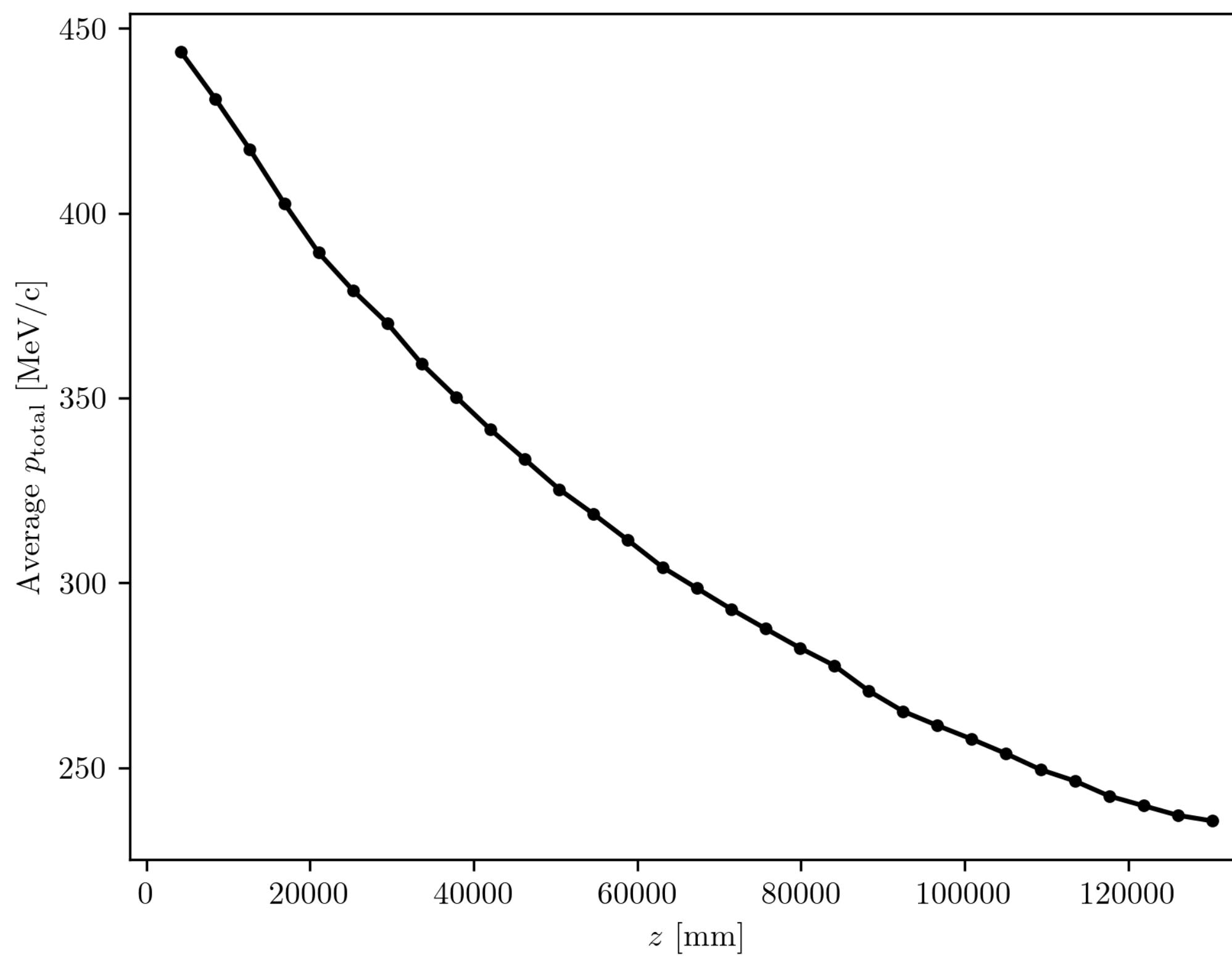


(Yuri's file)

MOMENTUM EVOLUTION

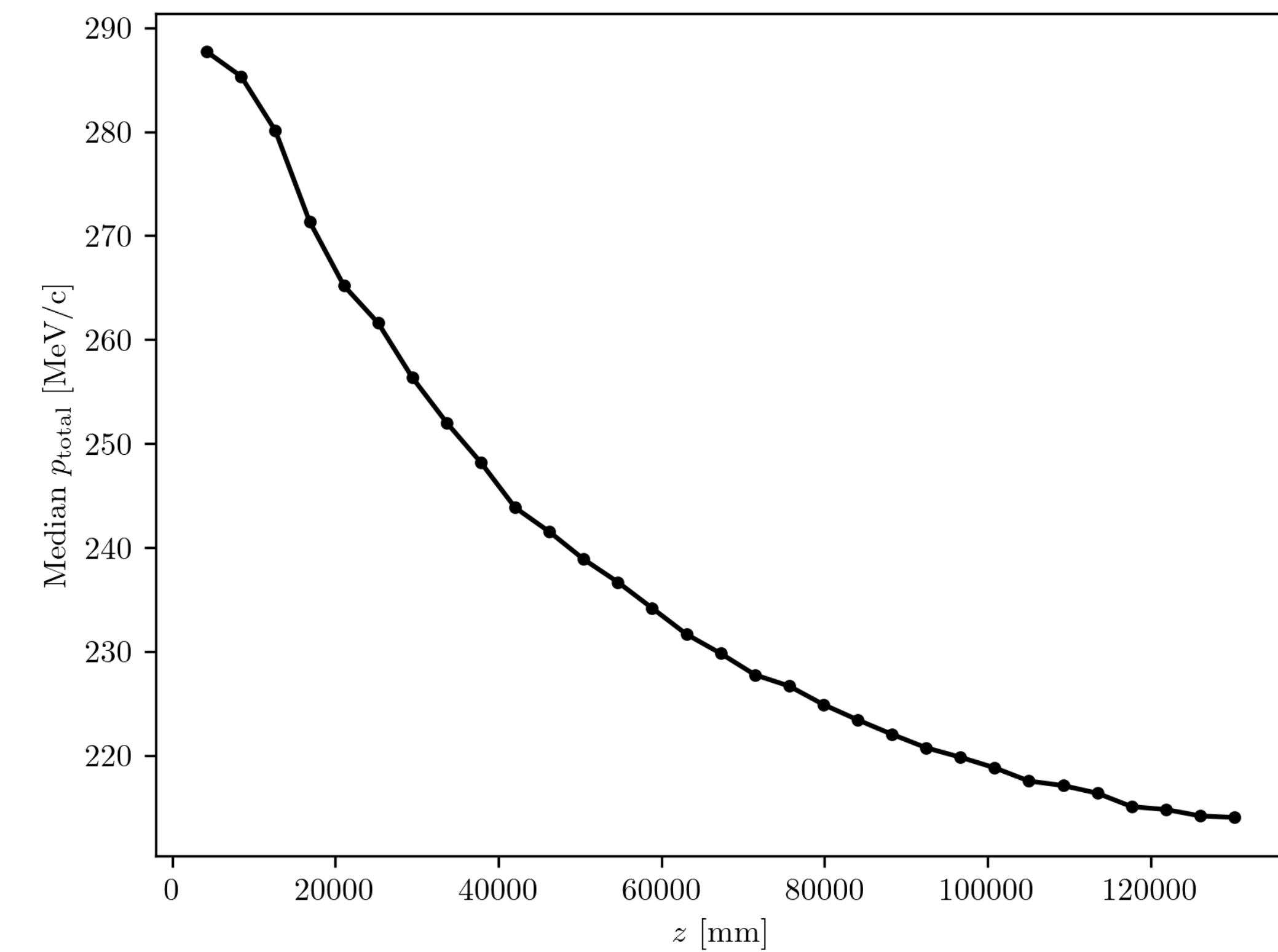
Initial: 444 MeV/c

Final: 236 MeV/c



Initial: 288 MeV/c

Final: 214 MeV/c



Note: It would be interesting to consider RMS values as well

(Yuri's file)

MAPPING THE MAGNETIC FIELD

For a single solenoid with HFOFO coil geometry, no tilts

ANALYTICAL APPROACH

ANALYTIC FIELD MAP - NO ROTATION

present. The field component for current sheets can be expressed in closed form in terms of elliptic integrals [16]. Let us define the functions

$$b_z(r, z) = \frac{\mu_0 I'}{\pi} \frac{za}{\zeta(a+r)} \left[K(k) + \frac{a-r}{2a} (\Pi(k, c) - K(k)) \right]$$

$$b_r(r, z) = \frac{\mu_0 I'}{\pi} \frac{\zeta}{4r} [2(K(k) - E(k)) - k^2 K(k)],$$

where I' is the current per unit length and we use the auxiliary quantities

$$k = \sqrt{\frac{4ar}{(a+r)^2 + z^2}} \quad \zeta = \sqrt{(a+r)^2 + z^2}$$

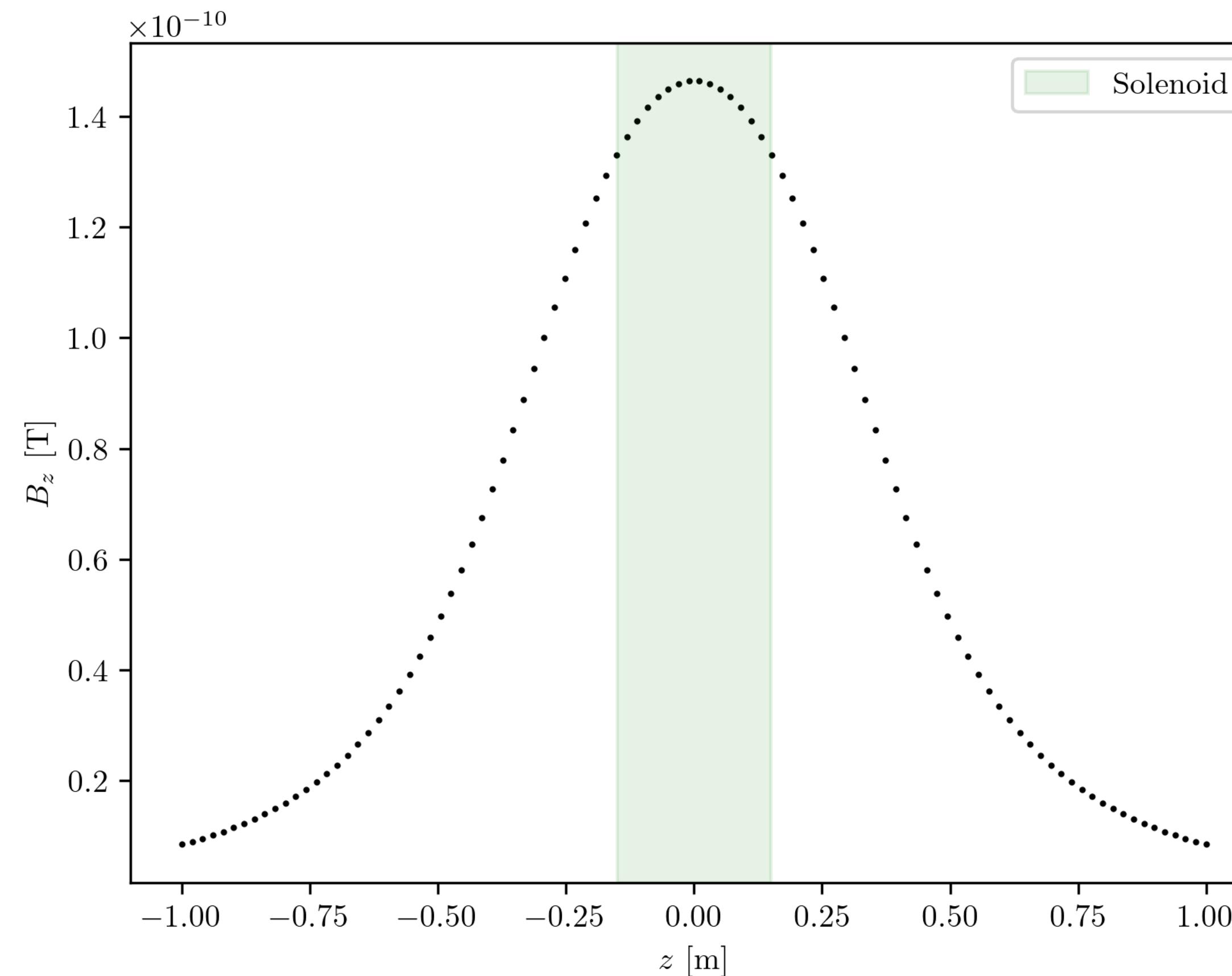
$$c = -\frac{4ar}{(a+r)^2}.$$

$K(k)$, $E(k)$, and $\Pi(k, c)$ are complete elliptic integrals. The magnetic field from the solenoidal sheet is given in terms of these functions by

$$B_z(r, z) = -b_z(r, z-L) + b_z(r, z+L)$$

$$B_r(r, z) = b_r(r, z-L) - b_r(r, z+L).$$

A quick sanity check of the on-axis B field:



ANALYTIC FIELD MAP - NO ROTATION

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present. The field component for current sheets can be expressed in closed form in terms of elliptic integrals [16]. Let us define the functions

$$b_z(r, z) = \frac{\mu_0 I'}{\pi} \frac{za}{\zeta(a+r)} \left[K(k) + \frac{a-r}{2a} (\Pi(k, c) - K(k)) \right]$$

$$b_r(r, z) = \frac{\mu_0 I'}{\pi} \frac{\zeta}{4r} [2(K(k) - E(k)) - k^2 K(k)],$$

where I' is the current per unit length and we use the auxiliary quantities

$$k = \sqrt{\frac{4ar}{(a+r)^2 + z^2}} \quad \zeta = \sqrt{(a+r)^2 + z^2}$$

$$c = -\frac{4ar}{(a+r)^2}.$$

$K(k)$, $E(k)$, and $\Pi(k, c)$ are complete elliptic integrals. The magnetic field from the solenoidal sheet is given in terms of these functions by

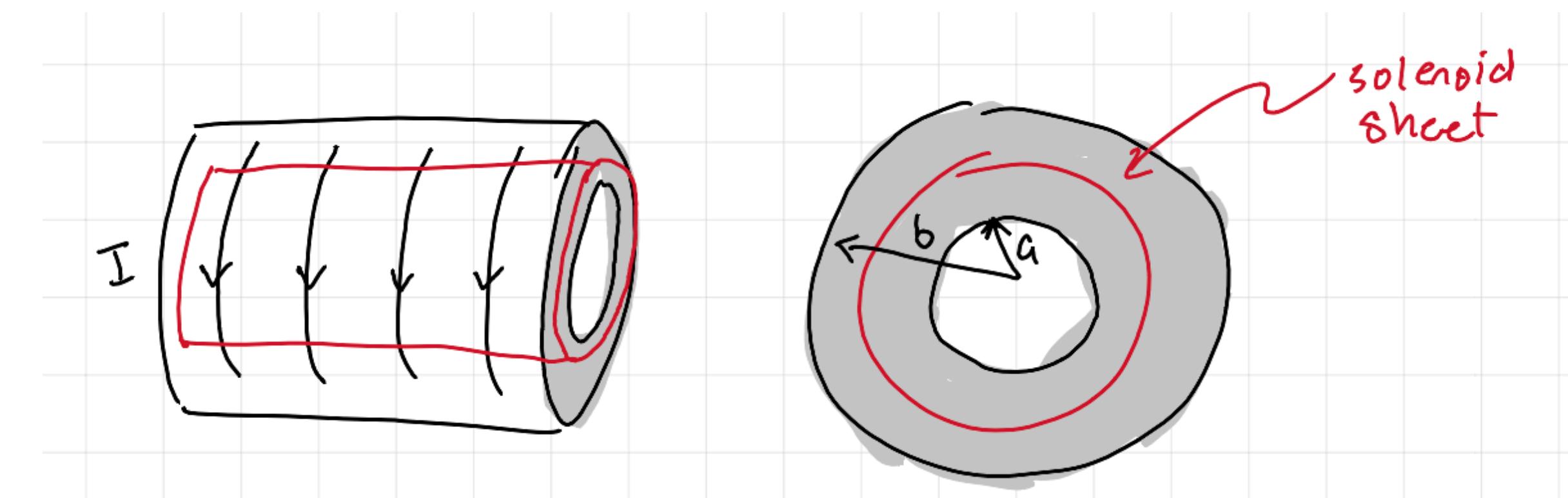
$$B_z(r, z) = -b_z(r, z - L) + b_z(r, z + L)$$

$$B_r(r, z) = b_r(r, z - L) - b_r(r, z + L).$$

A brief definitional note:

For a single solenoidal sheet, $I' = 2\pi r J$ where J is the current density and r is the radius

