

Short review of 2018 Campaign and new Senis probe

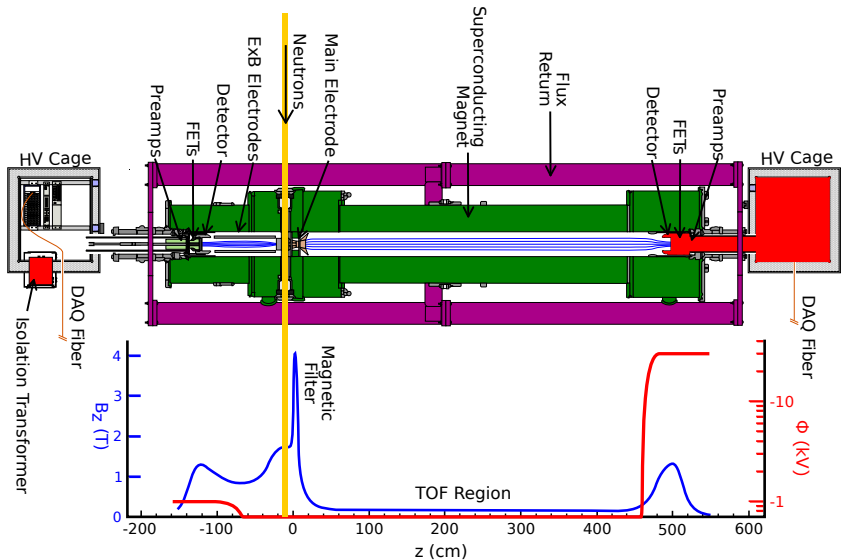
Jason Fry, for the magnetometry team

Eastern Kentucky University

December 16, 2025

2025 Co-Naboration

Nab Spectrometer Magnet



How do we relate proton momentum p_p to time of flight t_p ?

- Proton time of flight in B field:

$$t_p = L \frac{m_p}{p_p}, \quad \text{but...}$$

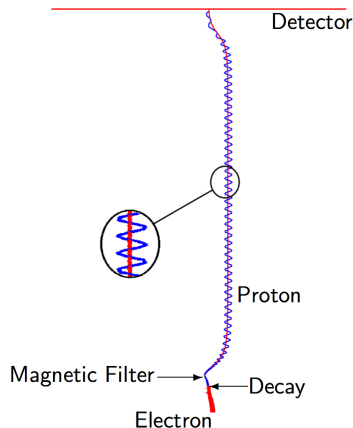
L depends on point at birth and the direction of momentum and field!

$$\cos \theta_{p,0} = \frac{\vec{p}_{p,0} \cdot \vec{B}}{p_{p,0} B} \Bigg|_{\text{decay pt.}}.$$

- For an adiabatically expanding field,

$$t_p = \frac{m_p}{p_p} \int_{z_0}^l \frac{dz}{\sqrt{1 - \frac{B(z)}{B_0} \sin^2 \theta_{p,0} + \frac{q(V(z) - V_0)}{E_{p,0}}}}$$

Geant4 simulation:



Magnetometry Campaign → Physics Analysis

- Not only do we need to satisfy the systematics table, but we need to implement magnetometry data into physics analysis and Nab geant4 simulation
- In the Nab geant4 simulation, we can calculate the field in two ways
 - Analytical routine from Ferenc Gluck
 - 1D expansion from the field and its derivatives on axis
- Purpose of the 2025-2026 Campaign is to complete the analysis of the magnetometry data and to re-check the field since 2019 (tie rod movement, multiple quenches, first map was in hybrid mode, etc)
- ASU group has shown the magnetic shield is negligible and can use analytic routines (definitely in the DV and F)

Radial series expansion and Zonal Harmonic Expansion

In order to determine the derivatives on-axis in terms of B_n^{cen} and ρ_{cen} , we can compare the radial expansion for B_z and B_r in equations

$$B_z(r, z) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2^n n!)^2} \frac{d^{2n} B_{0,z}}{dz^{2n}} r^{2n},$$

$$B_r(r, z) = - \sum_{n=0}^{\infty} \frac{(-1)^n}{n!(n+1)2^{2n+1}} \frac{d^{2n+1} B_{0,z}}{dz^{2n+1}} r^{2n+1}.$$

$$B_z = \sum_{n=0}^{\infty} B_n^{cen} \left(\frac{\rho}{\rho_{cen}} \right)^n P_n(u)$$

$$B_r = -s \sum_{n=1}^{\infty} \frac{B_n^{cen}}{n+1} \left(\frac{\rho}{\rho_{cen}} \right)^n P'_n(u)$$

1D Global FieldMap in the geant4 simulation

Now that we can compute the derivatives on-axis, which is basically matching the source terms from the Zonal Harmonic Expansion, we use these at 1 mm increments to calculate the field off-axis:

$$B_z(r, z) = B_z(0, z) - \frac{1}{4}r^2 \frac{d^2 B_{0,z}}{dz^2} + \frac{1}{64}r^4 \frac{d^4 B_{0,z}}{dz^4} - \frac{1}{2304}r^6 \frac{d^6 B_{0,z}}{dz^6}$$
$$B_r(r, z) = -\frac{1}{2}r \frac{dB_{0,z}}{dz} + \frac{1}{16}r^3 \frac{d^3 B_{0,z}}{dz^3} - \frac{1}{384}r^5 \frac{d^5 B_{0,z}}{dz^5}.$$

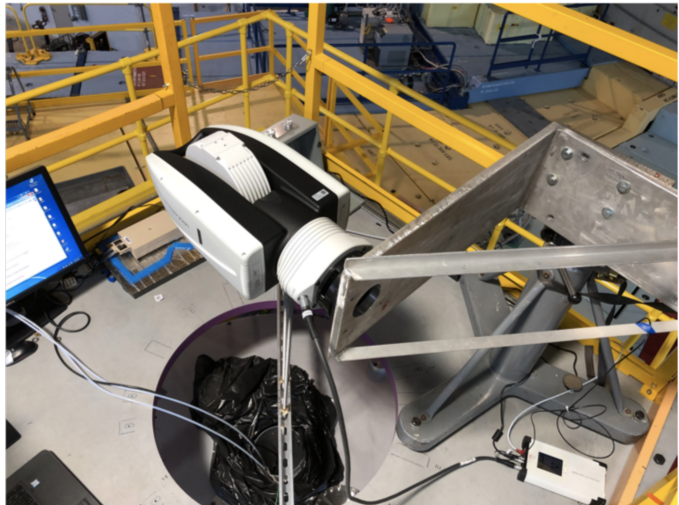
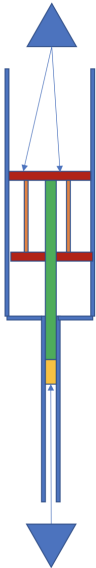
Bilinear interpolation of the last four points in a 1 mm voxel.

Comparing Ferenc routines (elliptical integrals) and a 1D expansion magnetic fields for the same initial kinematics for a proton gives a difference in the proton TOF to 0.03 ns and energy deposited in the upper detector of 0.1 eV.

How do we incorporate the data into the simulation/analysis?