∴ Integrating over solenoid thickness will give the total field for the solenoid

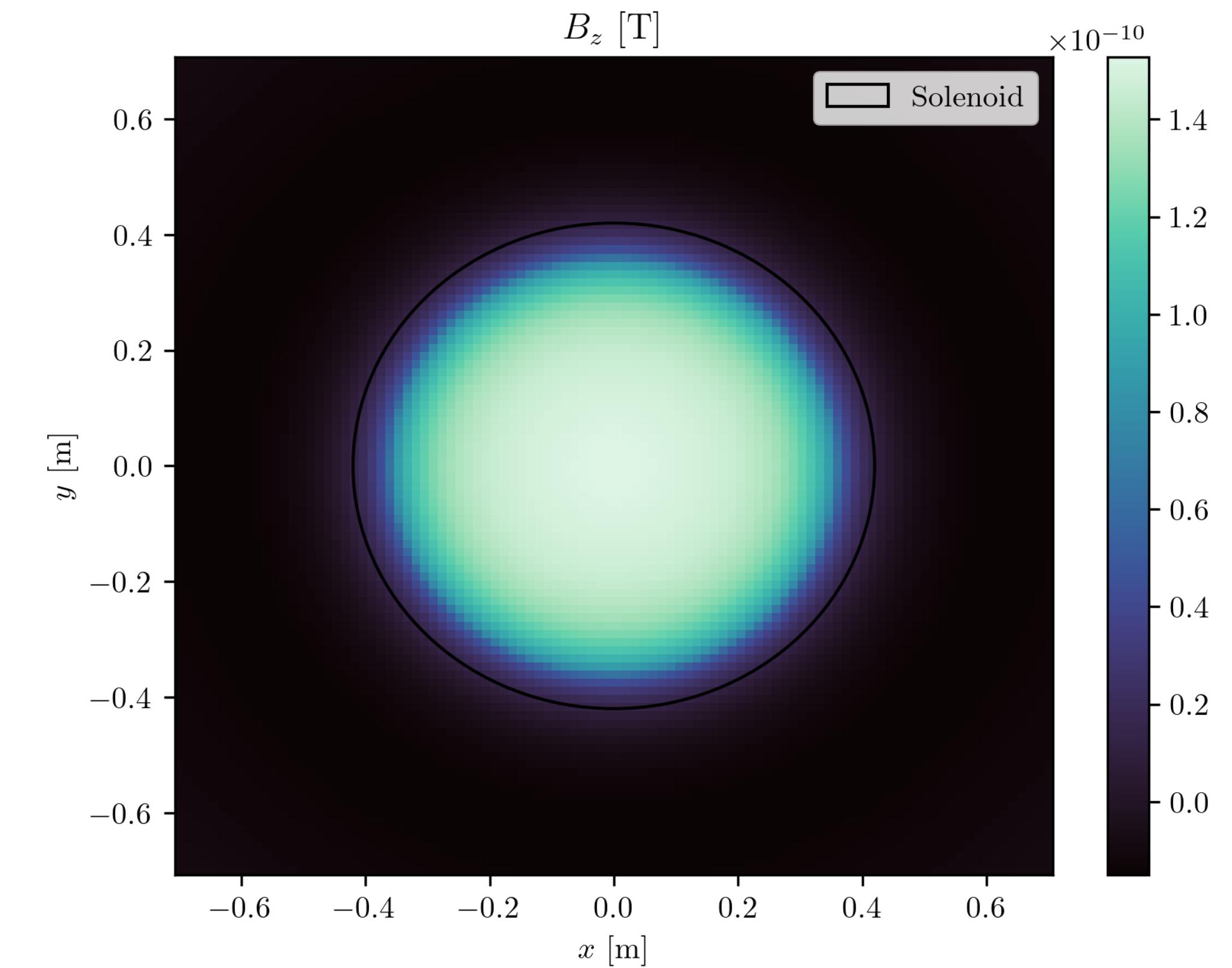
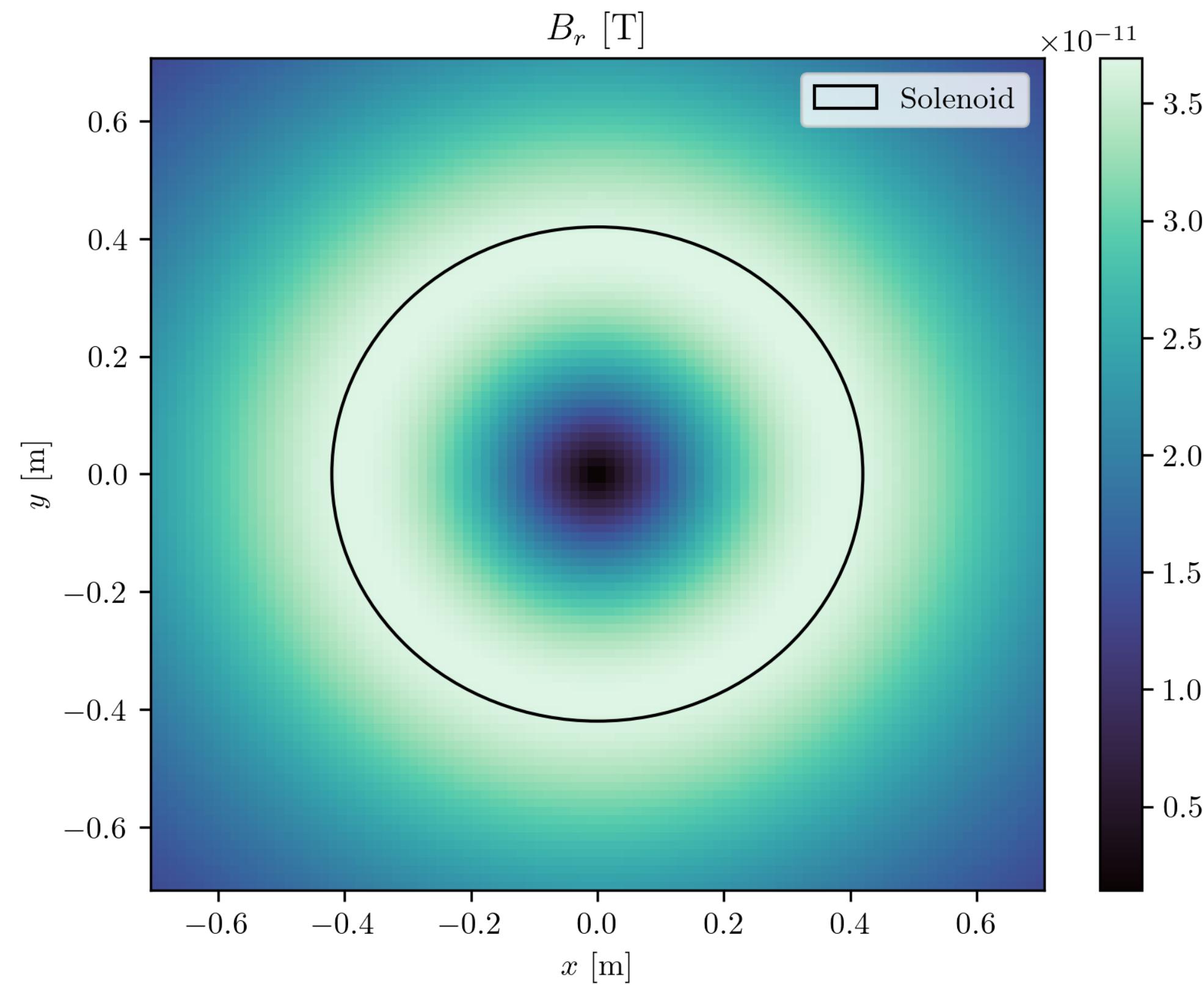


I have only used a single sheet to obtain the results here, but in the future I will add more to improve the calculation (G4beamline input uses 10)

ANALYTIC FIELD MAP - NO ROTATION

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Considering a transverse slice at $z=0$:

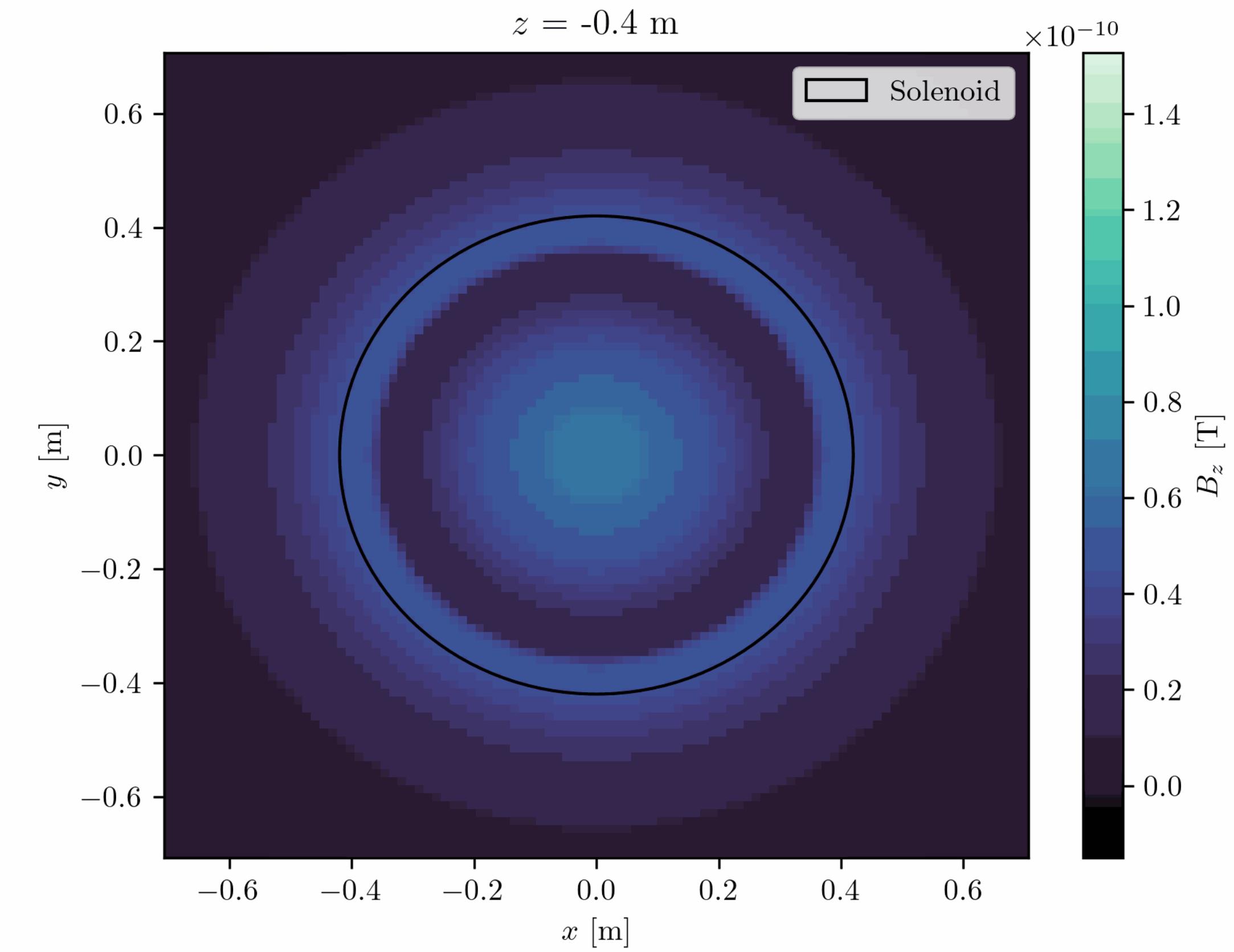
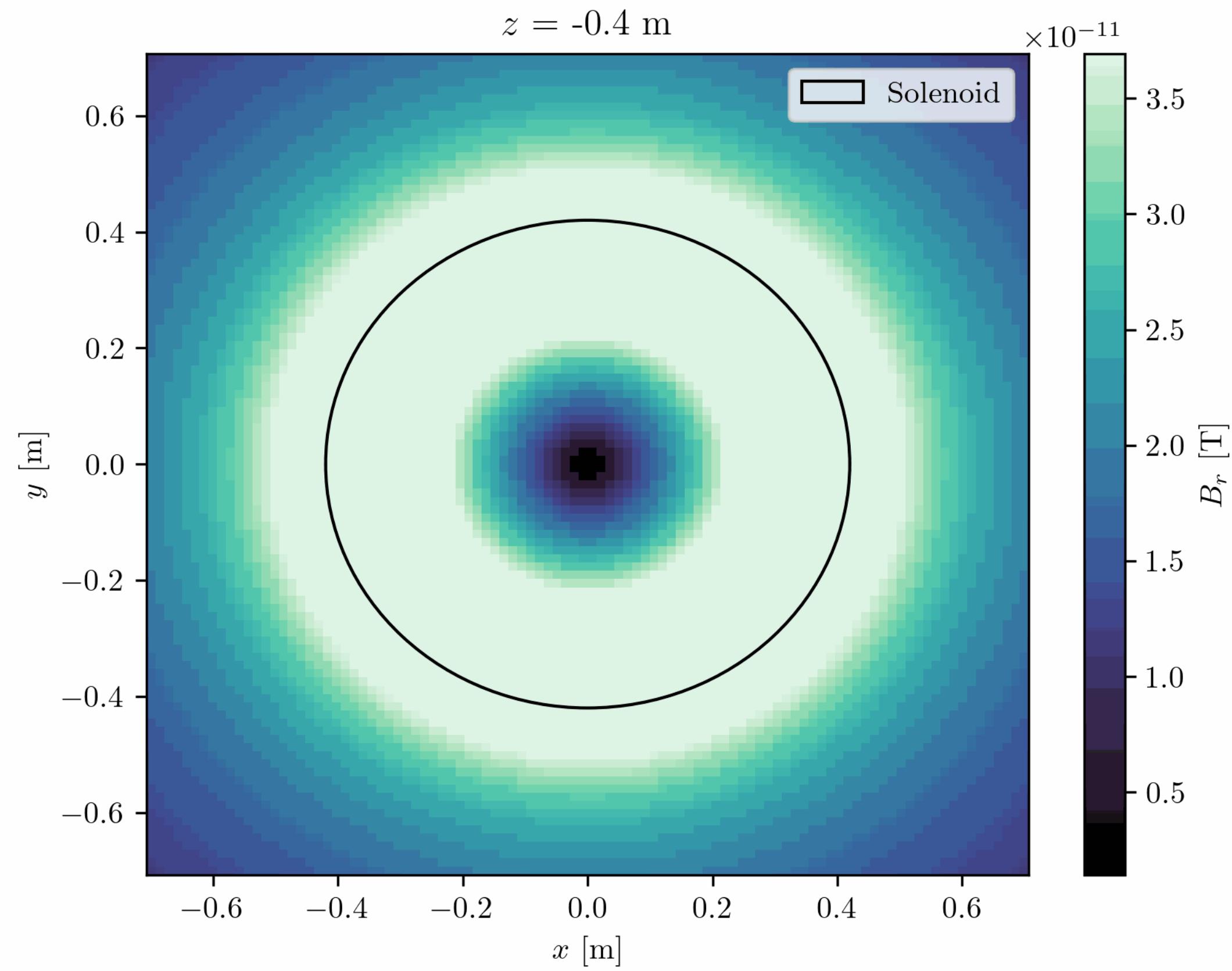


Why is B_r nonzero at the center of the solenoid?

ANALYTIC FIELD MAP - NO ROTATION

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Animation along z — note that solenoid is of length 0.3 m and is centered at $z=0$:

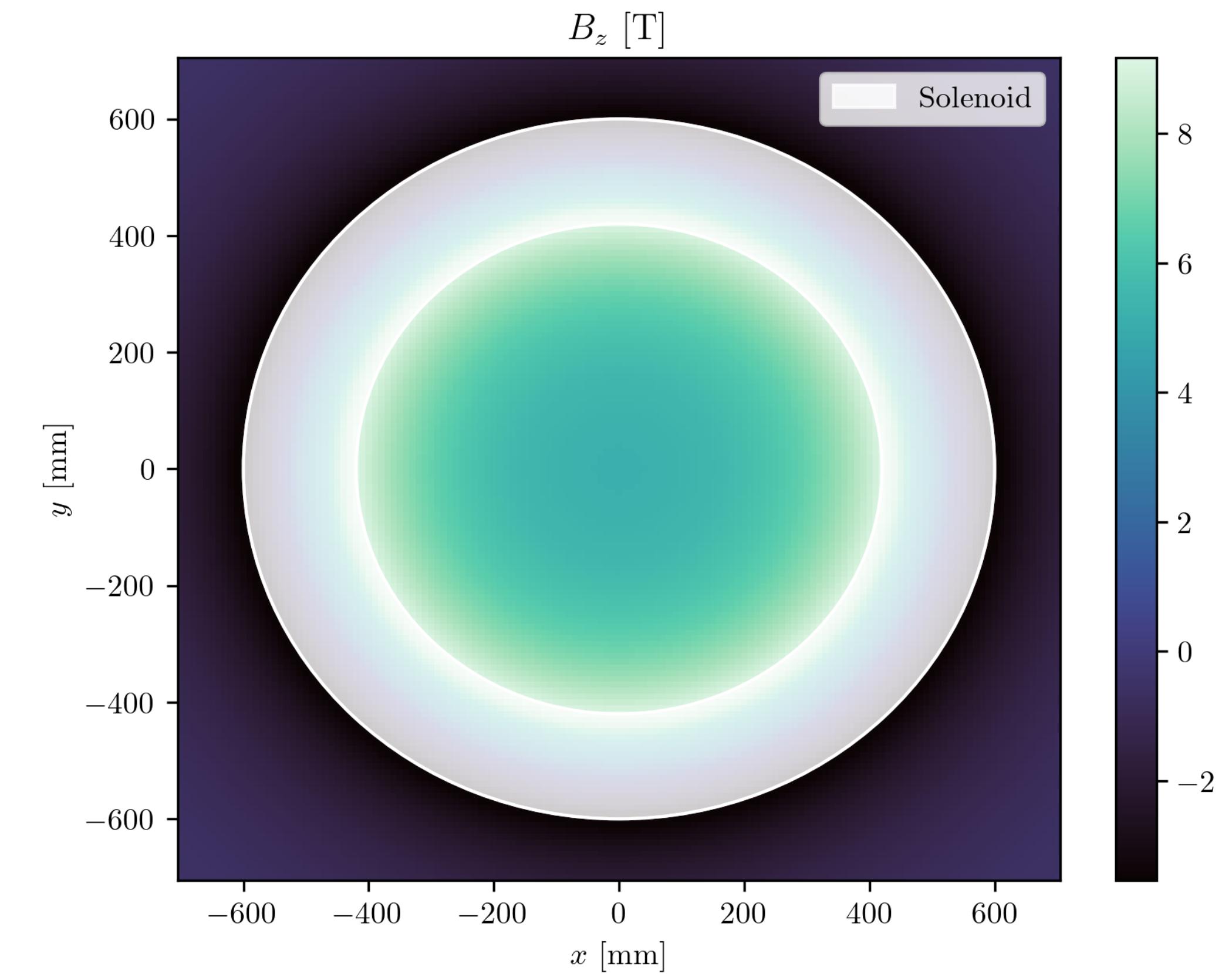
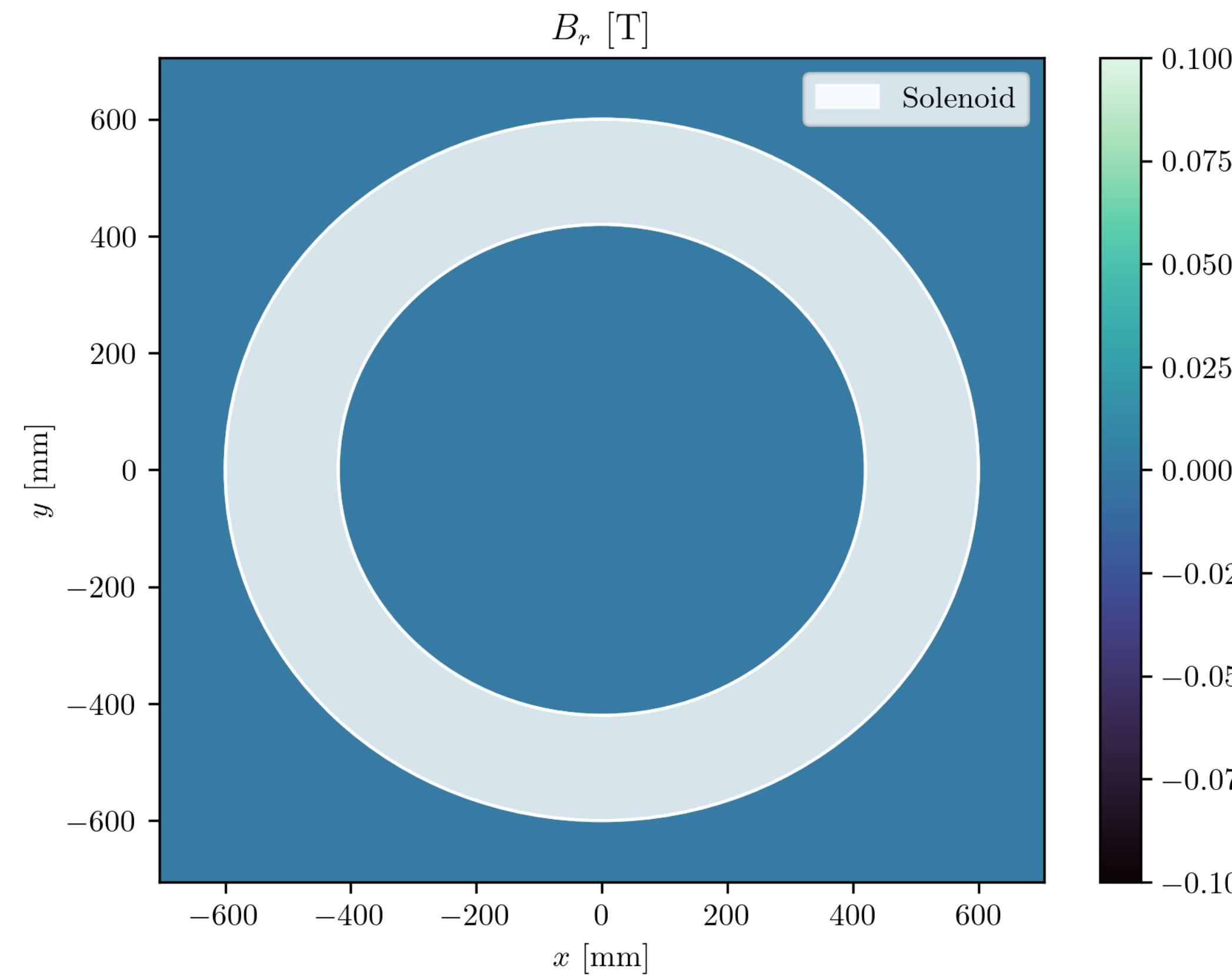