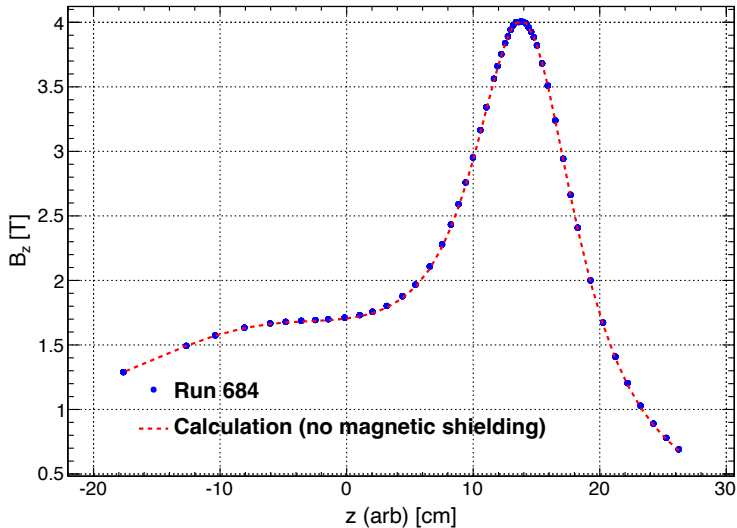
- Deliverables for Magnetometry:
 - produce a field map of the 2D/3D field everywhere
 - a method to fold the produced field map back into the simulation/analysis (I would like to get the field map back into the 1D expansion model)
 - need the simulation to be efficient when reading a fieldmap \rightarrow 1D fast, 3D slow

2018 Setup



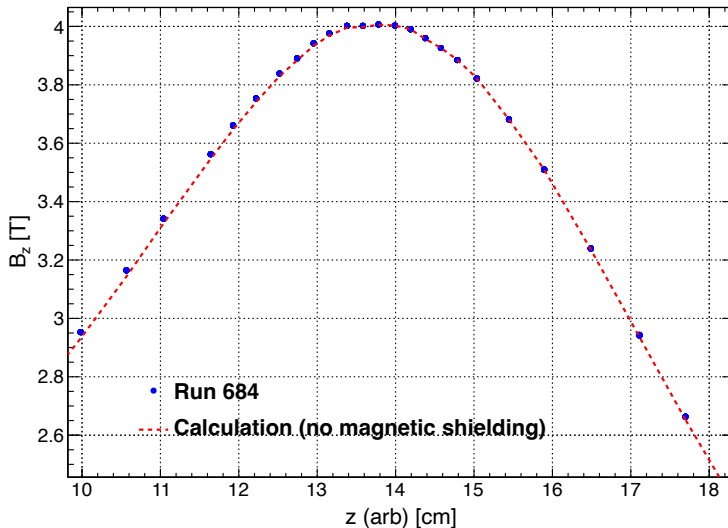
On-axis scans in DV and fitler

On-axis scan

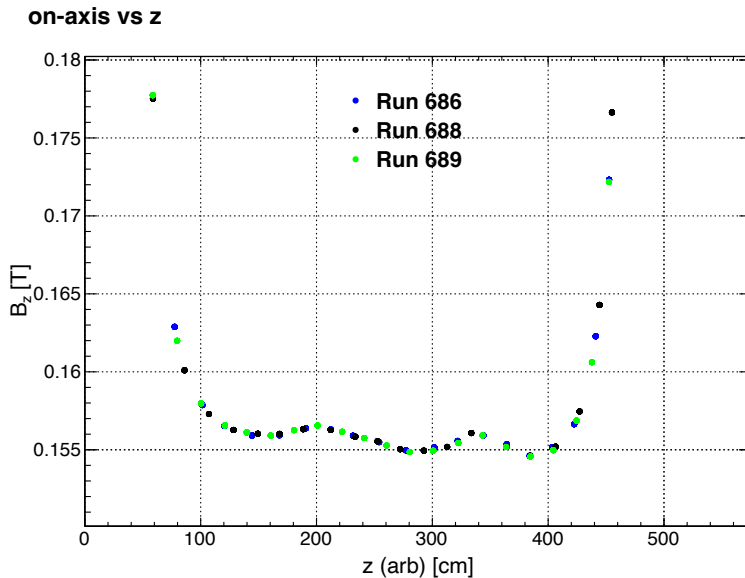


On-axis scans in DV and fitler

On-axis scan

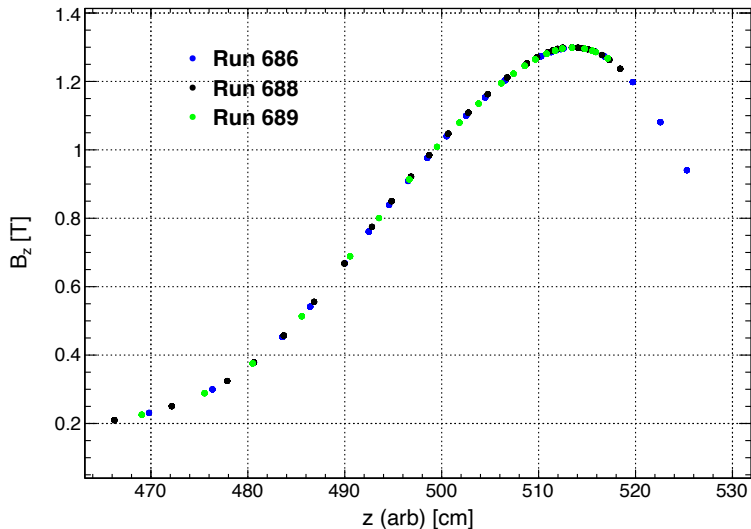


On-axis scans in TOF and UDet: 686, 688, 689



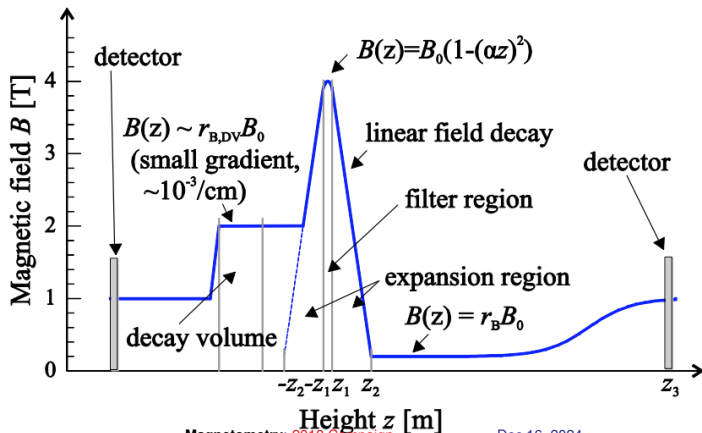
On-axis scans in TOF and UDet: 686, 688, 689

on-axis vs z

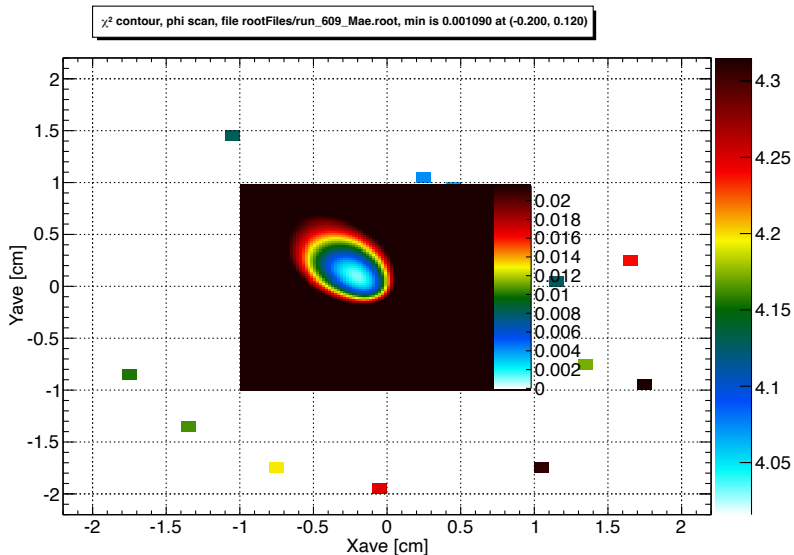


r_B , $r_{B,DV}$, α parameters in the systematic table

- CMS analysis from 2019
- preliminary on-axis results: $\alpha = 0.031 \text{ cm}^{-1}$ (analytical calculation 0.029 cm^{-1}), $r_B = 0.0386$ (analytical calculation 0.039), $r_{B,DV} = 0.424$ (analytical calculation 0.41). **Need off-axis and to complete this assessment**

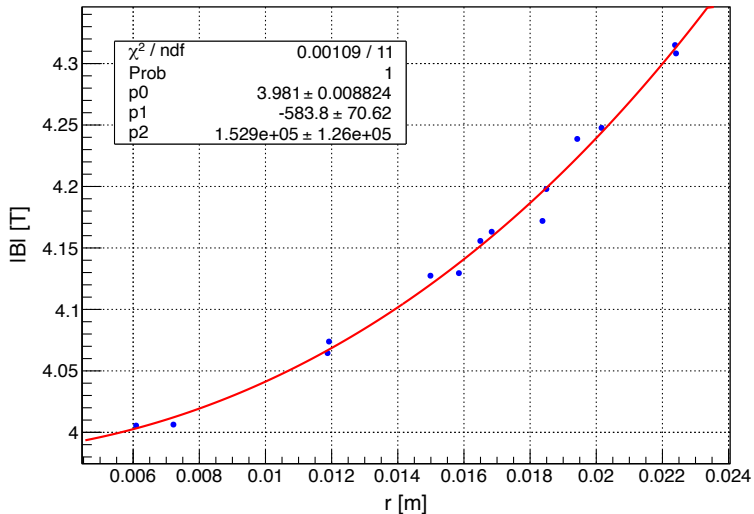


ϕ scans around Filter, e.g 609, 616, 634



ϕ scans around Filter, e.g 609, 616, 634

Run 609, Fit from offsetting the data by (-0.20,0.12)



Results from phi scans fit to magnetic center

From runs 609, 616, 634 the results are:

- $(-0.20, 0.12), (-0.18, -0.08), (-0.22, 0.06)$

Notes ($z=0$ is the theoretical center of the filter coil, 13.2 cm up from the center of the decay volume as measured by SA) :

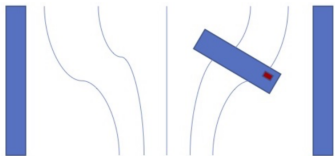
- run 609 is at $z = 0.12$ cm
- run 616 is at $z = -0.9$ cm
- run 634 is at $z = 0.9$ cm

In the peak of the filter, the tilt angle is zero (the theoretical minimum is $z = -0.2$ cm, at which B_r vanishes for all r).

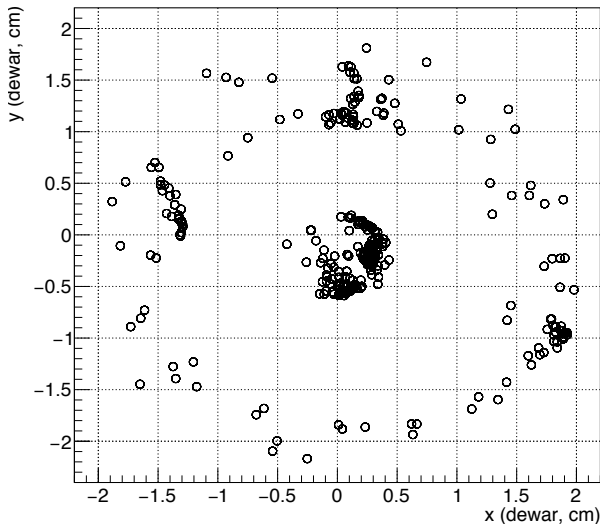
- What are the B_r contributions at these z positions off-axis? Do they need to be included in the fit? **Yes**
- The other phi scan runs 610, 613, 633 need the full fit

All points in the DV and filter

Did we tilt correctly?
Consistently?



- B_z fit seems ok for z values where B_r is small
- Need 3D fit and include B_r (physically allowed fields)



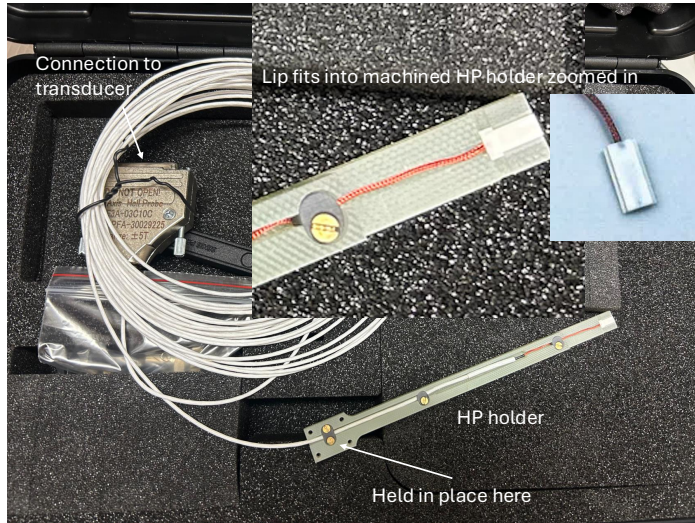
3-Axis B field Transducer F3A for 2025-2026 Campaign

Hall Probe and Holder



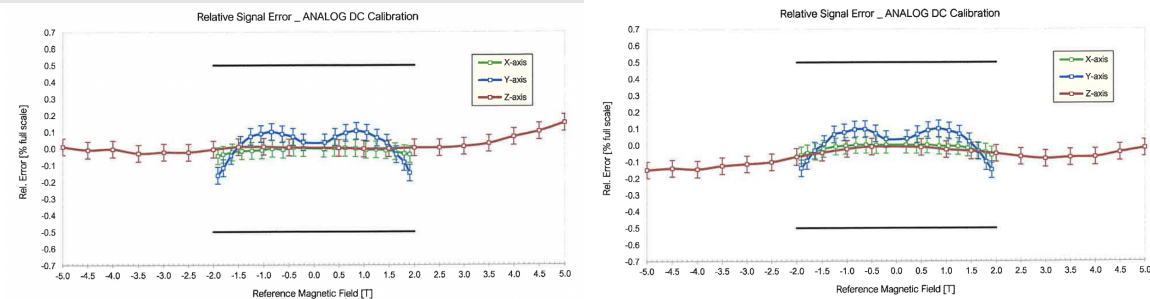
- Accuracy up to 0.05%
- active temperature compensation, “virtually” no planar Hall Effect

3-Axis magnetic field Transducer 3FA with fully integrated HP



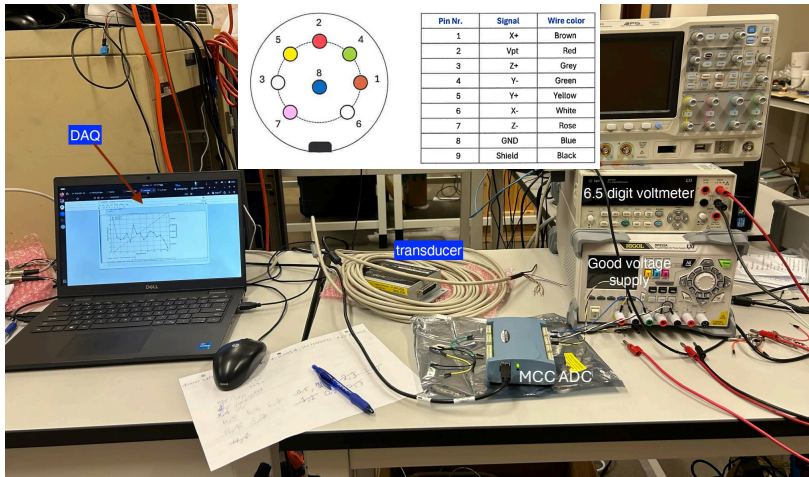
- we have two 3D transducer/HP's that have been calibrated to 2 T in x,y and 5 T in z
- each transducer/HP combo measures differential channels for B_x , B_y , B_z
- measure the temperature voltage with respect to signal ground

High field 5T calibration



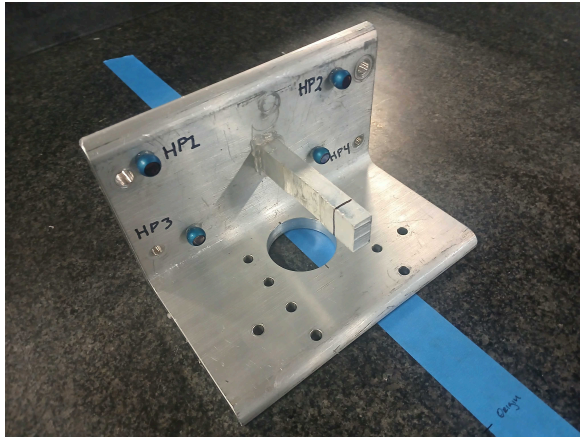
Reference Magnetic Field B [mT]	Reference Magnetic Field B [T]	Measured Output Voltage Vz [V]	Magnetic Sensitivity Sz [V/T]	Relative Error [% of BZFS]
4999.878	4.999878	10.01320	2.003	0.15
4499.784	4.499784	9.00797	2.002	0.10
3999.891	3.999891	8.00491	2.002	0.07
3499.817	3.499817	7.00074	2.001	0.03