SIMULATION

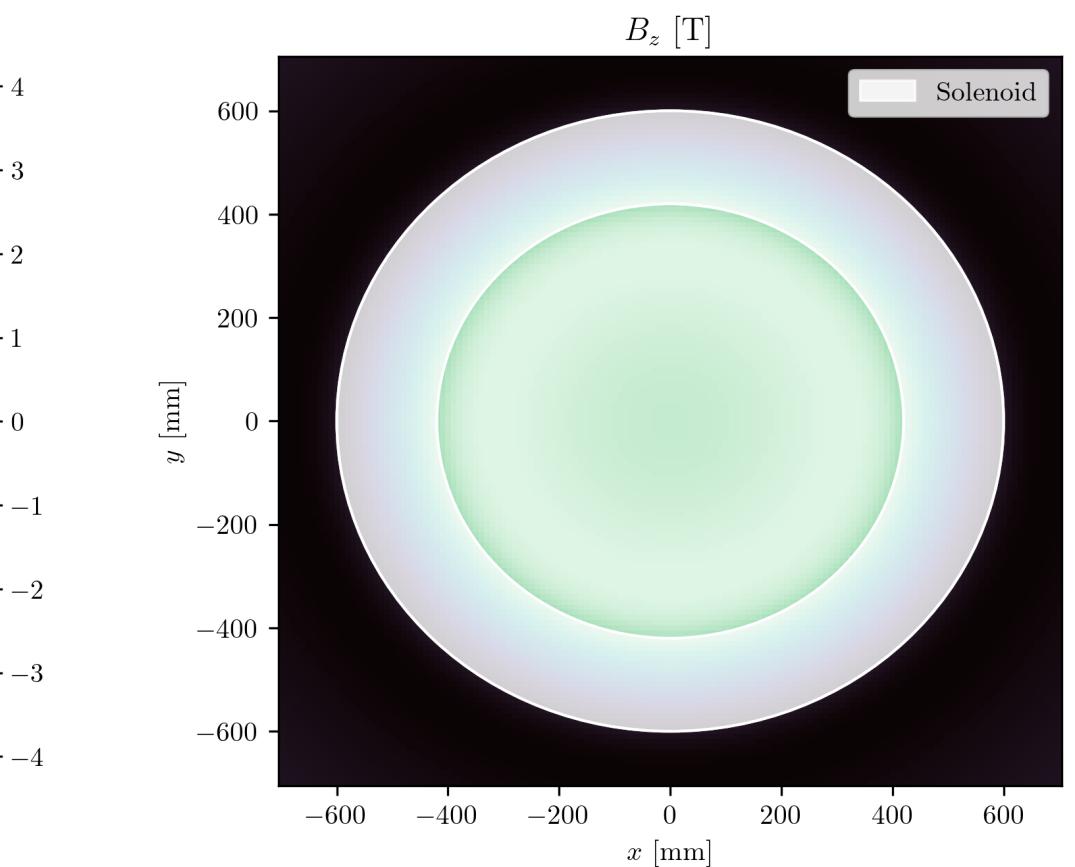
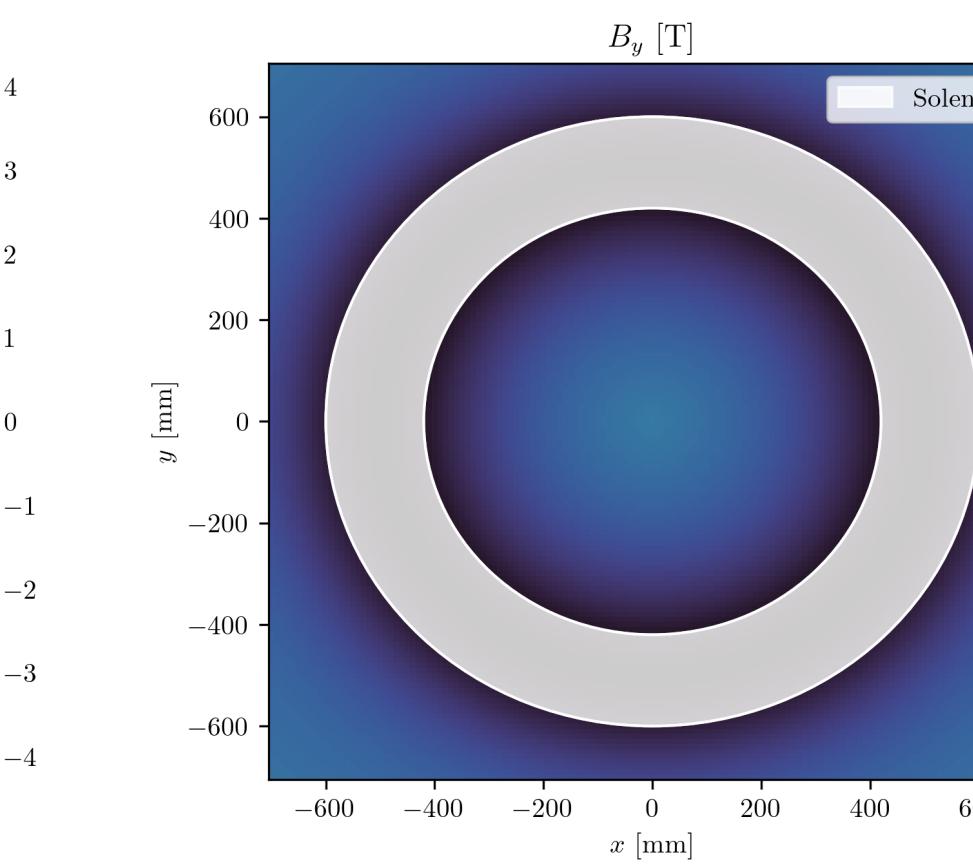
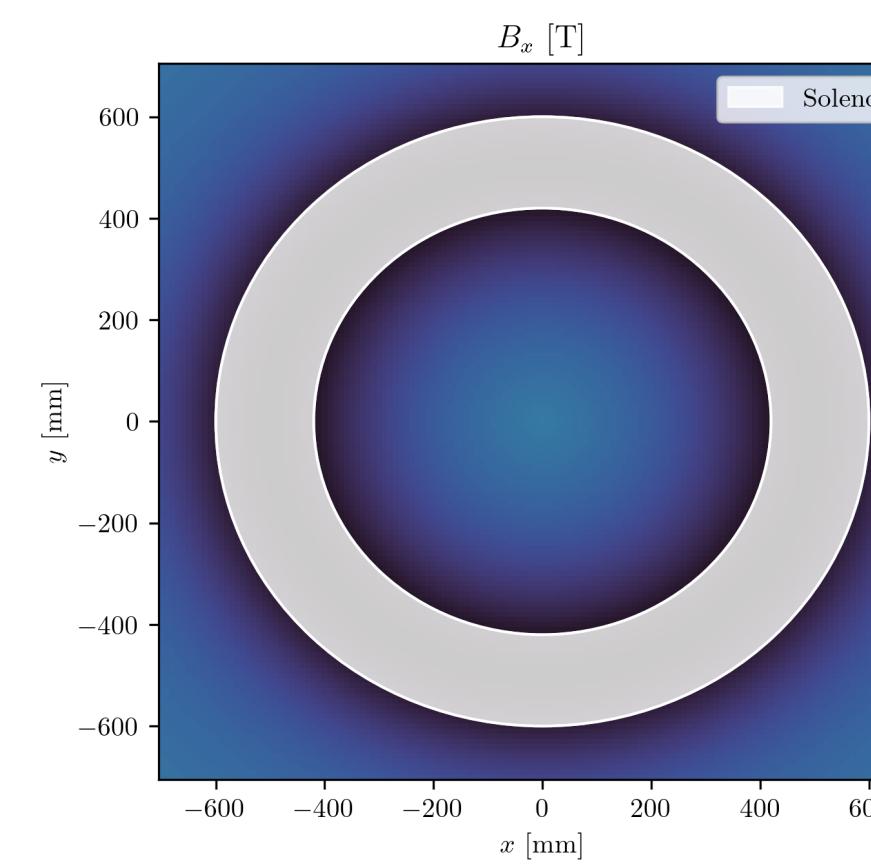
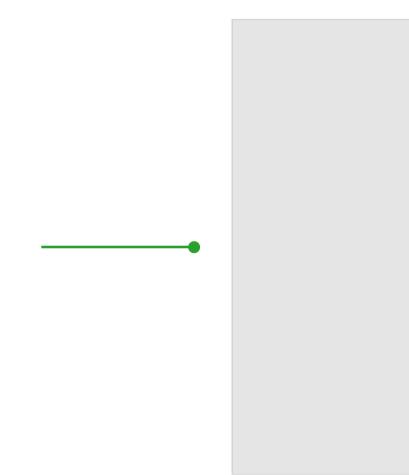
FIELD MAP FROM G4BL - NO ROTATION

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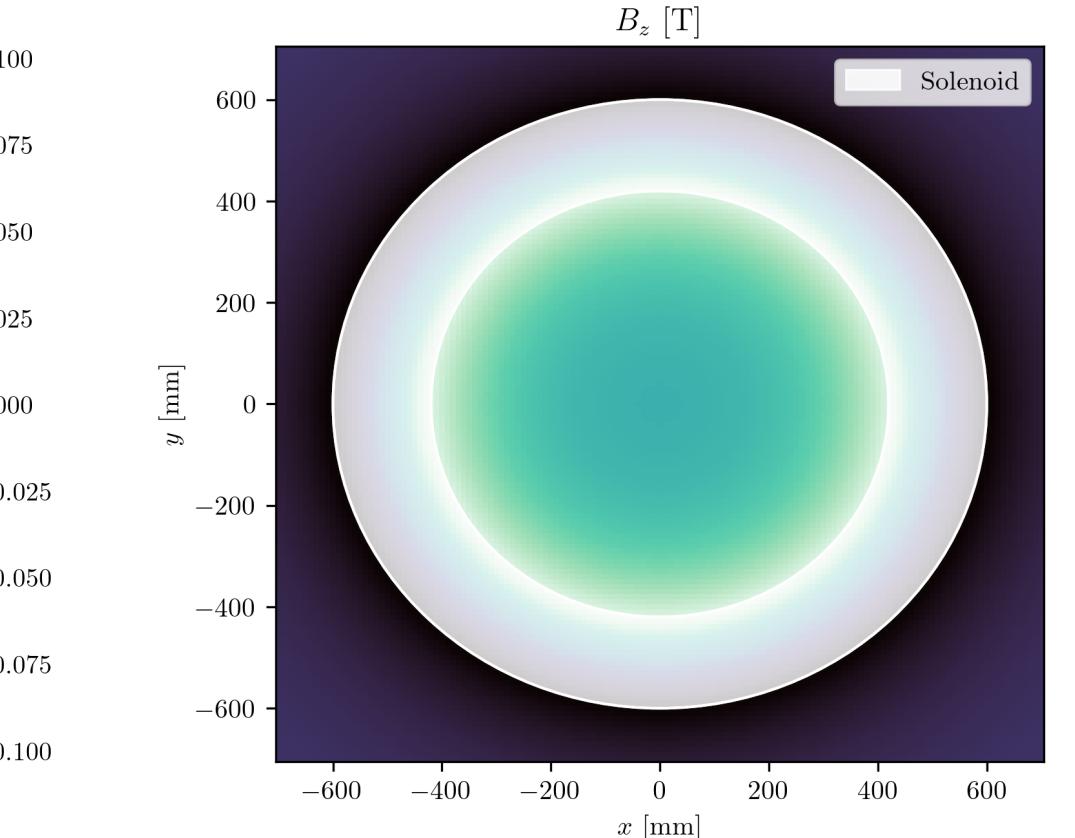
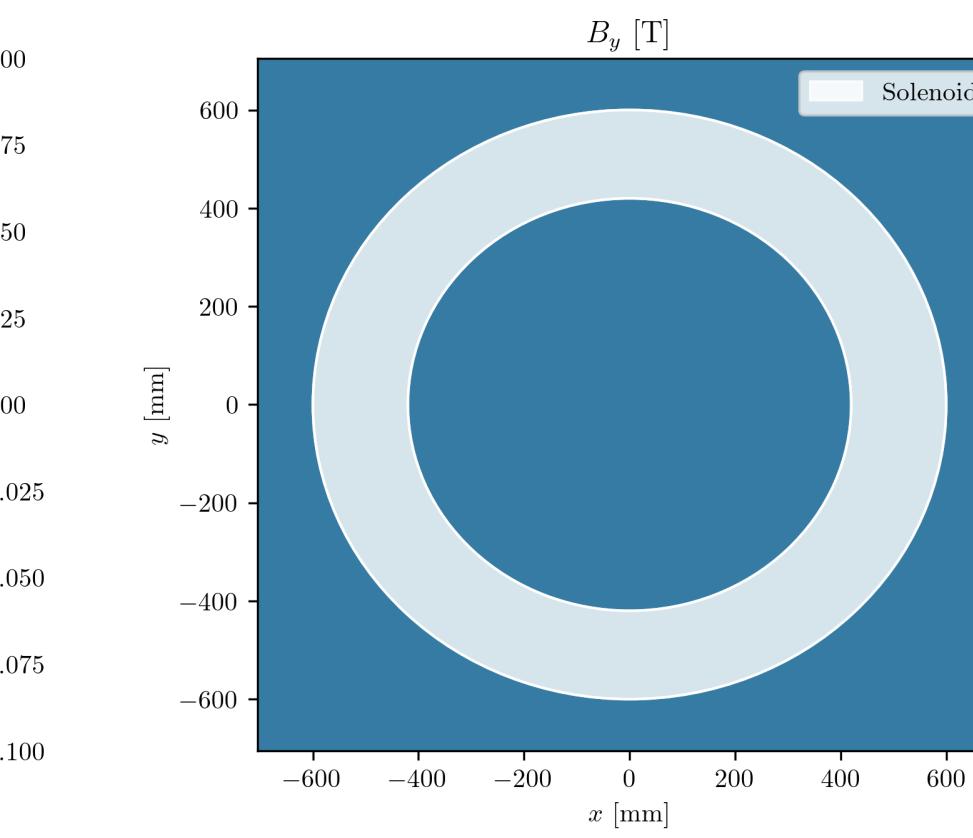
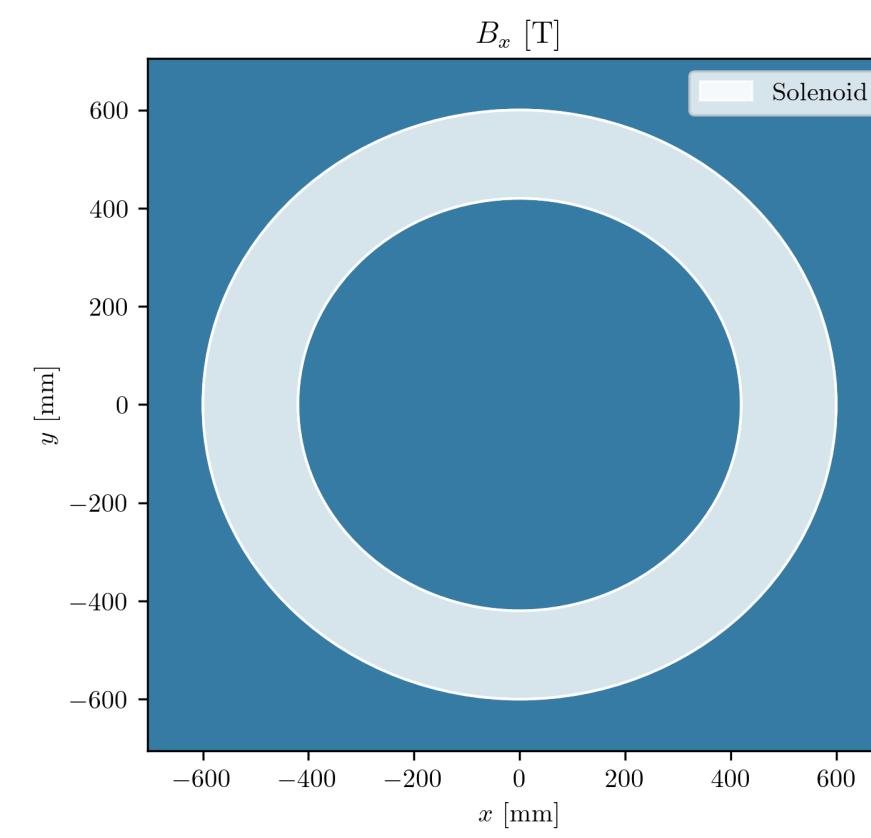
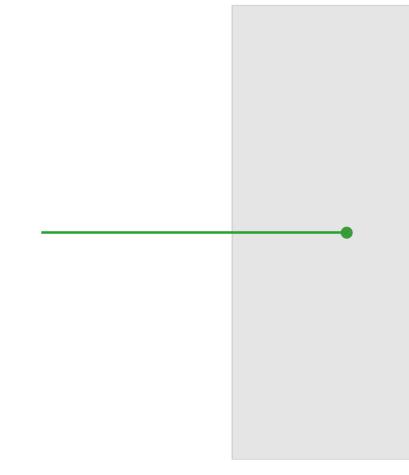
Considering a transverse slice at $z=0$:



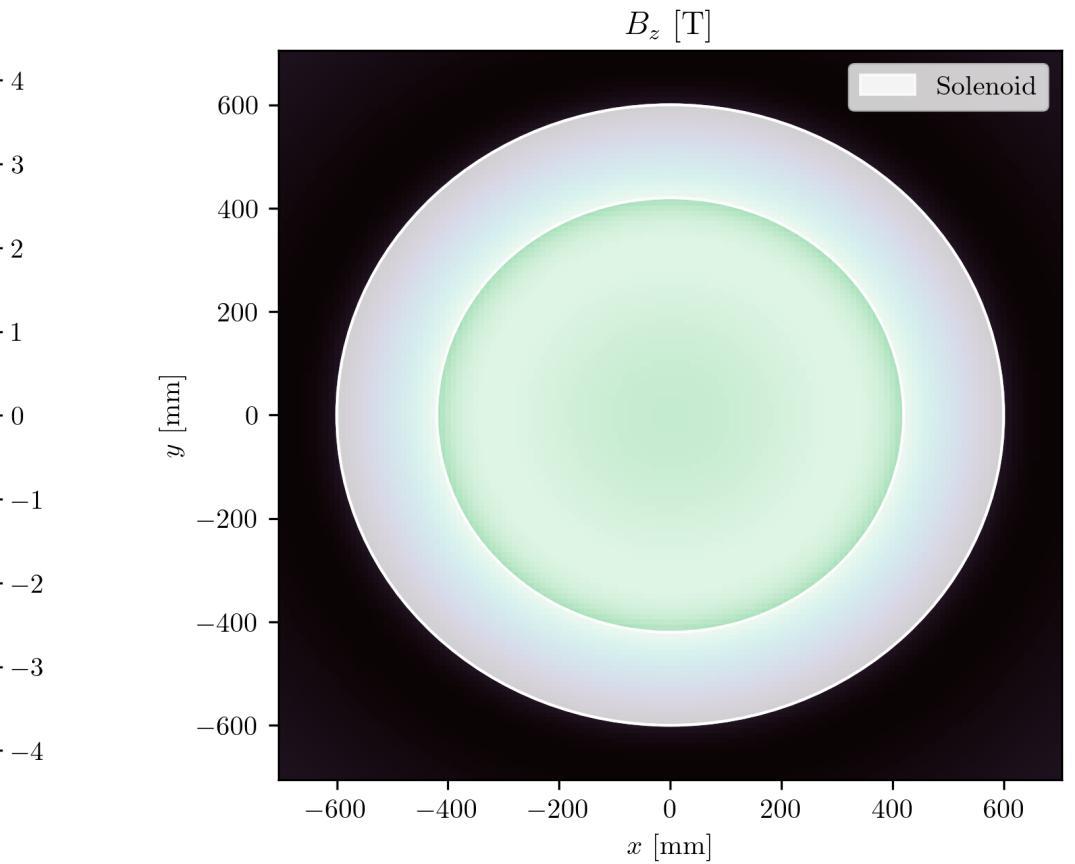
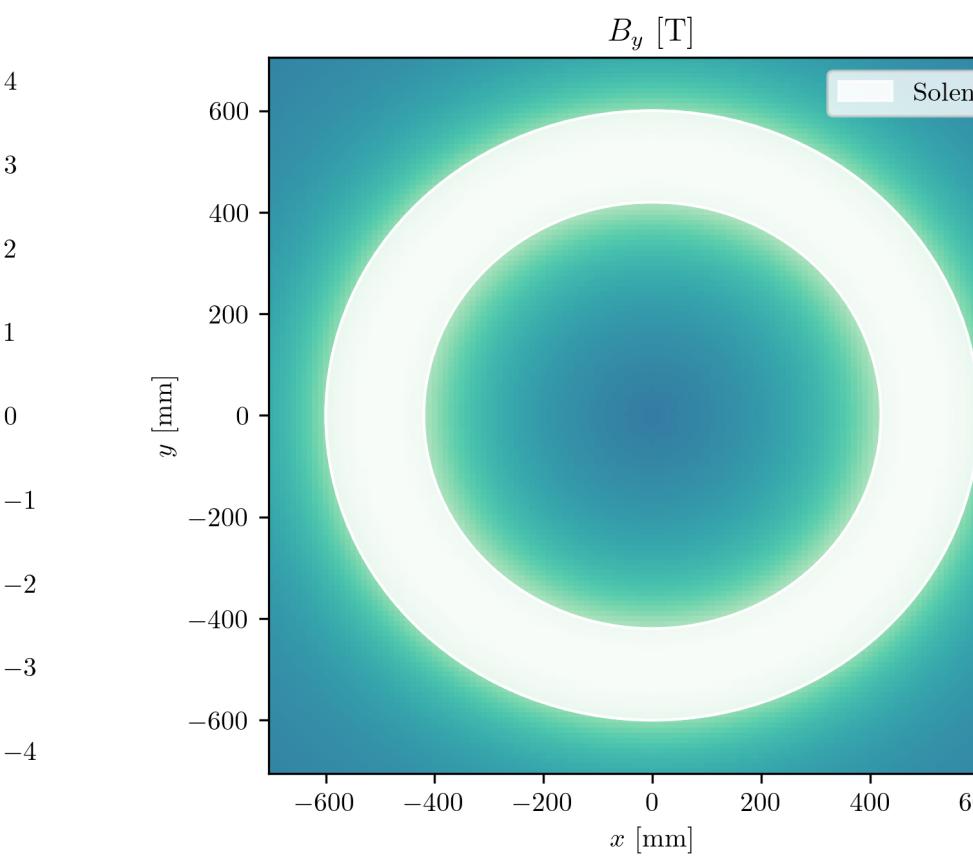
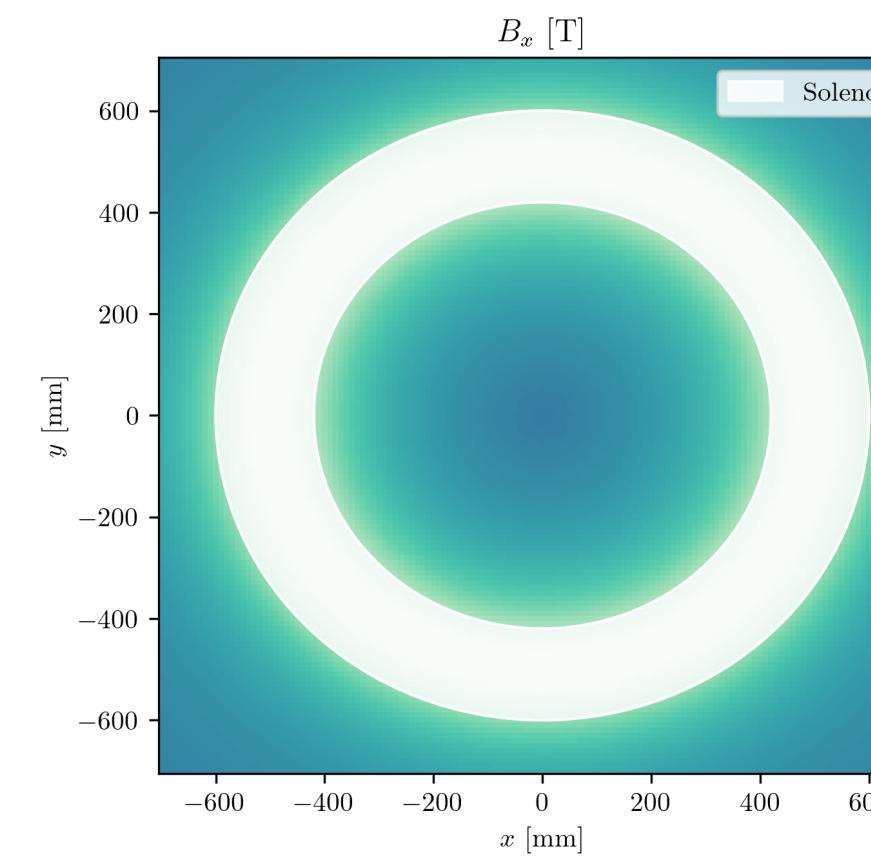
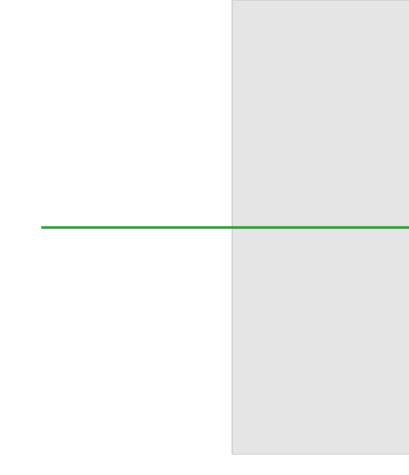
$z = -200$



$z = 0$



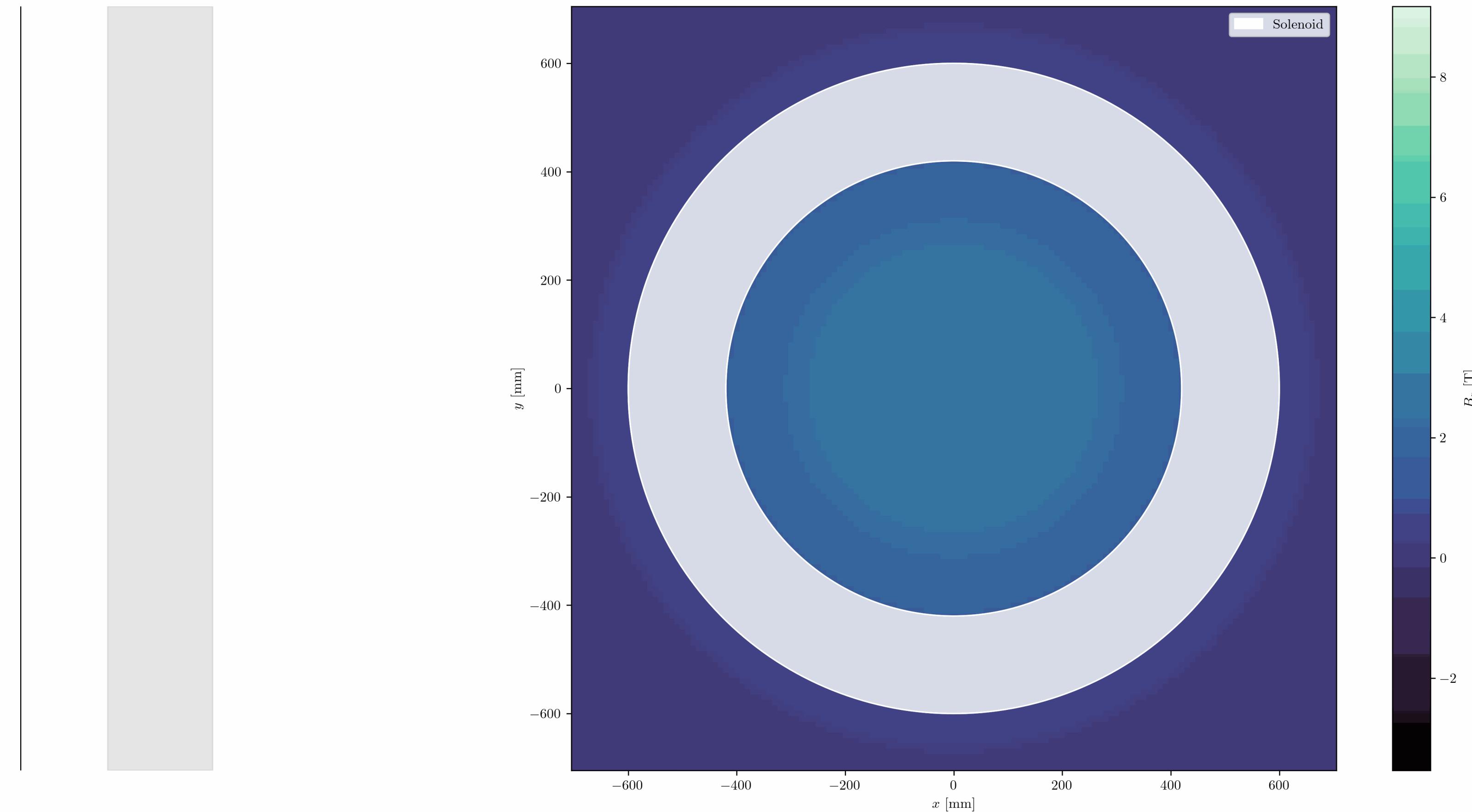
$z = 200$



FIELD MAP FROM G4BL - NO ROTATION

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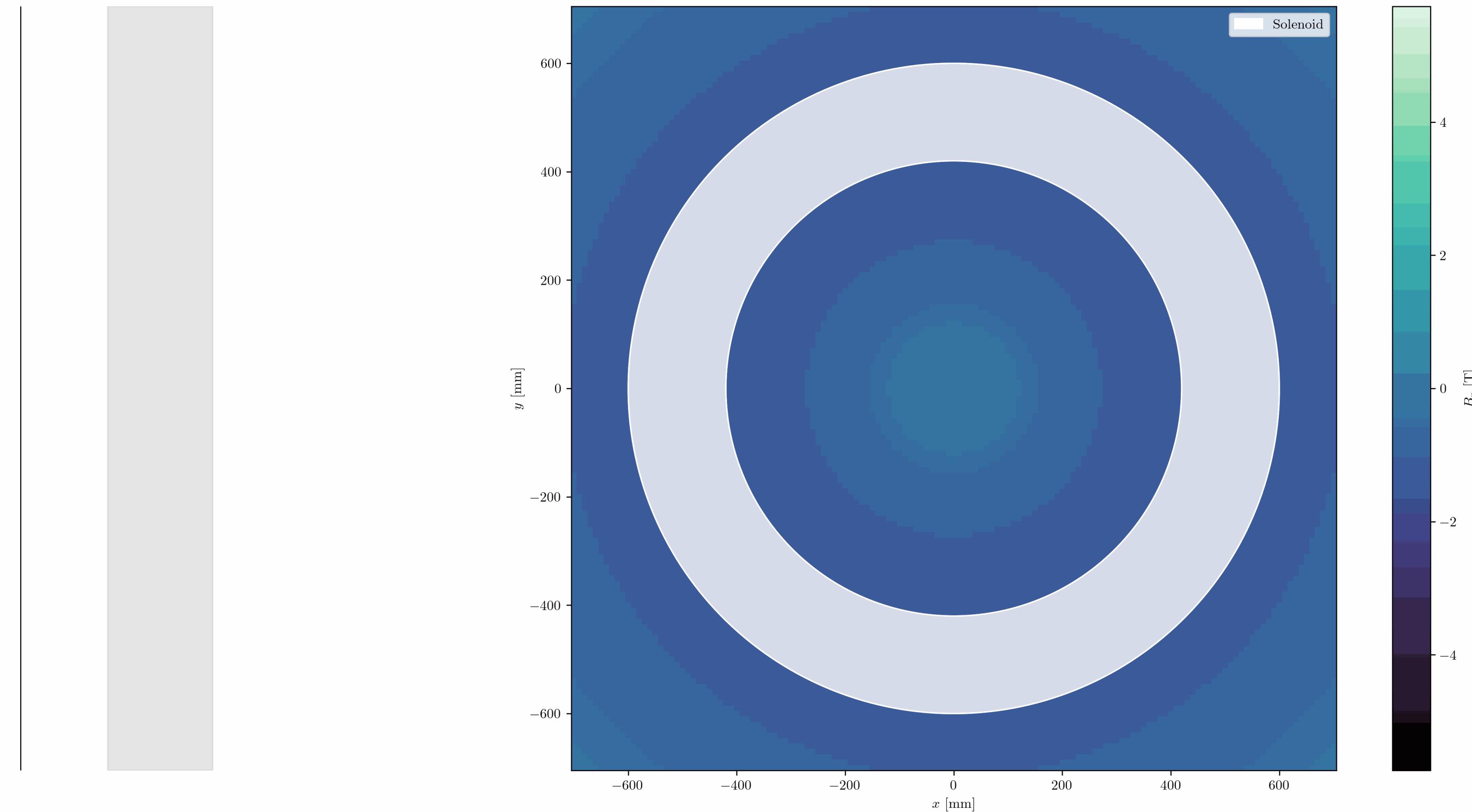
Animation of longitudinal field along z:



FIELD MAP FROM G4BL - NO ROTATION

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Animation of radial field along z:

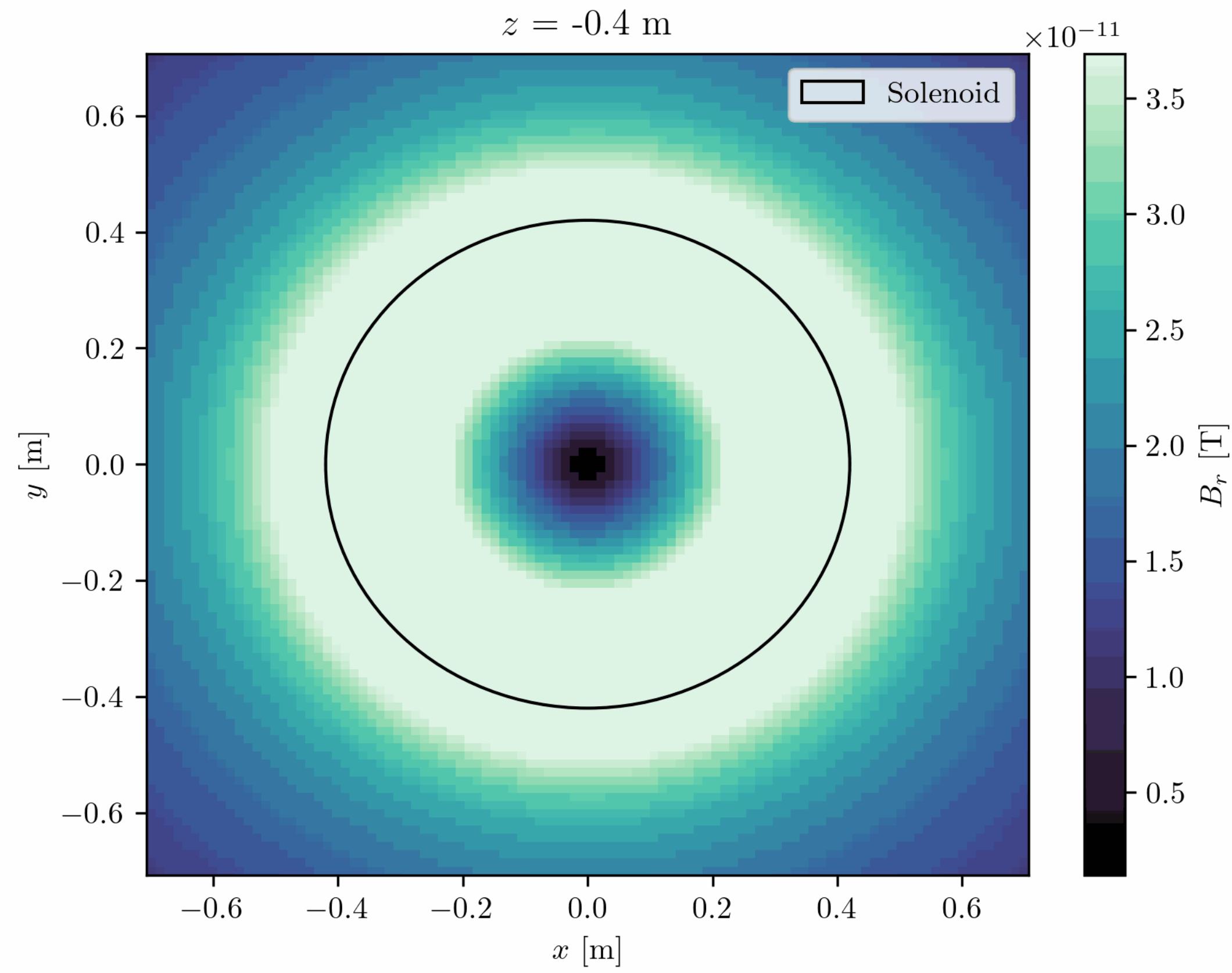


COMPARING APPROACHES

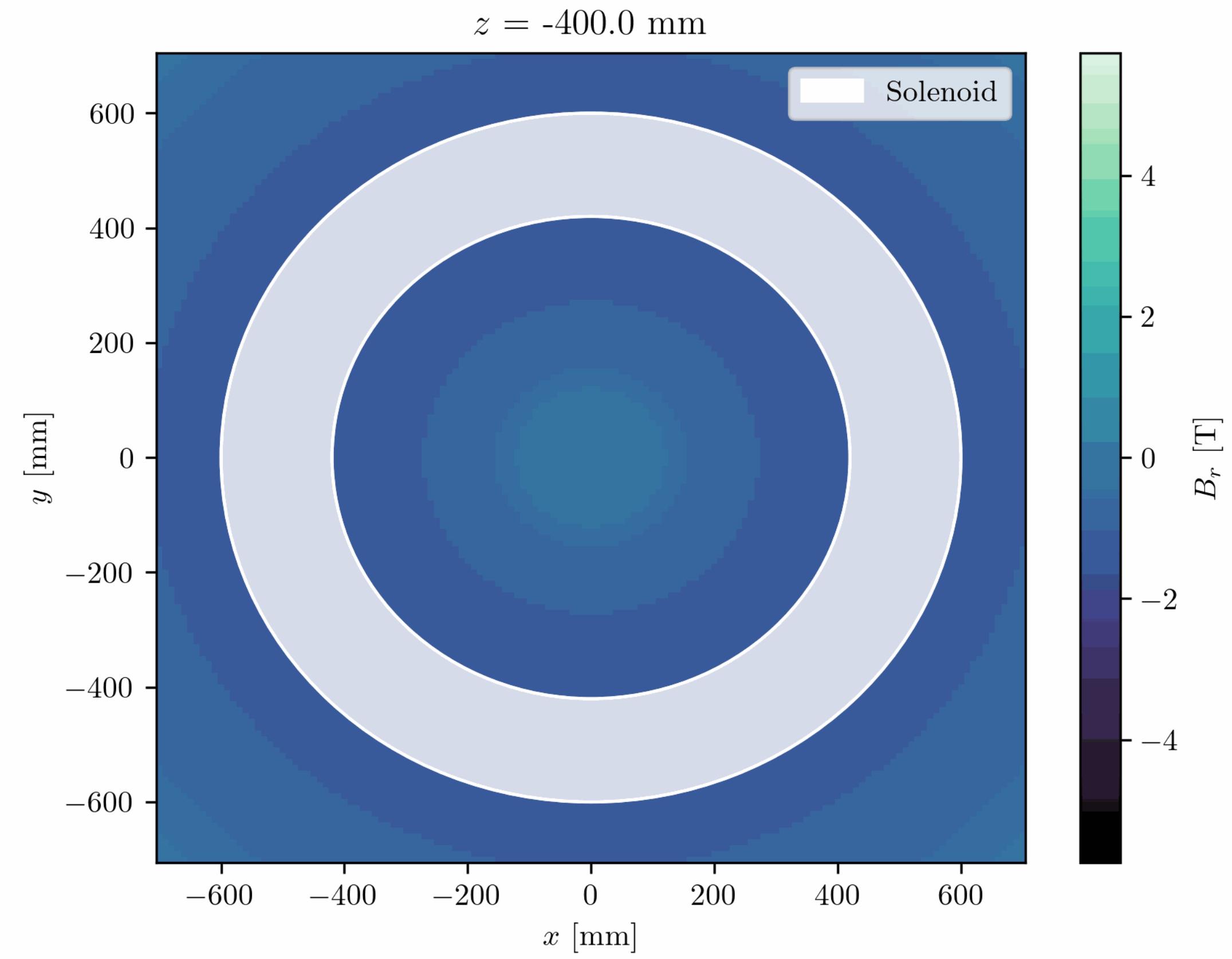
ANALYTIC VS. G4BL FIELD MAP - NO ROTATION

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Analytic:



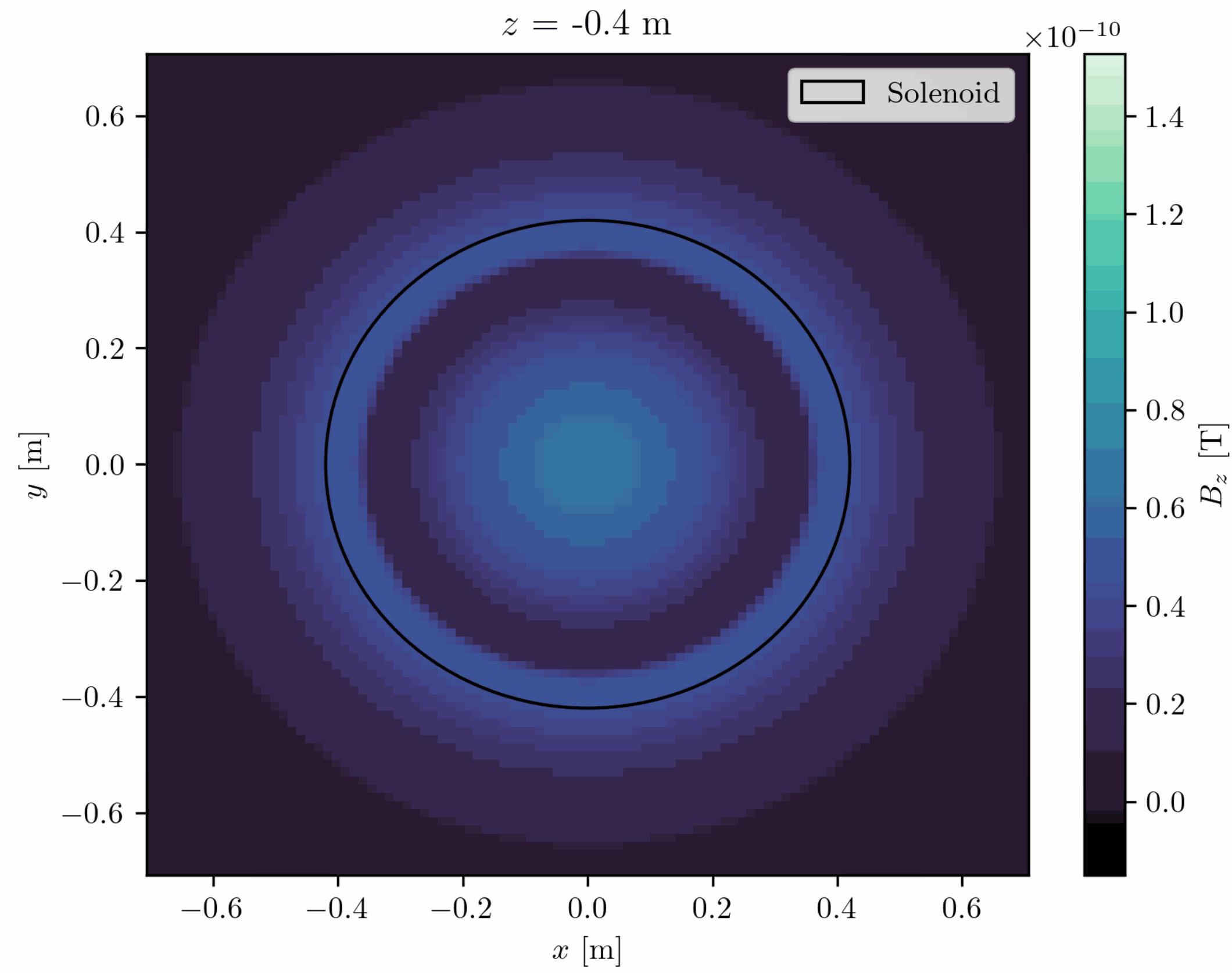
Simulation:



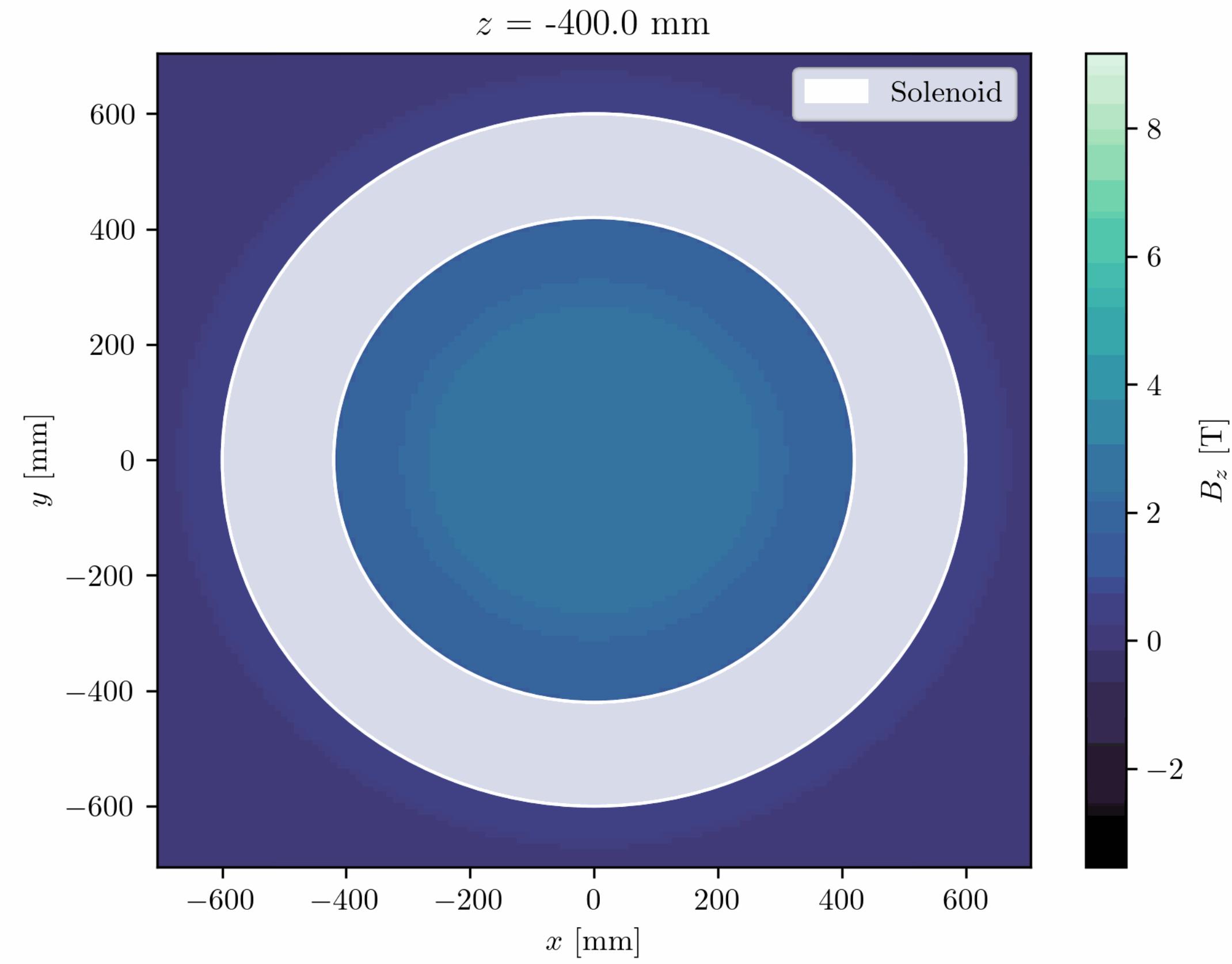
ANALYTIC VS. G4BL FIELD MAP - NO ROTATION

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Analytic:



Simulation:



MAPPING THE MAGNETIC FIELD

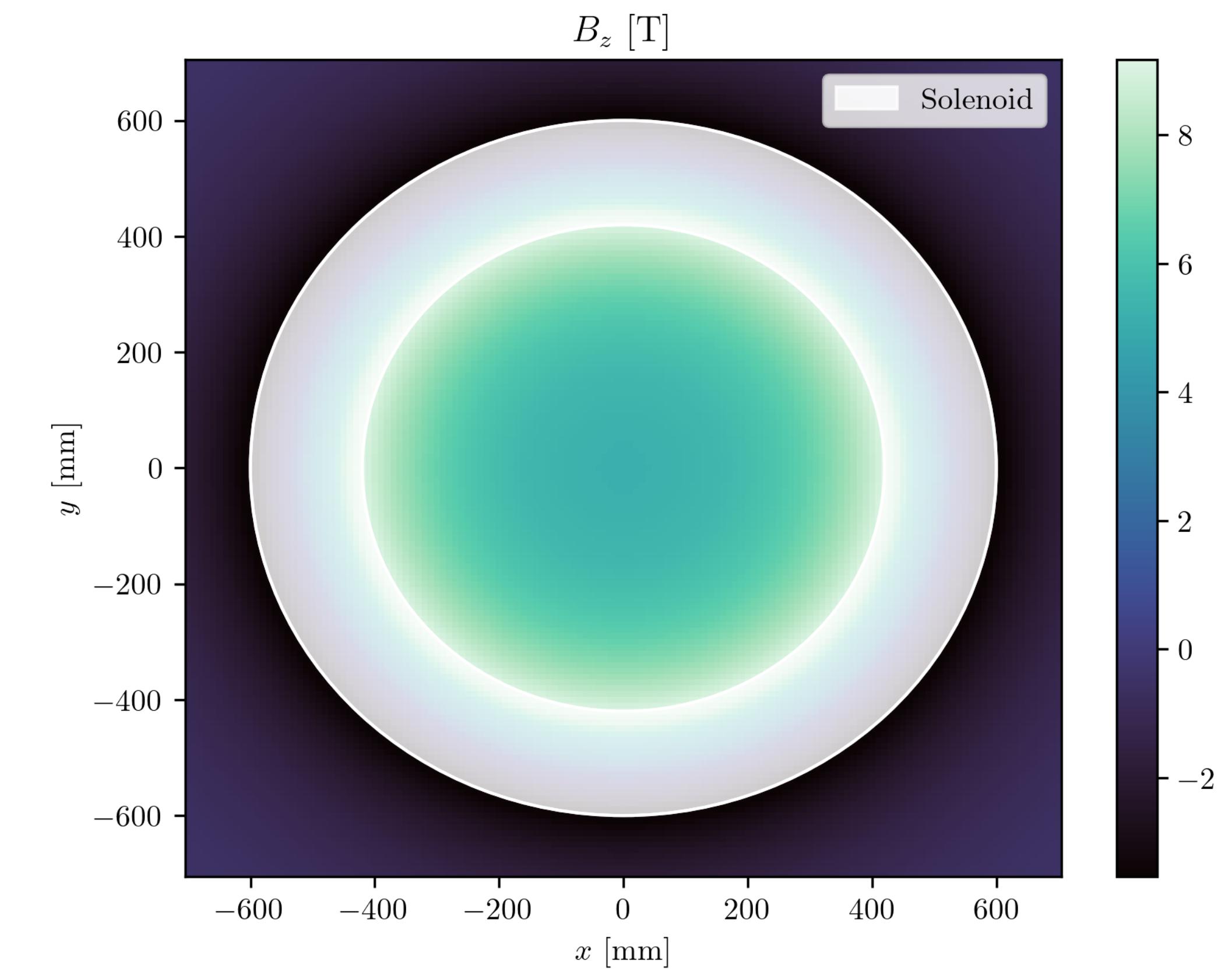
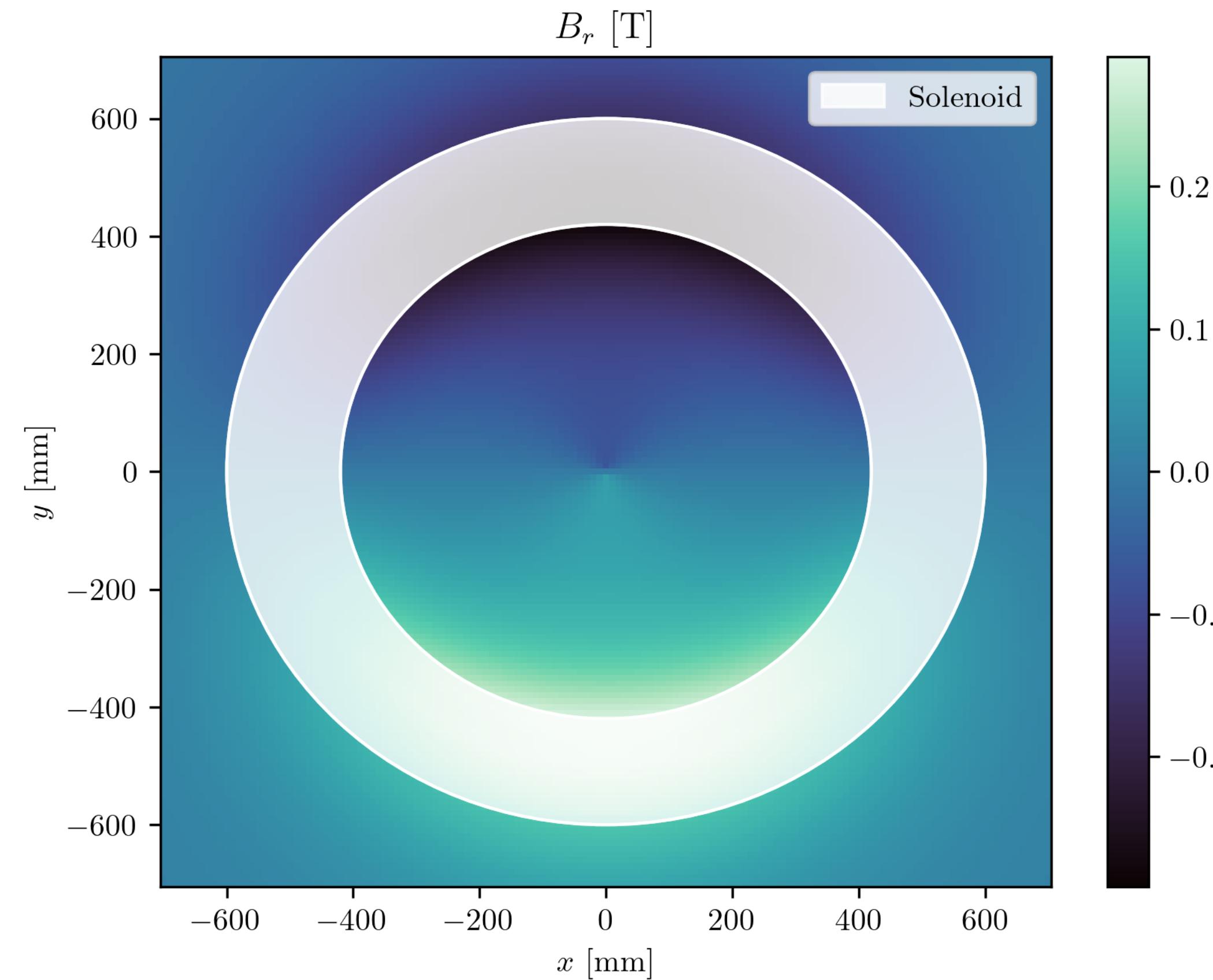
For a single solenoid with HFOFO coil geometry and pitch