F3A Hall Probe Testing



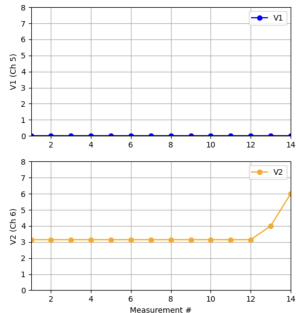
- grounding: each probe/transducer combo needs separate ground. So we need an ADC for each of the two probes
- cross talk: checked between all differential channels to find the least noisy of the channels. Only need 4/8 differential channels
- compared to quality voltmeter

mapdaq2 and S&A



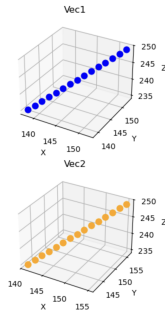
- S&A is graciously delivering a program that sights 3-4 retroreflectors on the trolley (see Richard) and gives us the position of the HP with respect to the “center” of the decay volume
- Converts voltage from $B_{x,y,z}$ to field through interpolating calibration table
- On-line plots of B_z vs z for each x, y, z that also calculates the analytical field (Ferenc routine) for checks

mapdaq2 and S&A



Take Measurement

Tristin Ingram and Josiah Miller



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- Converts voltage from $B_{x,y,z}$ to field through interpolating calibration table
- On-line plots of B_z vs z for each x, y, z that also calculates the analytical field (Ferenc routine) for checks

Pictures of dewar installation



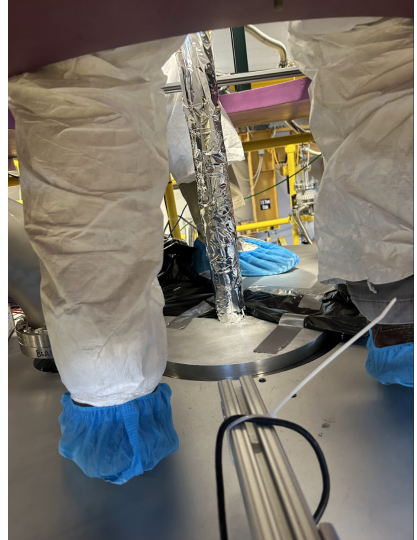
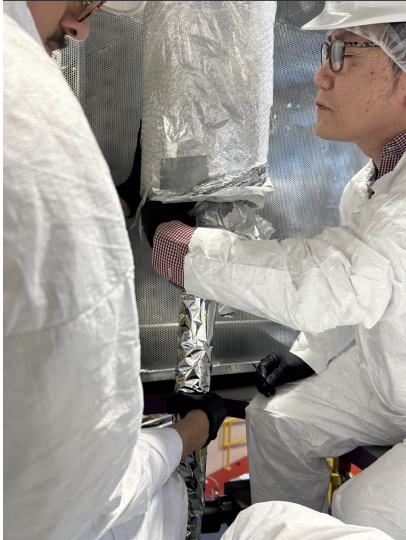
Pictures of dewar installation



Pictures of dewar installation



Pictures of dewar installation





Thermal Stabilization for Hall Probe

| Fahat Hossain | Department of Physics and Astronomy | University of Kentucky |

Motivation

❑ To maintain the probe near 15°C to preserve calibration accuracy and minimize sensitivity drift

- Probe was calibrated at 15°C .
- Probe temperature was between $17 - 21^{\circ}\text{C}$ in filter and DV last magnetometry campaign.



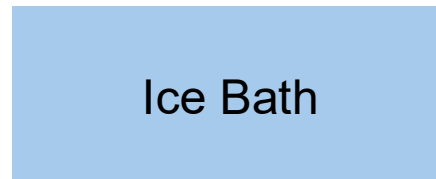
Temperature Coeff. Sensitivity $< \pm 100 \text{ ppm}/^{\circ}\text{C}$ ($\pm 0.01 \text{ } \%/^{\circ}\text{C}$)

[From Hall Probe datasheet]

Nitrogen Gas
from Bottle



Heat Exchange in
Thermal Bath



$\sim 15^{\circ}\text{C}$ gas



Bore



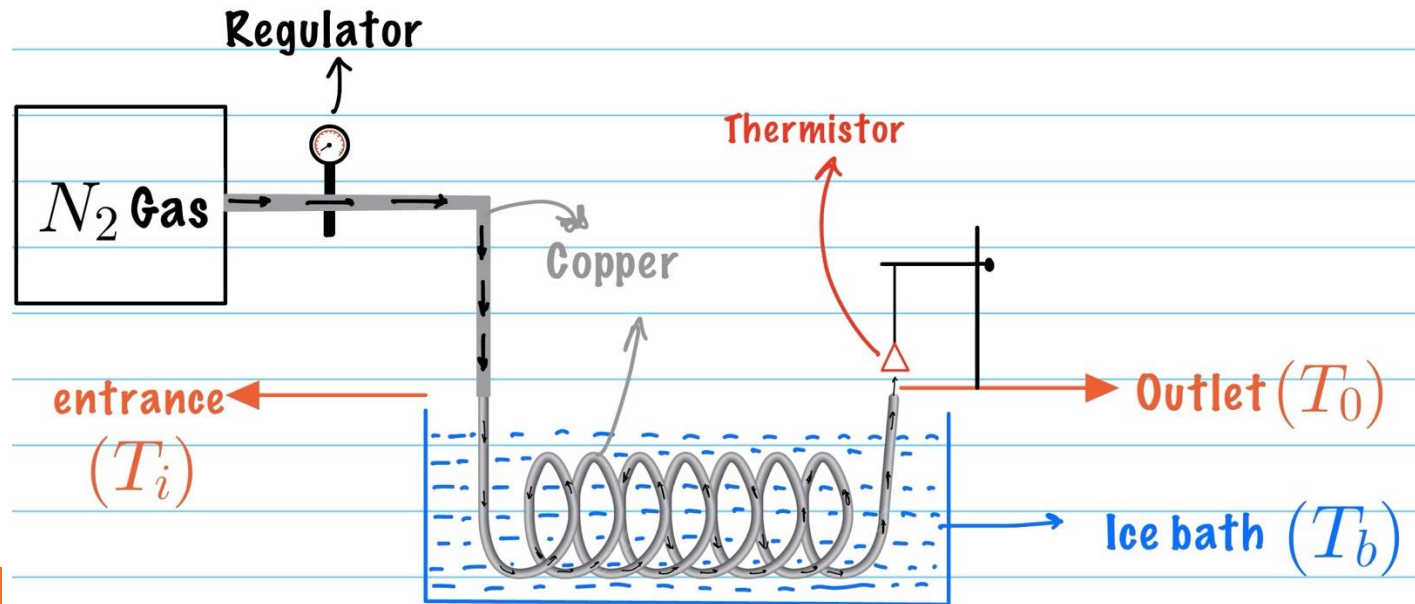
$\sim 15^{\circ}\text{C}$ gas

[Brad Plaster, UK]

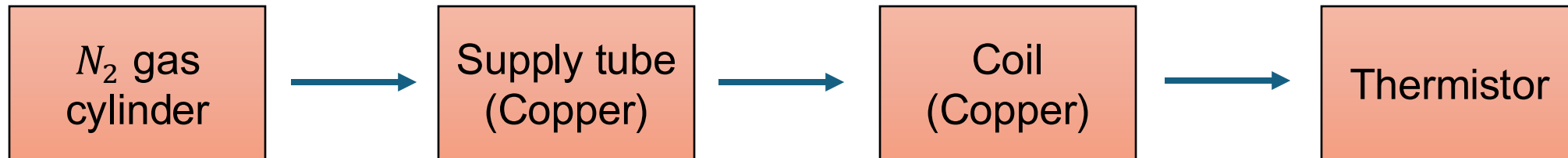
Probe



Toy-model of the Heat Exchanger for Lab Test



Block Diagram



Outlet temperature,

$$T_o = T_b + (T_i - T_b) \exp\left(-\frac{U\pi D_i L}{\dot{m}c_p}\right)$$

$U \rightarrow$ Overall heat transfer coefficient

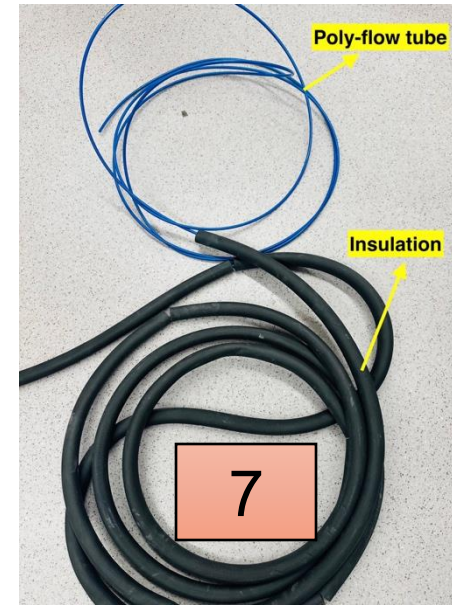
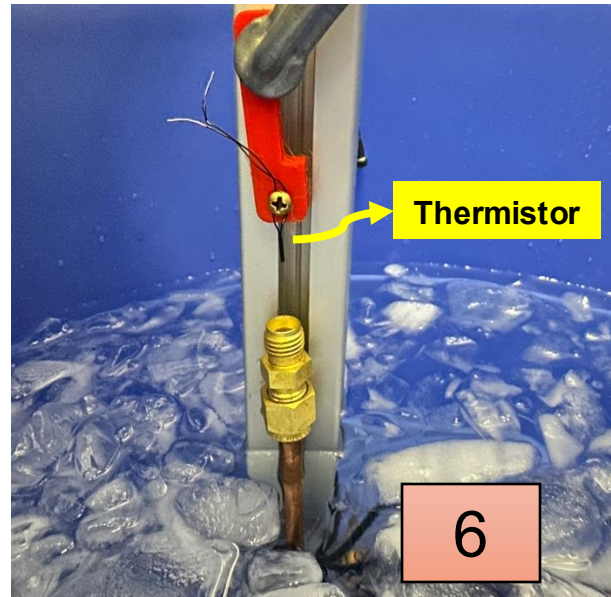
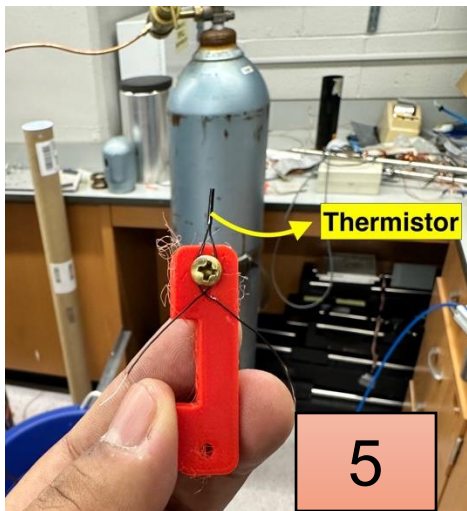
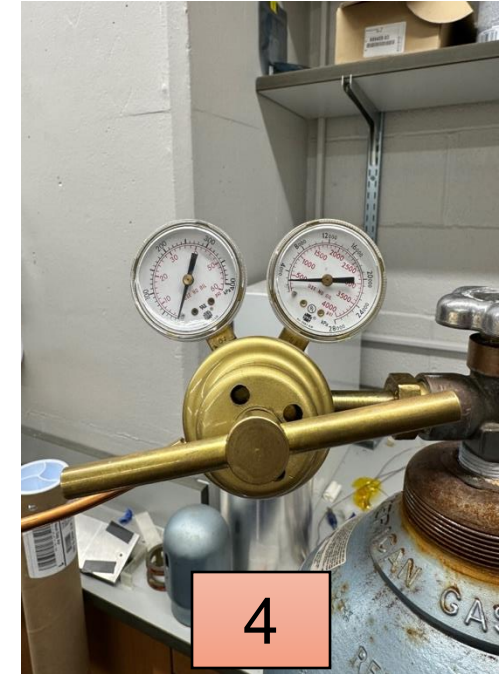
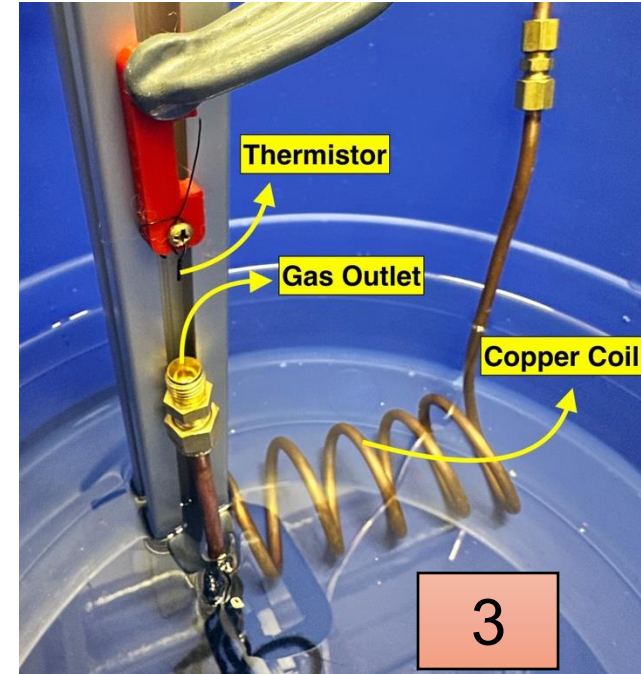
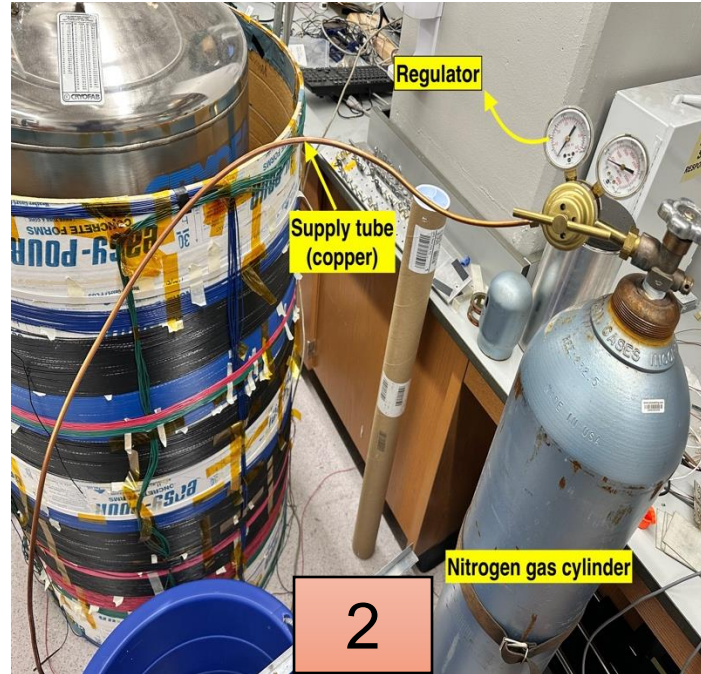
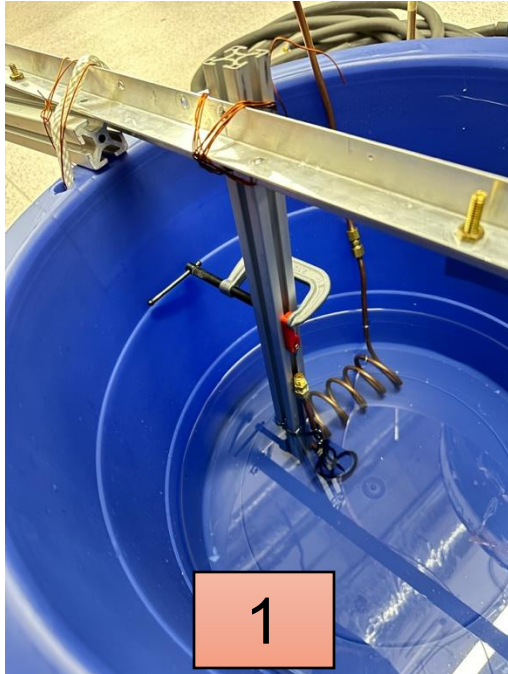
$D_i \rightarrow$ inner diameter of the copper