SIMULATION

FIELD MAP FROM G4BL - WITH X ROTATION

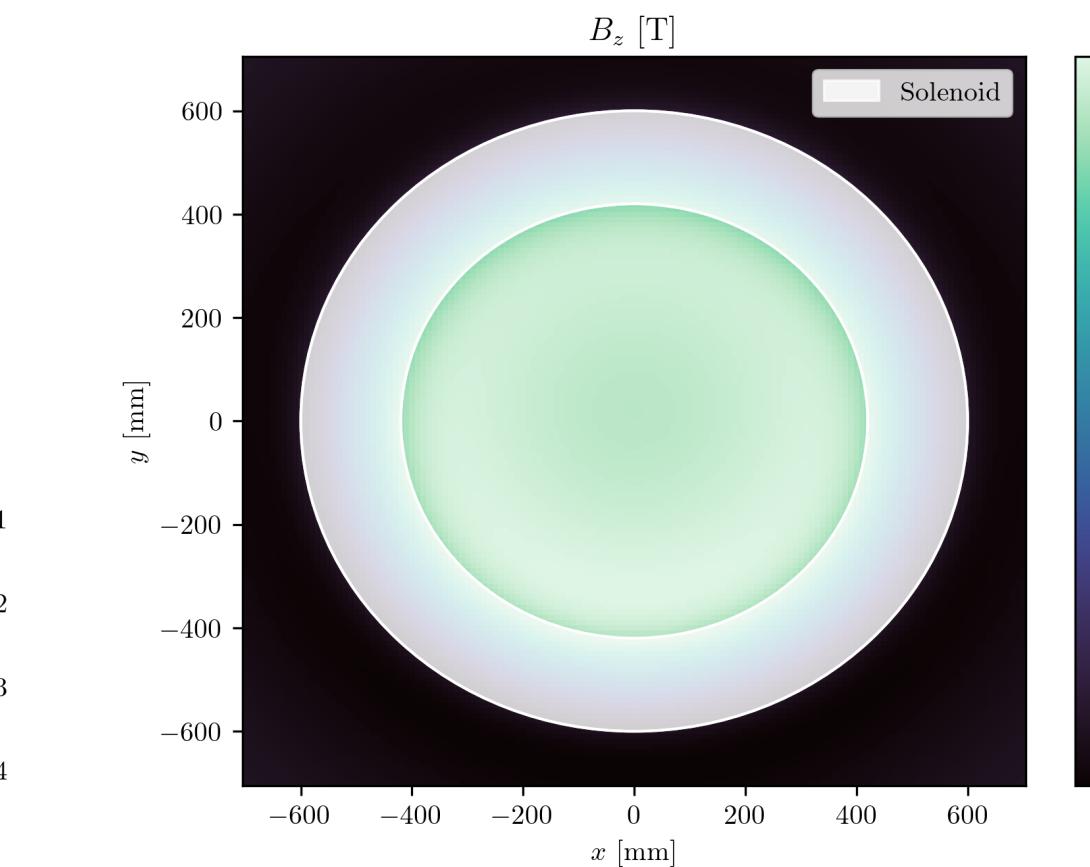
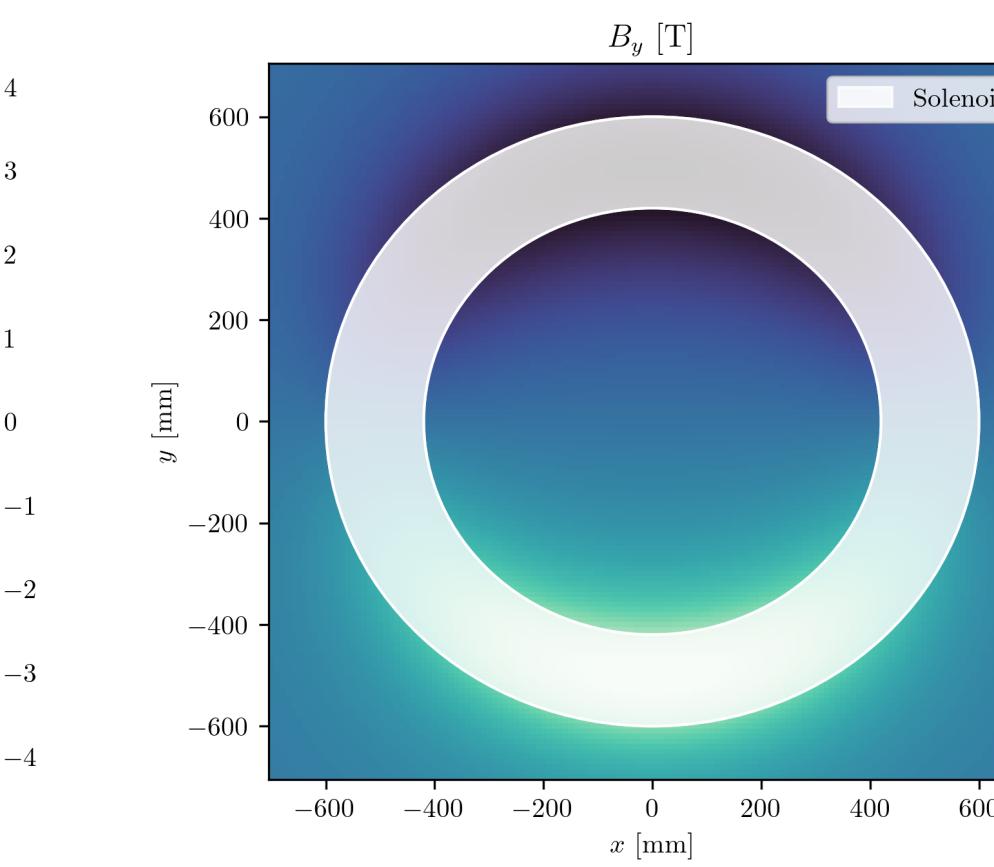
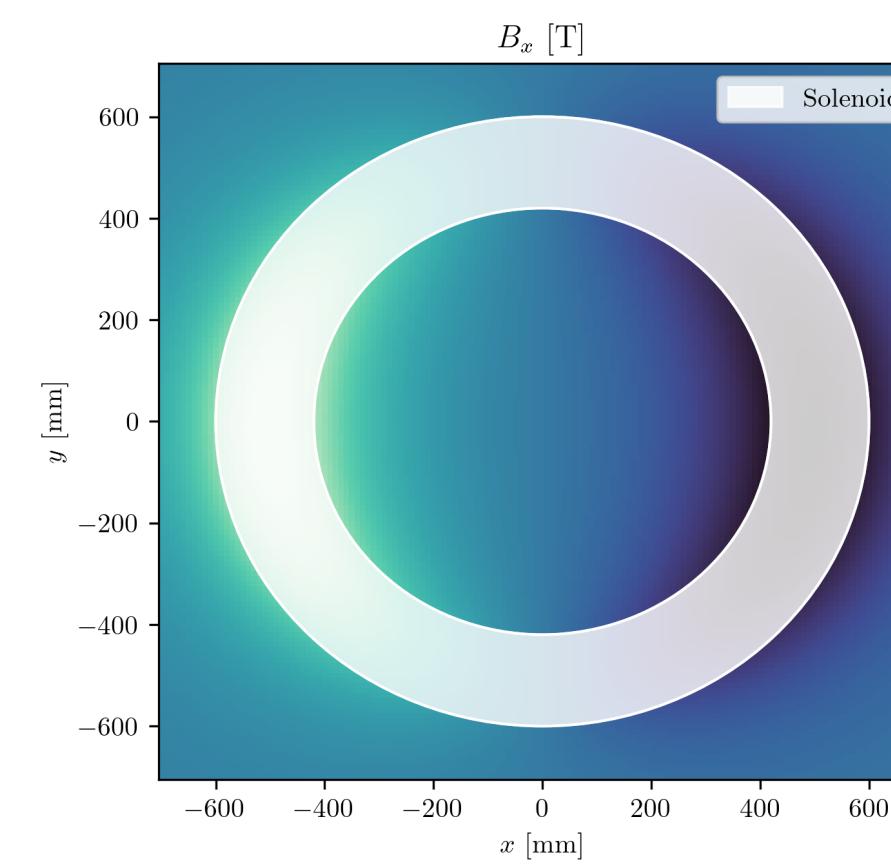
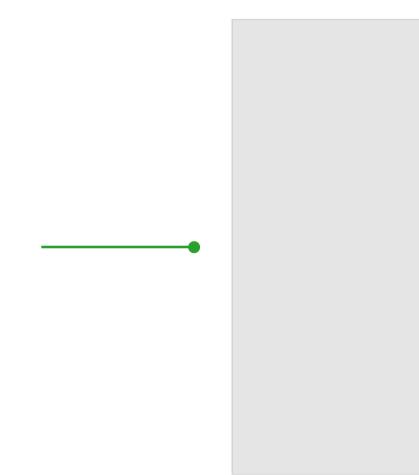
22

Considering a transverse slice at $z=0$:

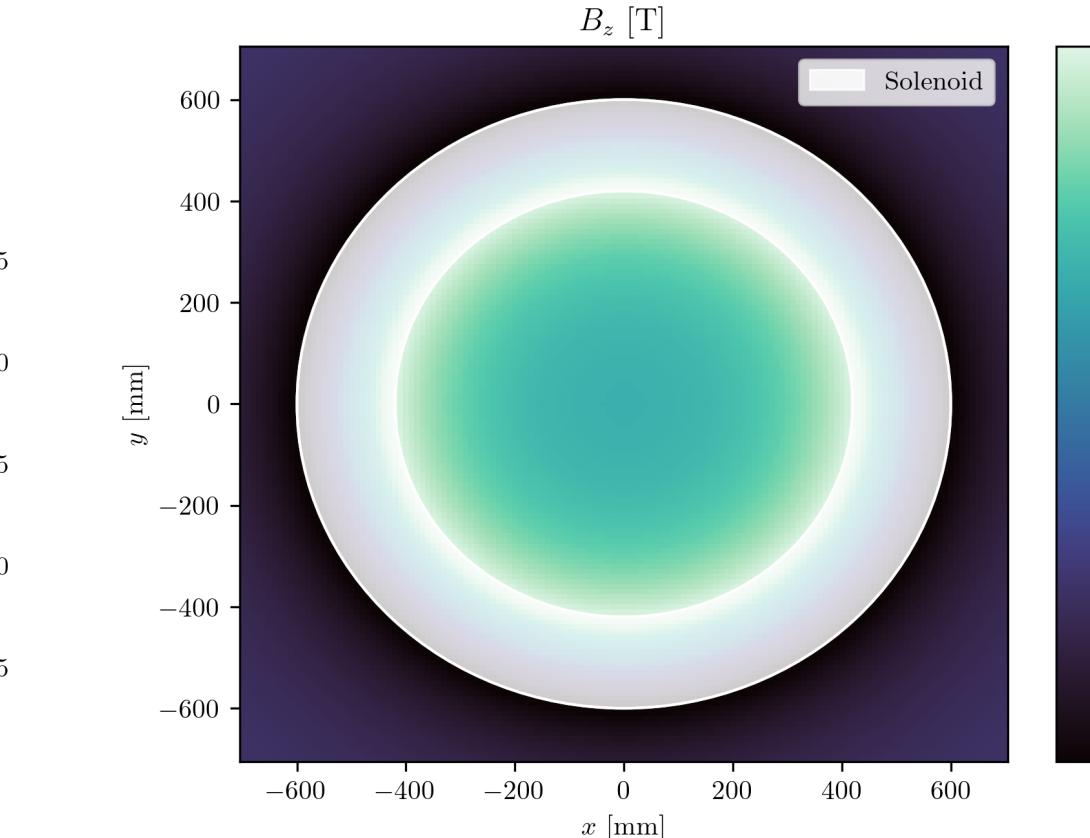
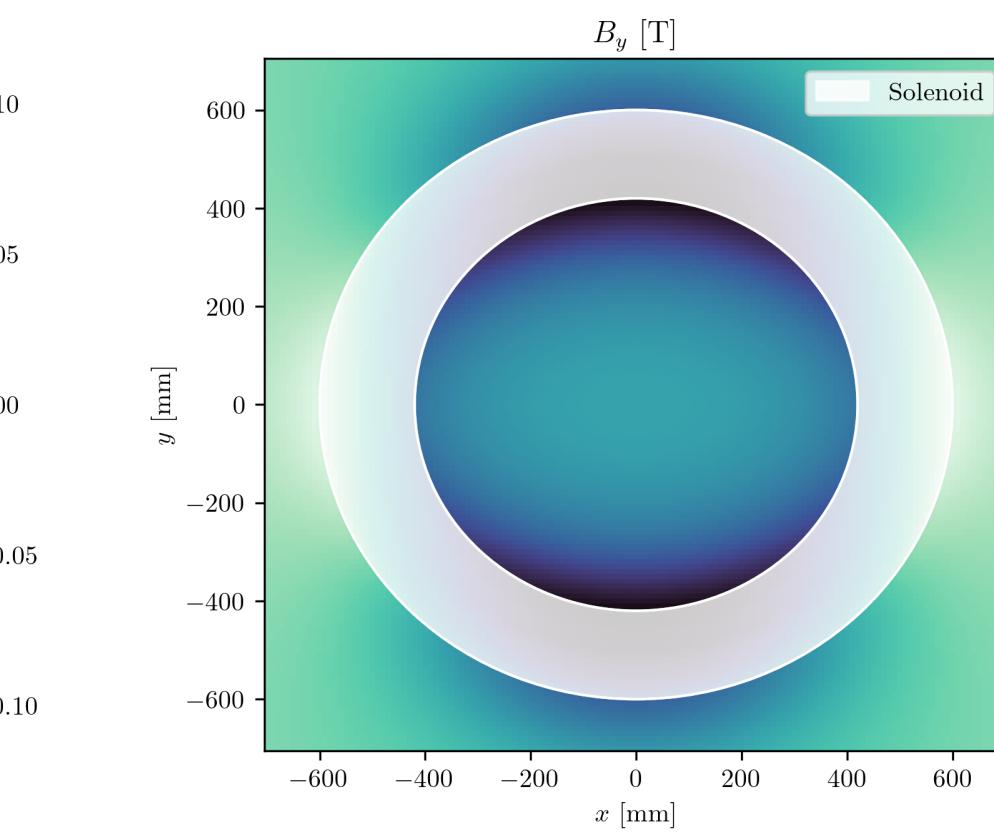
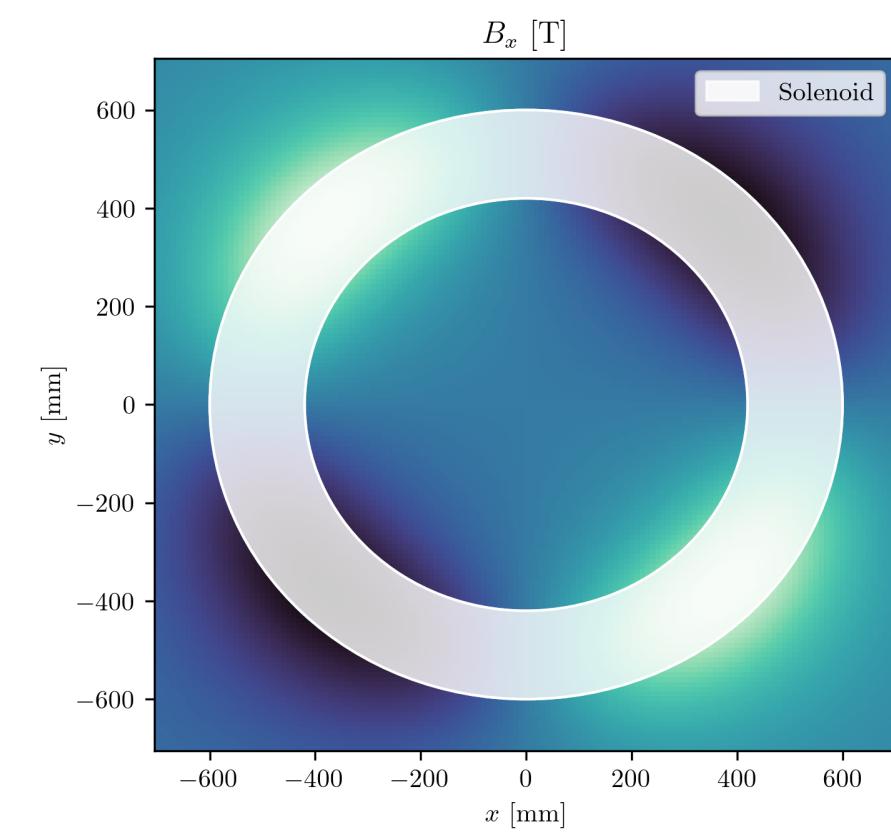
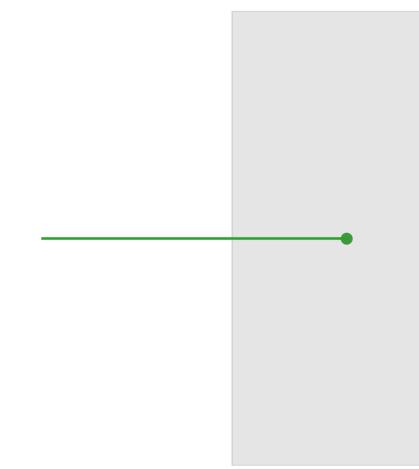


* Note that the solenoid visualization does not account for the (sub-degree) tilt