$\dot{m} \rightarrow$ mass flow rate,

$c_p \rightarrow$ Specific heat

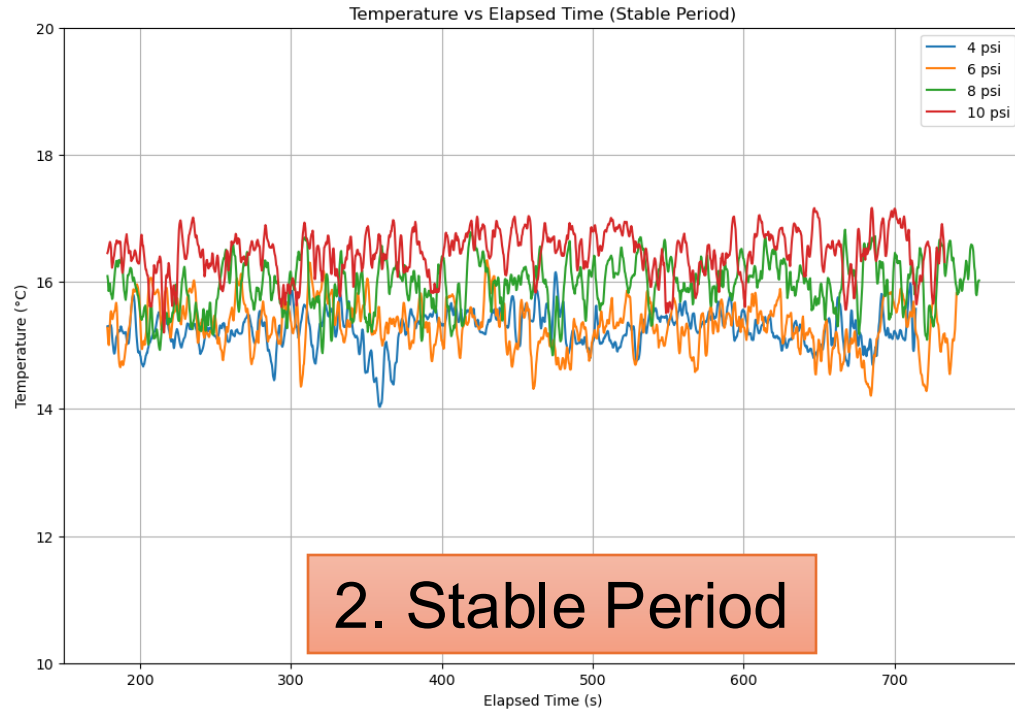
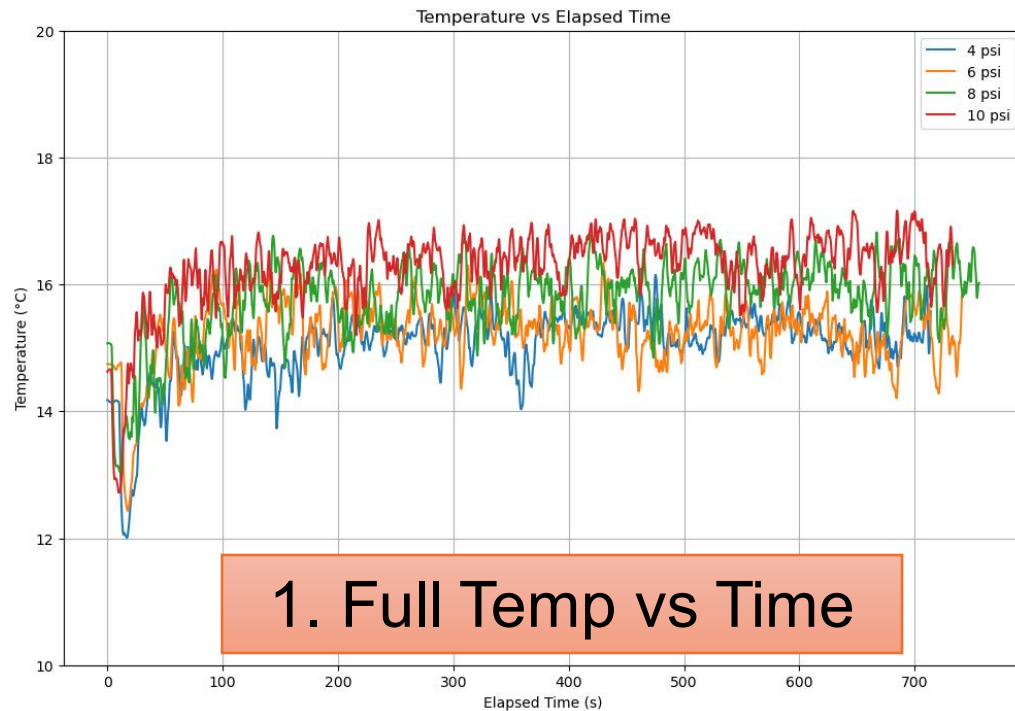
$L \rightarrow$ length of the coil

Heat Exchanger Setup at UK

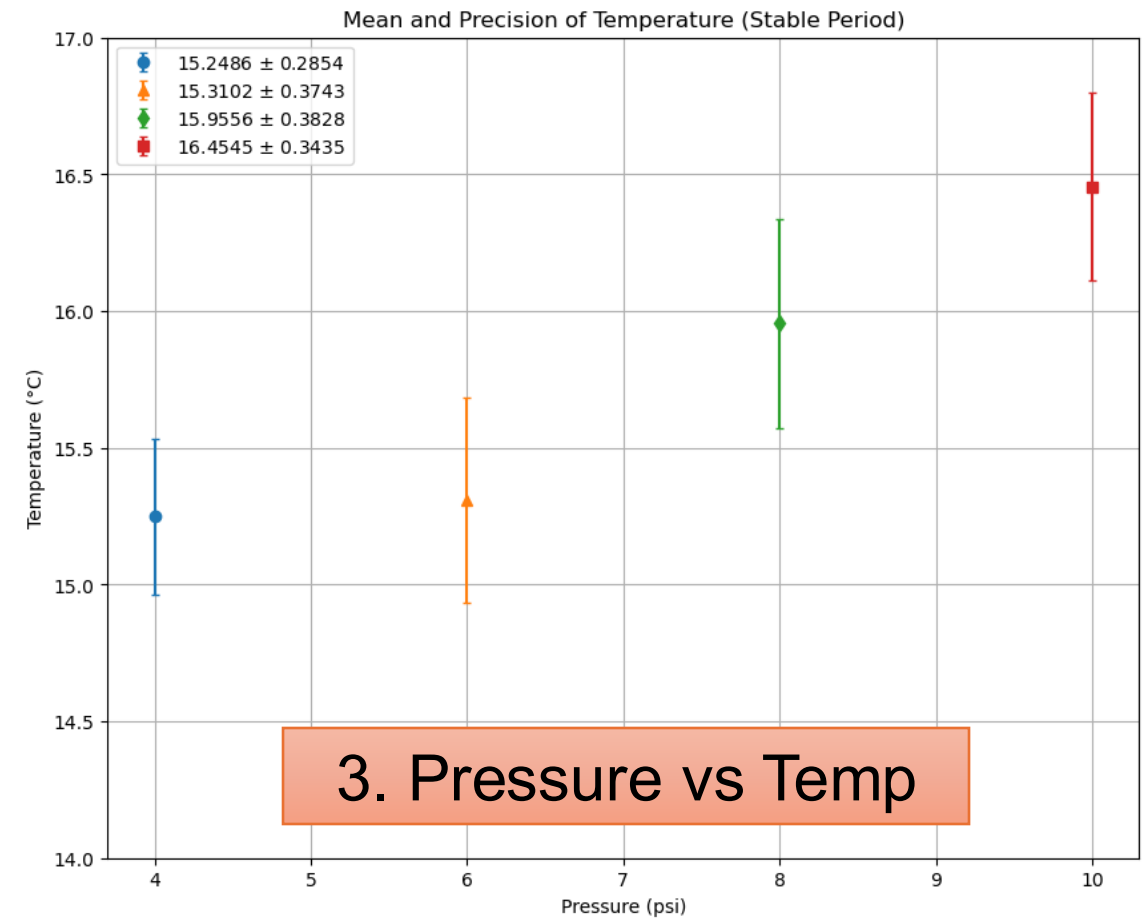


Method

- ❑ Measured initial room temperature.
- ❑ Prepared an ice–water bath and allowed the coil to reach thermal equilibrium.
- ❑ Began gas flow and recorded temperature for different regulator pressures.
- ❑ Allowed the thermistor temperature to stabilize between trials before collecting the next data set.
- ❑ In this experiment, we measured temperature of the gas flow just coming out from the coil.



- Room temperature was $\sim 24^{\circ}\text{C}$
- Stable temperature after ~ 3 minutes.



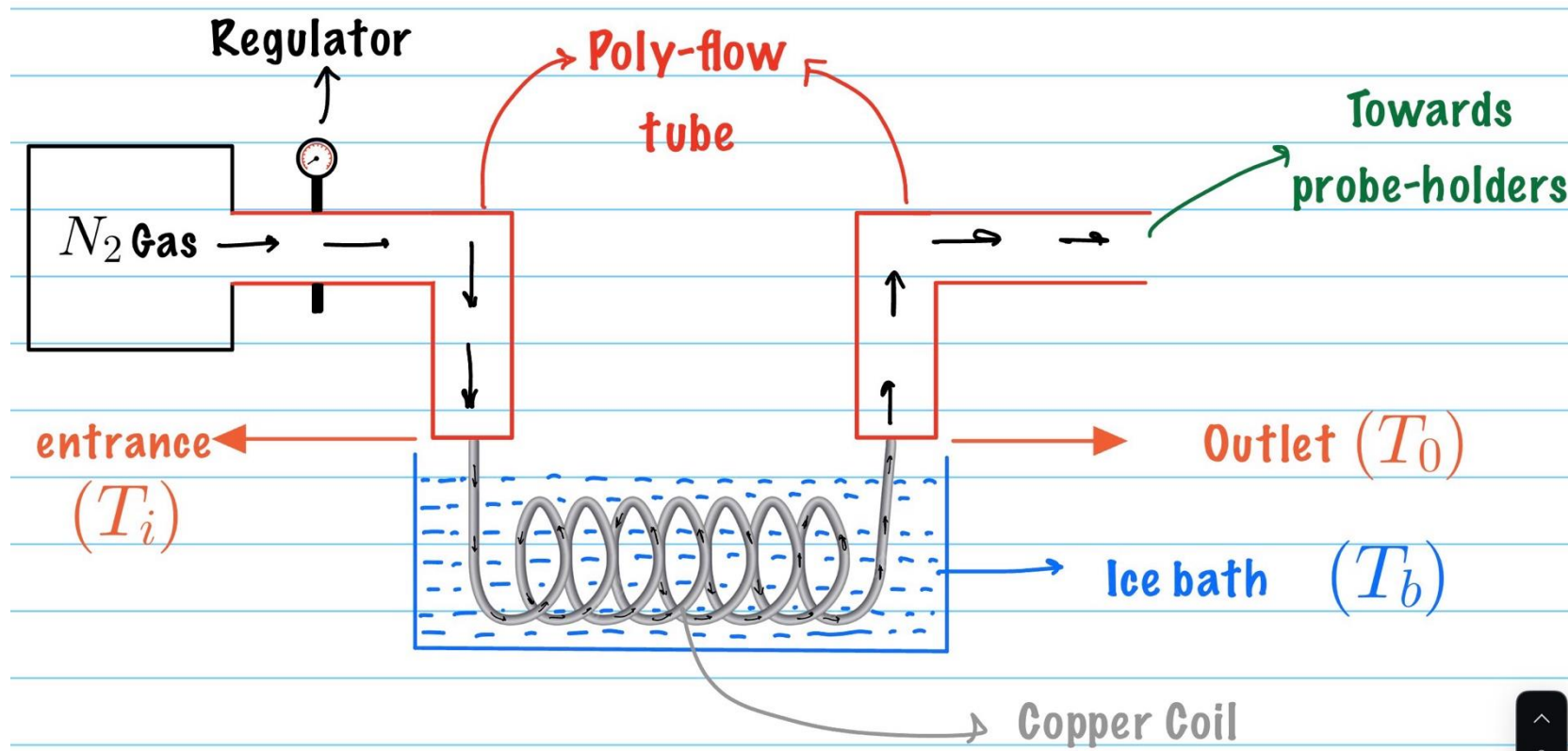
Takeaways/Future Work

- ❑ Across different pressure settings, the measured thermistor temperature remained stable within $< \pm 0.4\text{ }^{\circ}\text{C}$ indicating good thermal control of the probe during operation.
- ❑ We are going to implement further refinements on this experiment.

Thanks to...

- Karl Belin (UK)
- Gene Baber (UK)
- Nab Magnetometry Team
- Our group members

Backup



- Poly-flow and copper tubes both have the same inner diameter.

Heat loss by gas $\rightarrow \dot{m} C_p dT$

The heat across the copper = $U \Delta T$

where, $U \rightarrow$ Overall heat transfer co-efficient ($W m^{-2} K^{-1}$)

$$\Delta T \rightarrow T(z) - T_b$$

Then, the total heat transferred through the tube for

a length $dz = U A \Delta T$

$$= U \pi D_i dz \Delta T \quad [D_i \rightarrow \text{inner diameter of copper tube}]$$

Then from 1st law of thermodynamics,

$$\dot{m} C_p dT = -U \pi D_i dz \Delta T$$

$$\Rightarrow \frac{1}{\Delta T} dT = - \frac{U \pi D_i}{\dot{m} C_p} dz$$

$$\Rightarrow \frac{dT}{T - T_b} = - \frac{U \pi D_i}{\dot{m} C_p} dz$$

$$\Rightarrow \frac{df}{f} = - \frac{U \pi D_i}{\dot{m} C_p} dz \quad [\text{taking } f = T - T_b]$$

$$\Rightarrow \int_{f_i}^{f_o} \frac{df}{f} = - \frac{U \pi D_i}{\dot{m} C_p} \int_0^L dz \quad \left[\begin{array}{l} \text{and } f_i = T_i - T_b \\ f_o = T_o - T_b \end{array} \right]$$