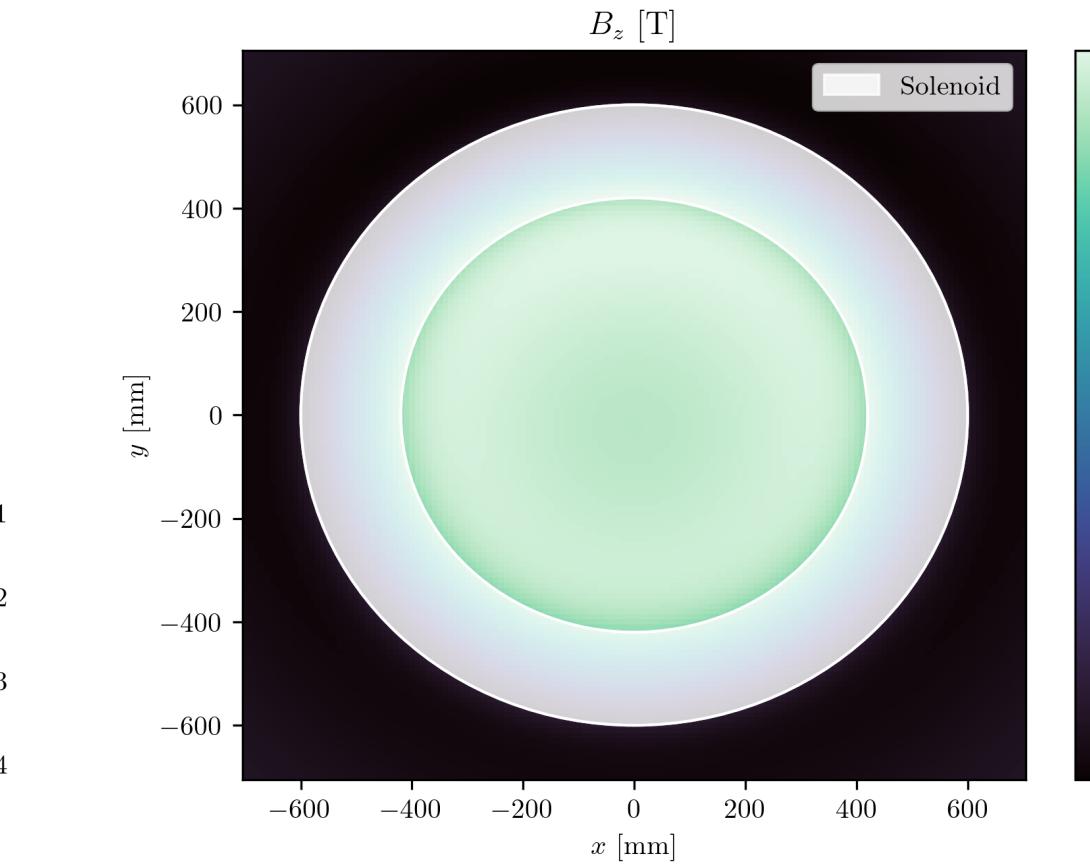
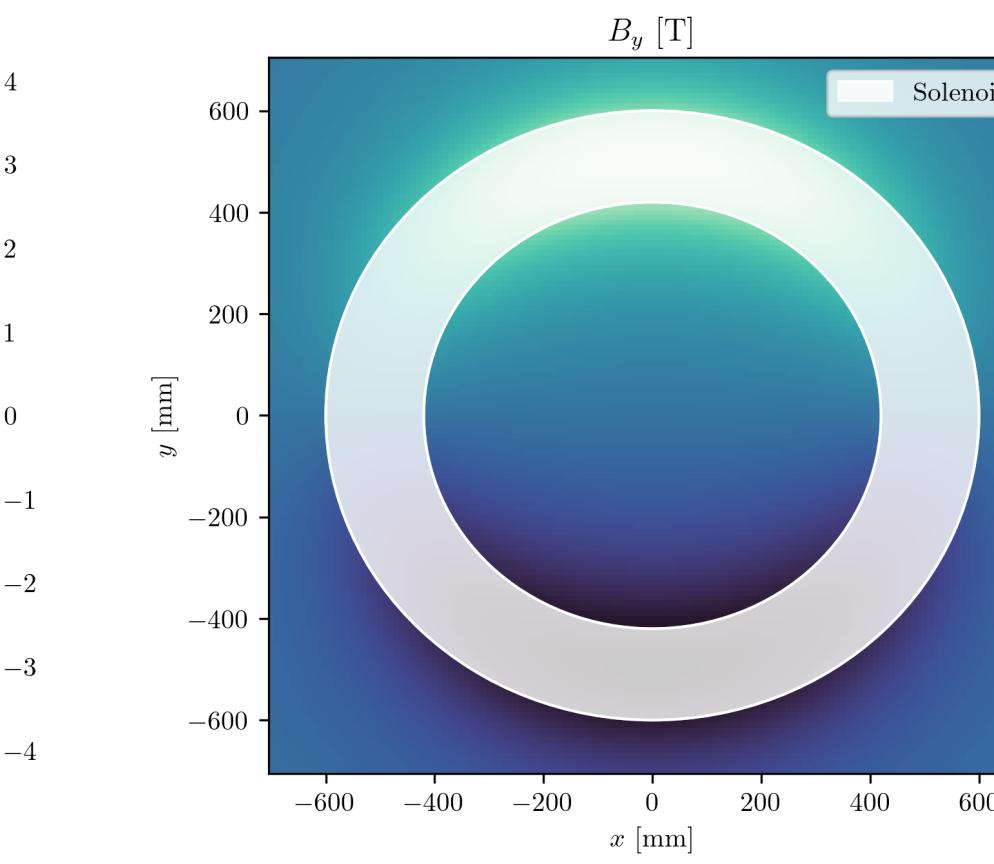
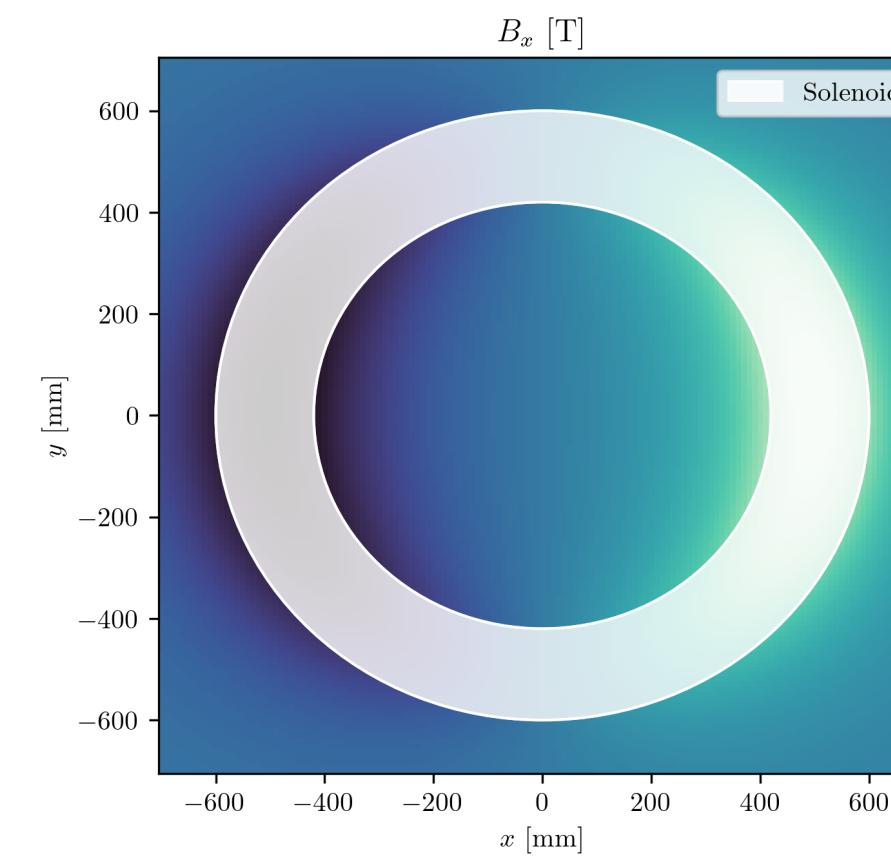
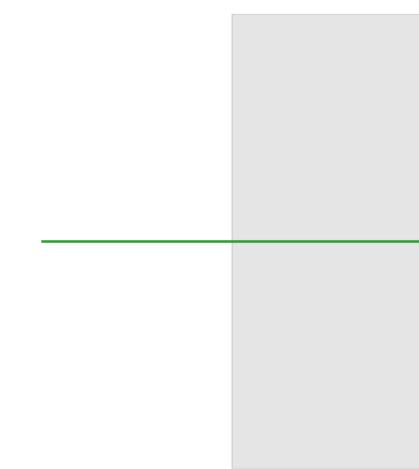
$z = -200$



$z = 0$



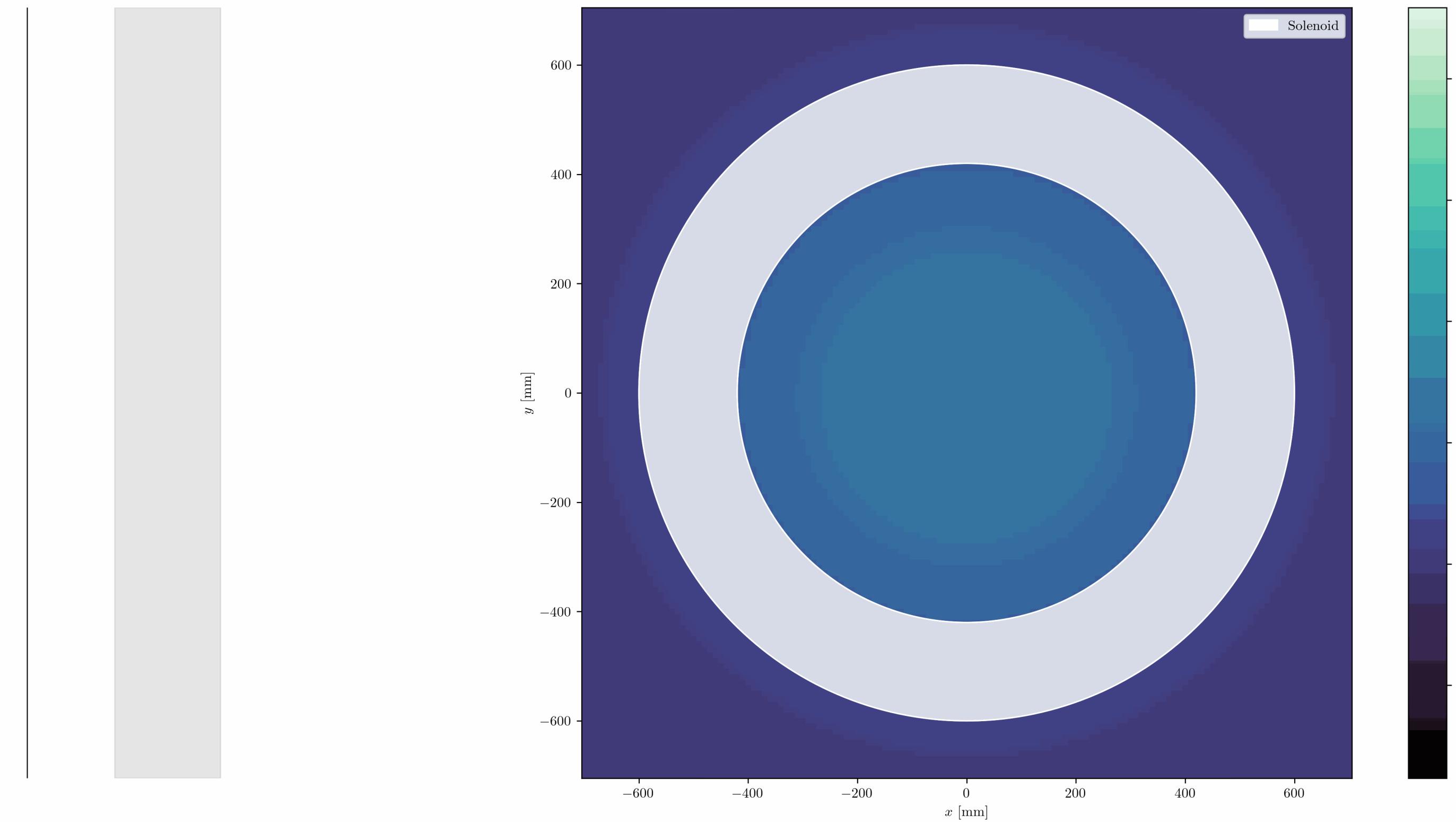
$z = 200$



FIELD MAP FROM G4BL - WITH X ROTATION

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Animation of longitudinal field along z:

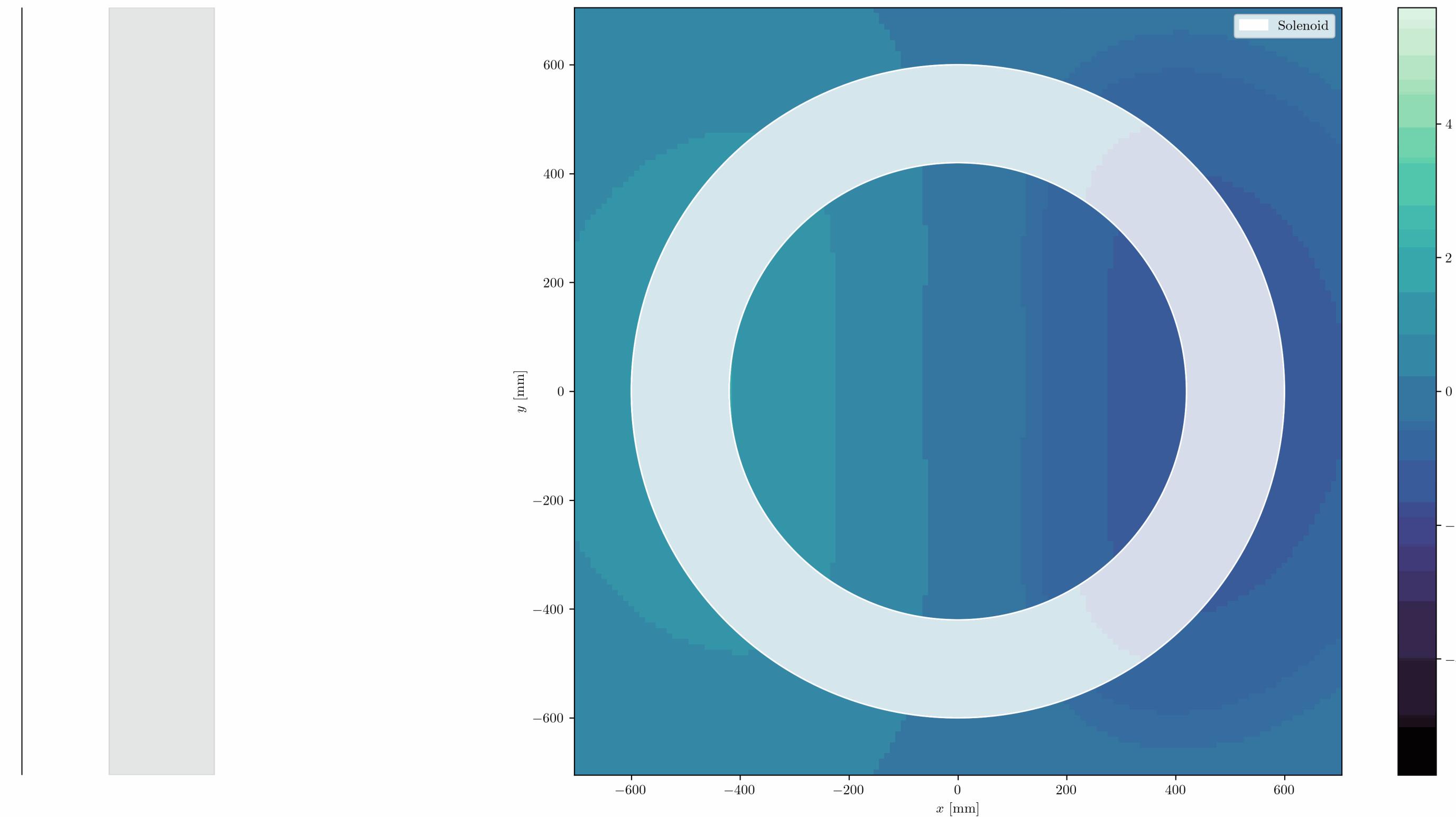


* Note again that neither visualizations of the solenoid account for the tilt

FIELD MAP FROM G4BL - WITH X ROTATION

25

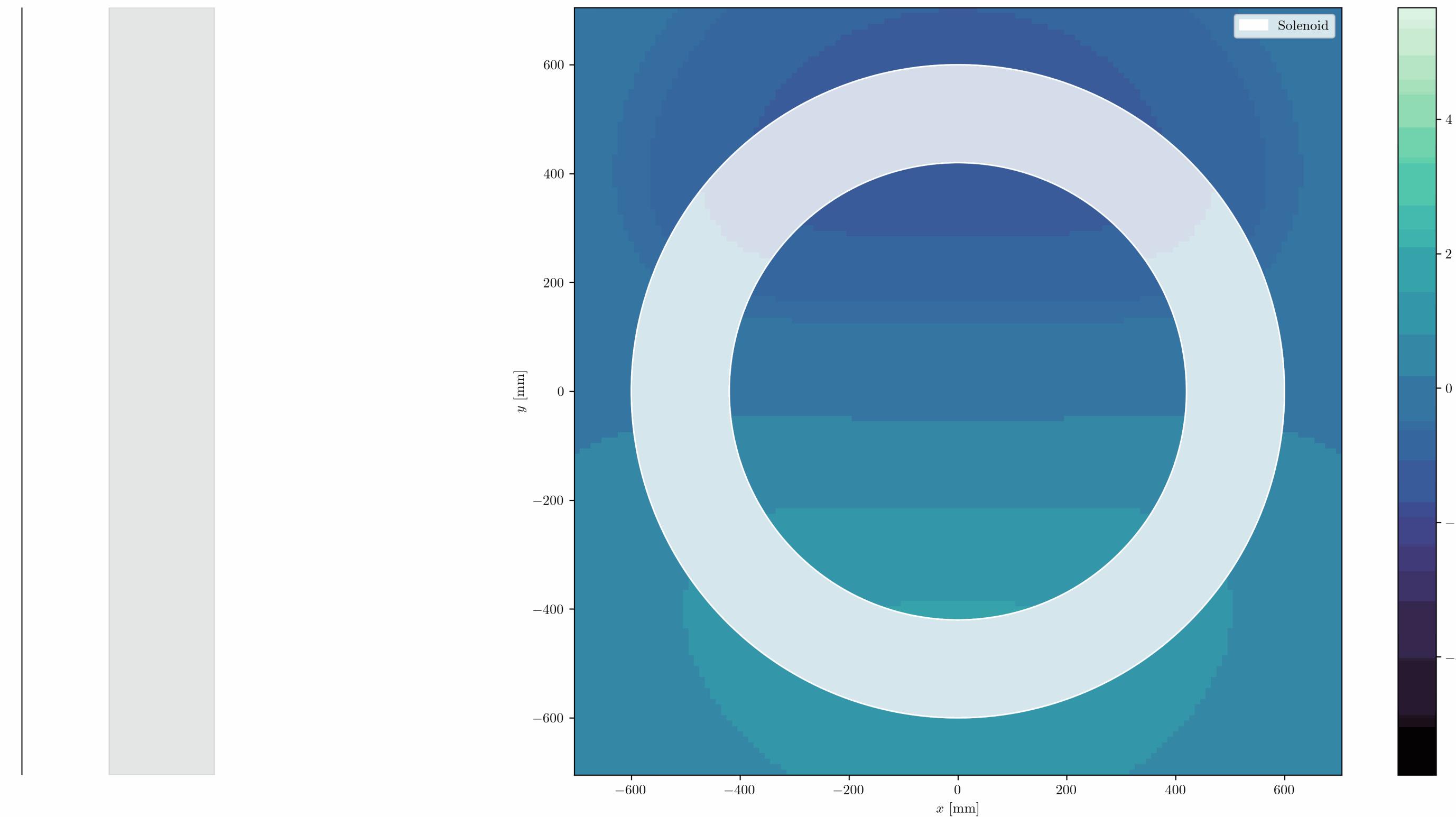
Animation of x-component of field along z:



FIELD MAP FROM G4BL - WITH X ROTATION

26

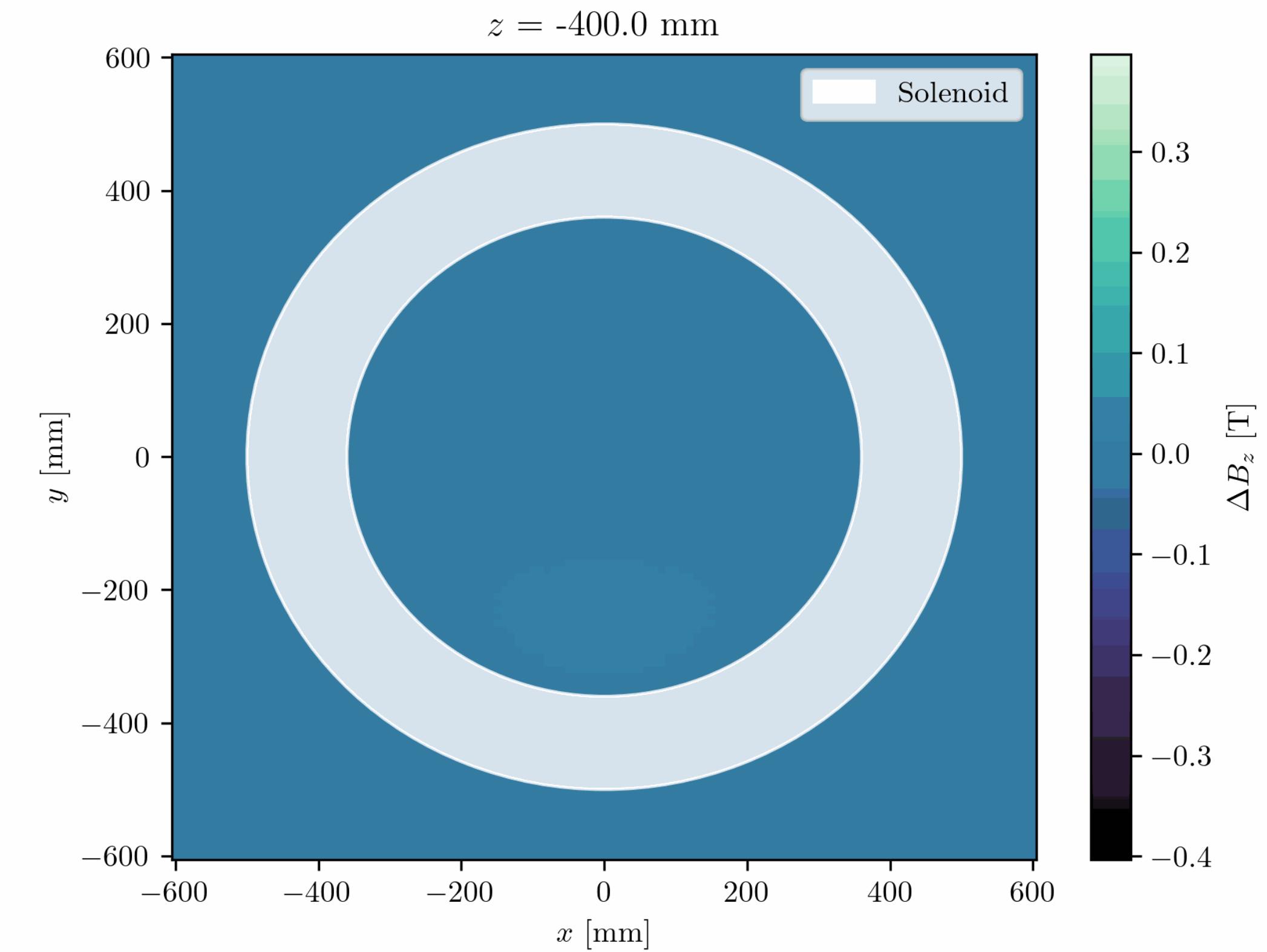
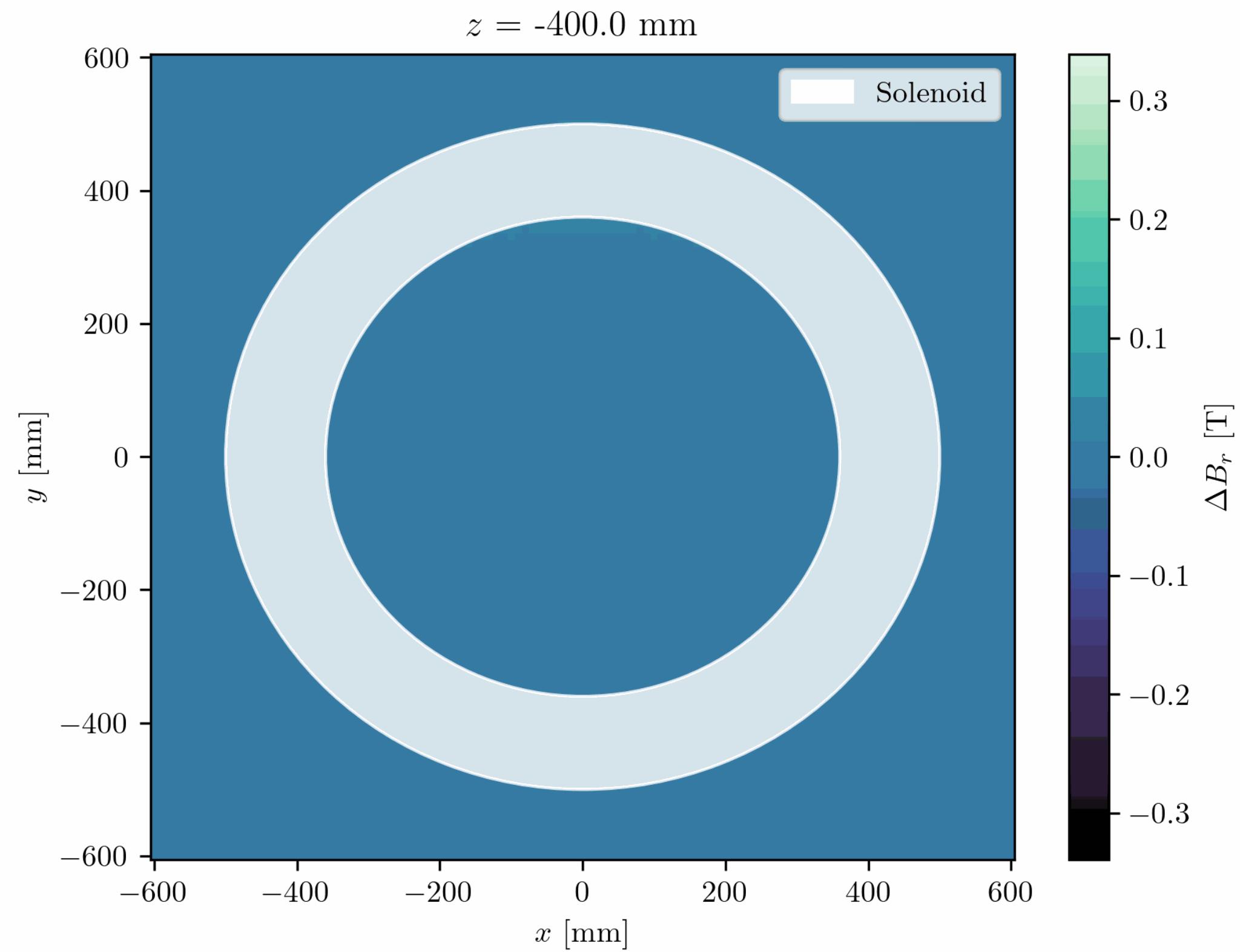
Animation of y-component of field along z:



FIELD MAP FROM G4BL - WITH X ROTATION

27

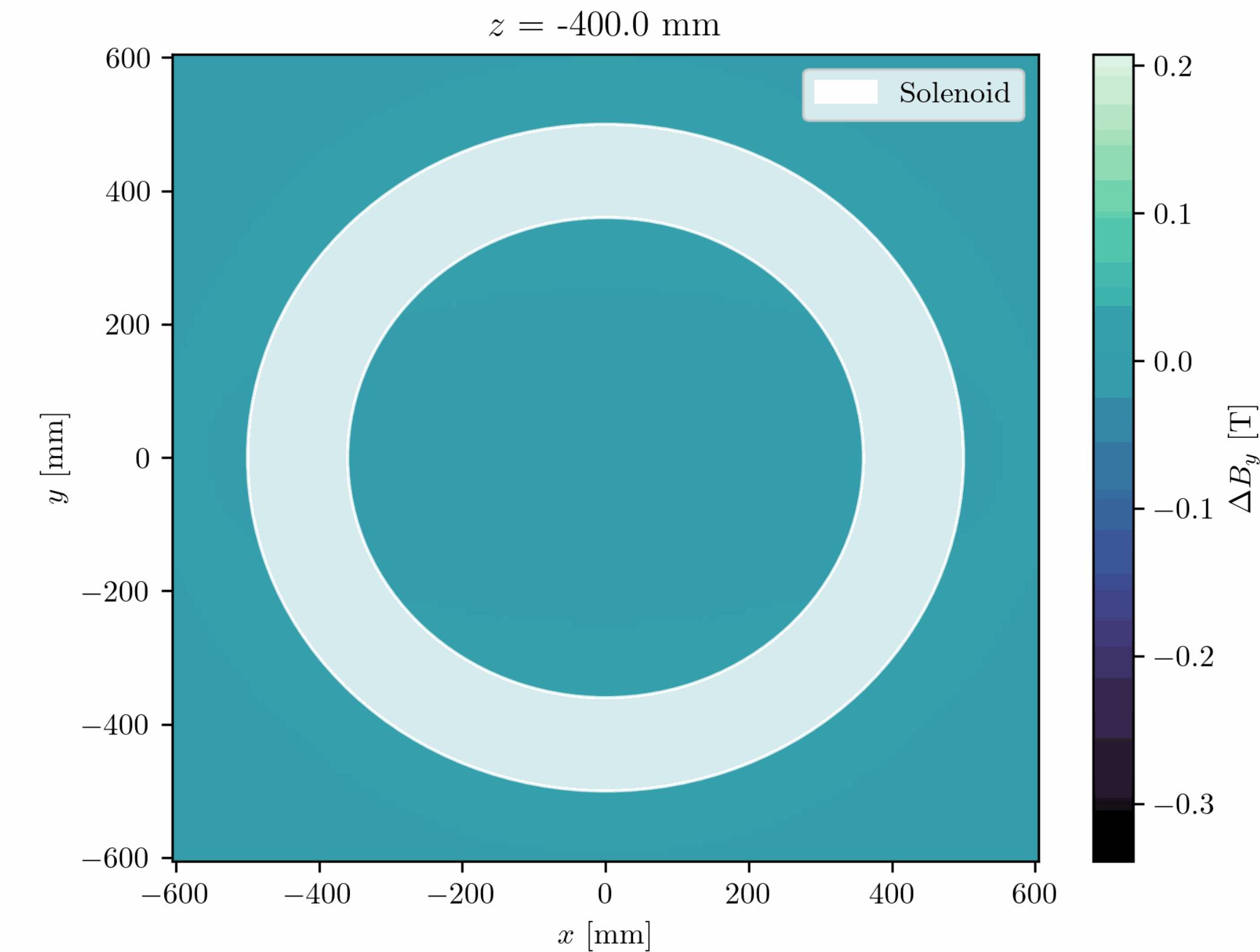
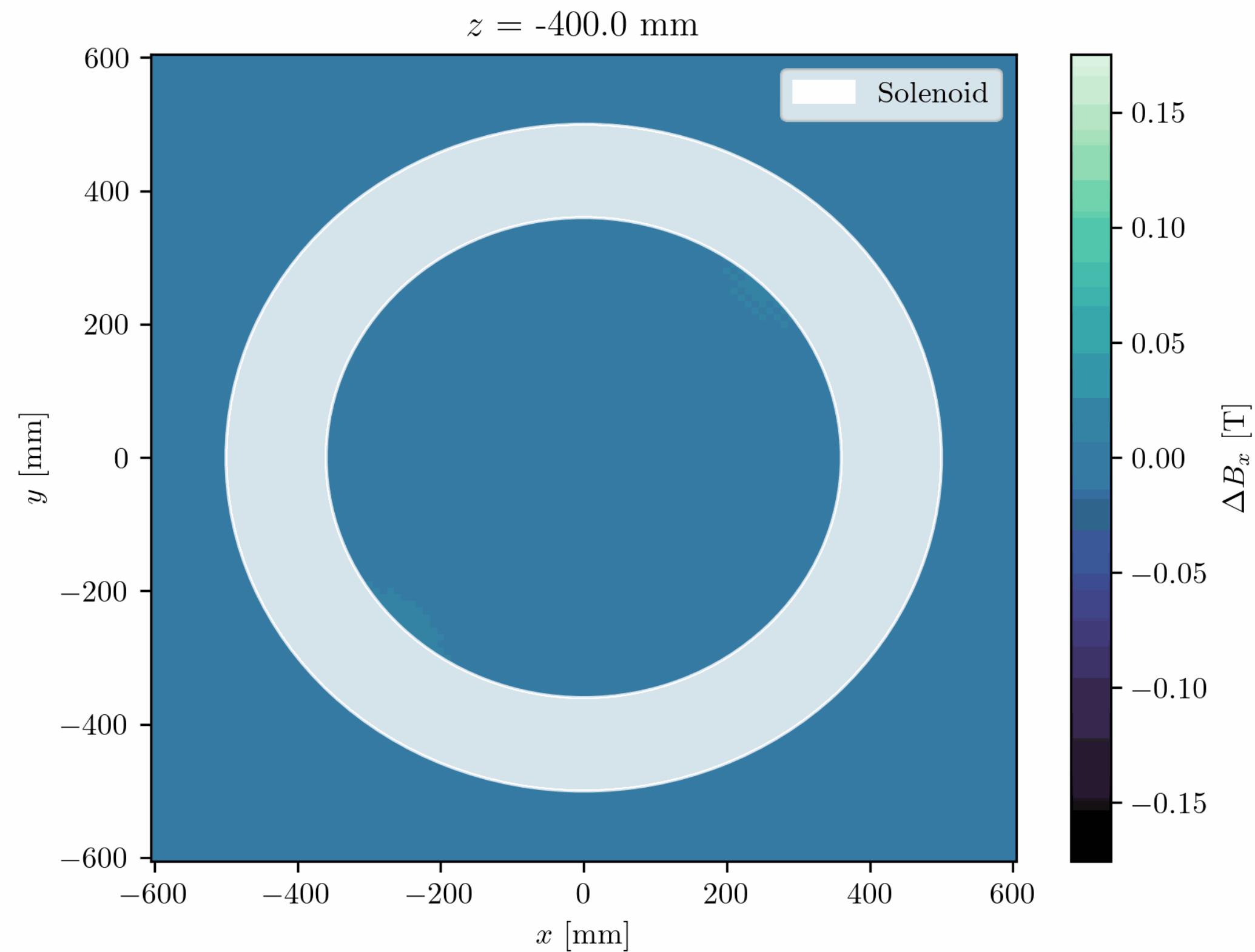
Animation of difference between pitch and no-pitch cases along z:



FIELD MAP FROM G4BL - WITH X ROTATION

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Animation of difference between pitch and no-pitch cases along z:

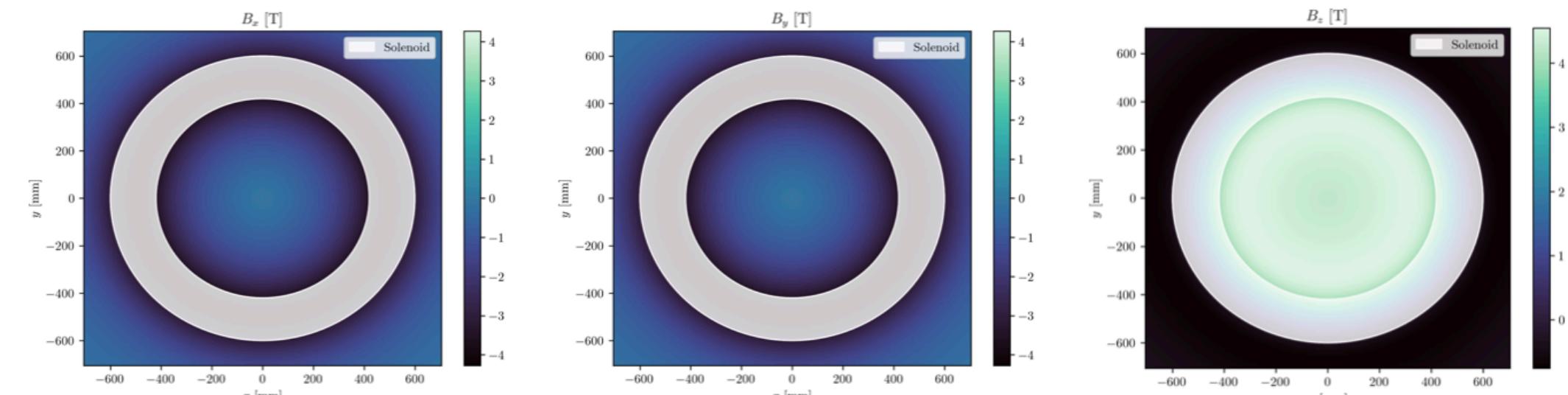


MAPPING THE MAGNETIC FIELD

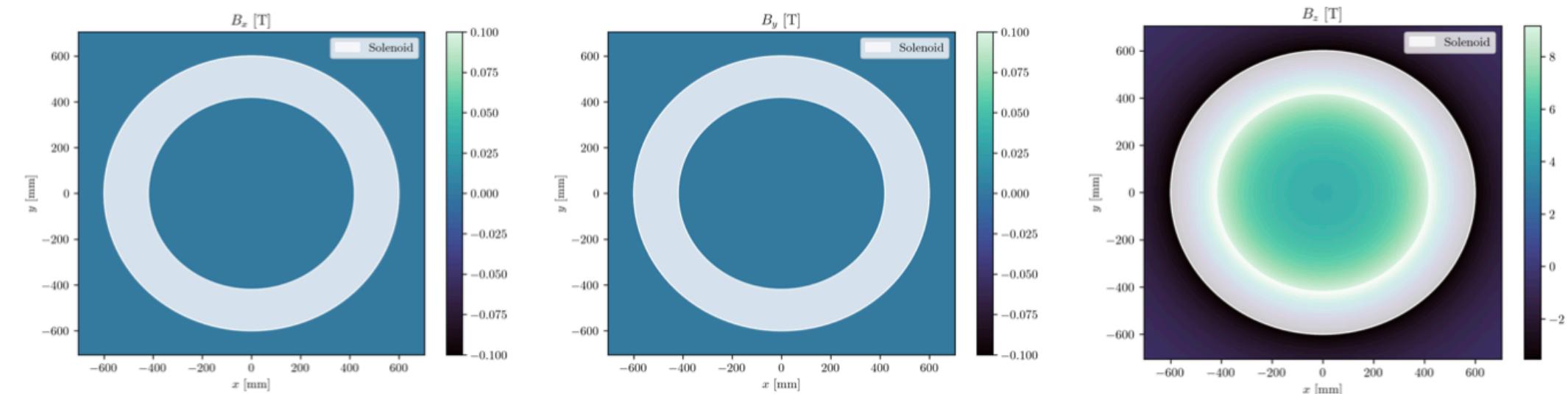
Comparing rotated to not-rotated case

No rotations

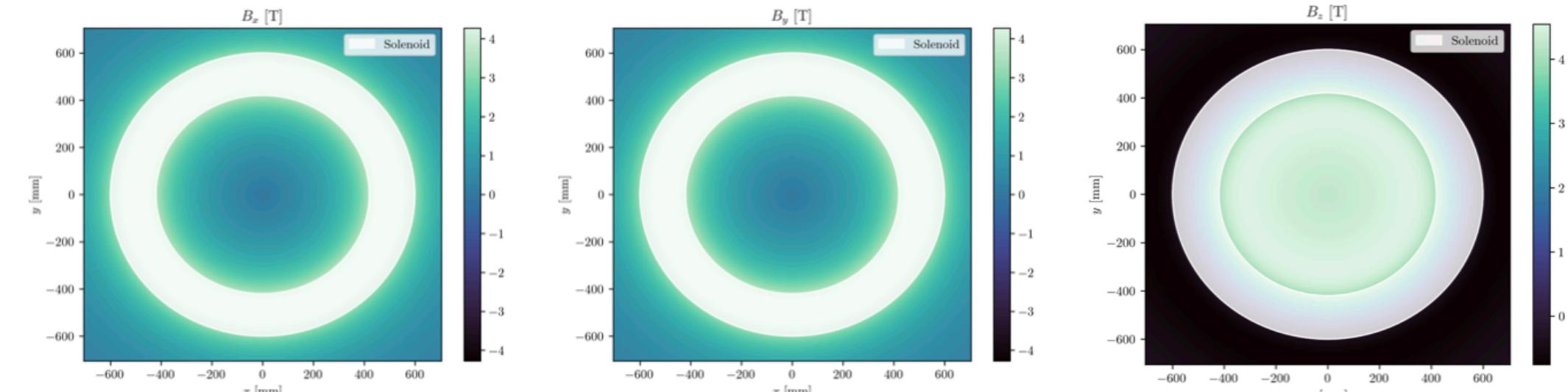
$z = -200$



$z = 0$



$z = 200$



Rotations