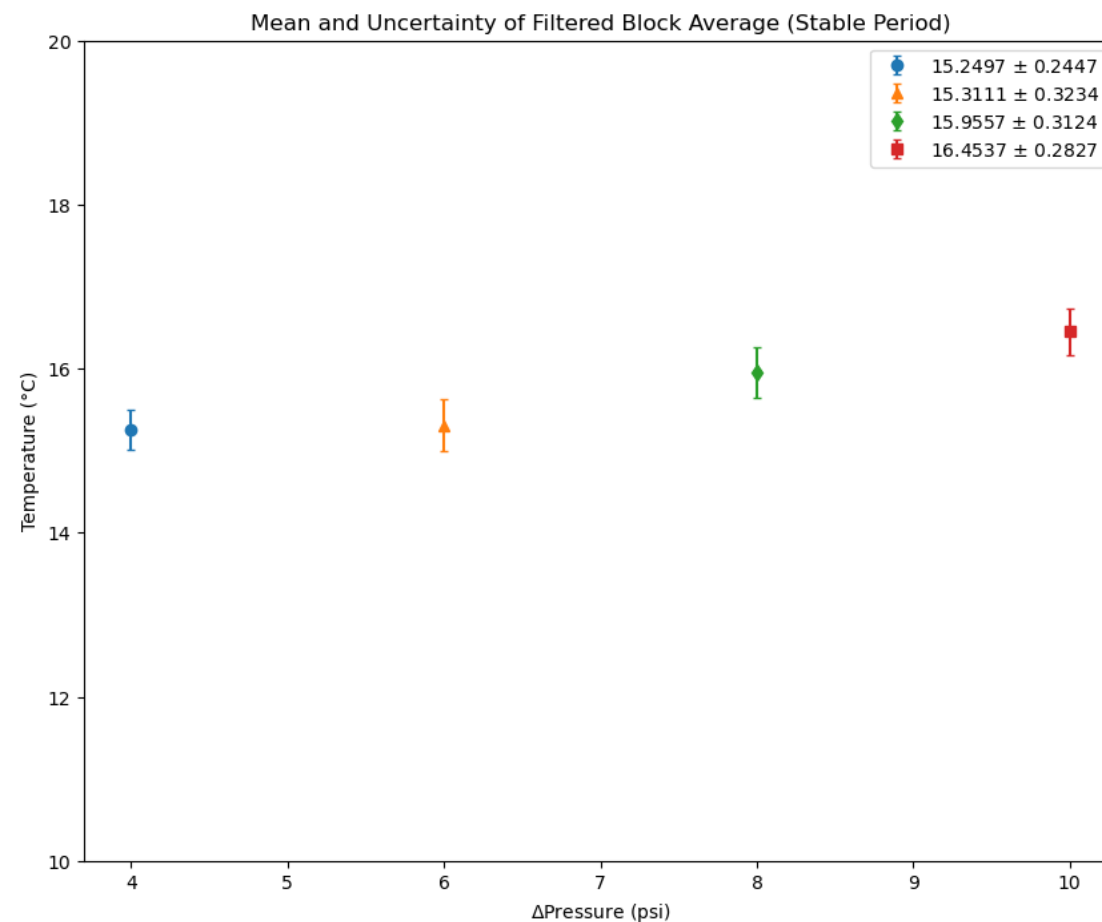
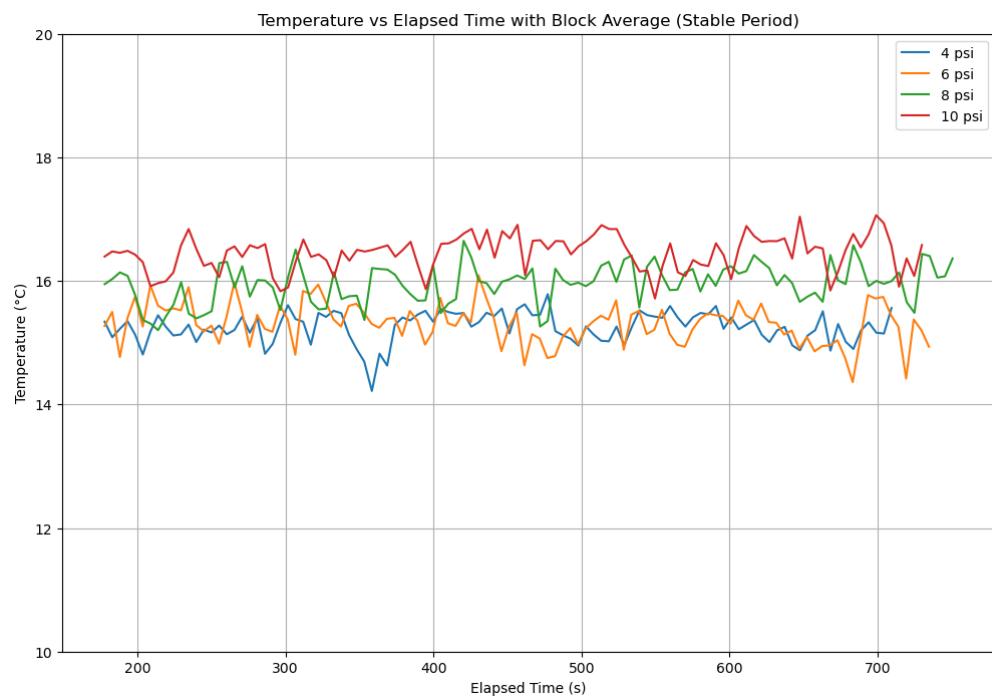
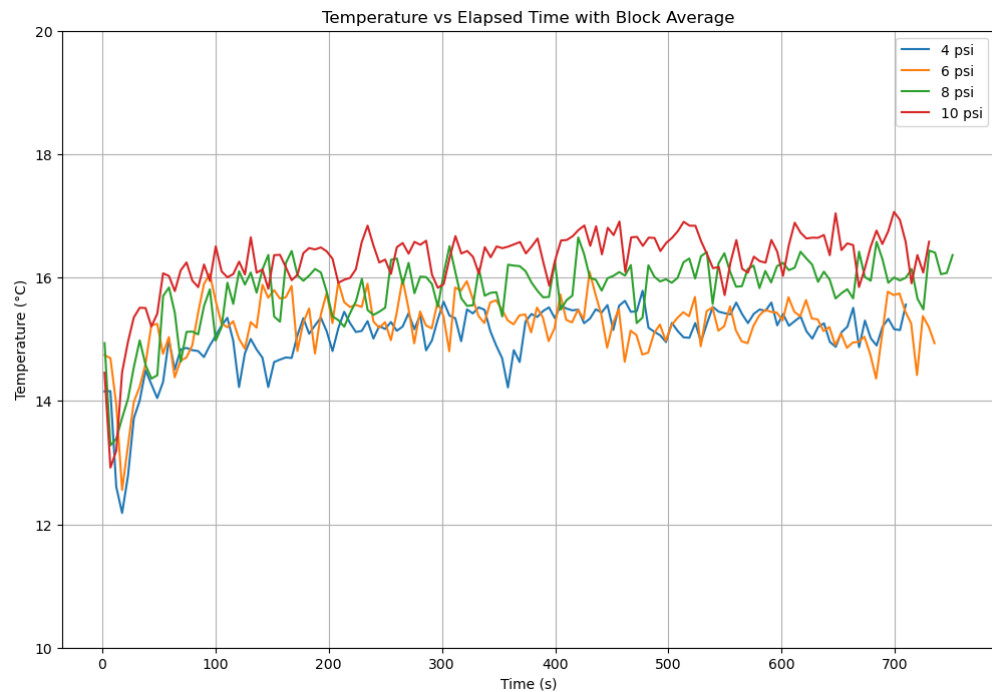
$$\Rightarrow \ln \left(\frac{f_o}{f_i} \right) = - \frac{U \pi D_i}{\dot{m} C_p} L \quad [L \rightarrow \text{length of coil}]$$

$$\Rightarrow \dot{m} = \frac{-U \pi D_i L}{C_p \ln \left(\frac{f_o}{f_i} \right)} = \frac{U \pi D_i L}{C_p \ln \left(\frac{T_i - T_b}{T_o - T_b} \right)}$$

For a given gas flow, the outlet temperature will be,

$$\frac{T_o - T_b}{T_i - T_b} = \exp \left(- \frac{U \pi D_i L}{\dot{m} C_p} \right)$$

$$\therefore T_o = T_b + (T_i - T_b) \exp \left(- \frac{U \pi D_i L}{\dot{m} C_p} \right)$$



Magnetometry Trolley System Co-*Nab*-oration 2025

In contribution to:

Oak Ridge National Laboratory

FNPB

Magnetometry Team

Prepared by:

University of Kentucky

College of Arts & Sciences

Department of Physics & Astronomy

Richard E. McDonald IV

Coming up

- Motivation
- Trolley Overview
- Materials
- Motions
- Initial Tests @ ECU
- Most Recent / Work In progress

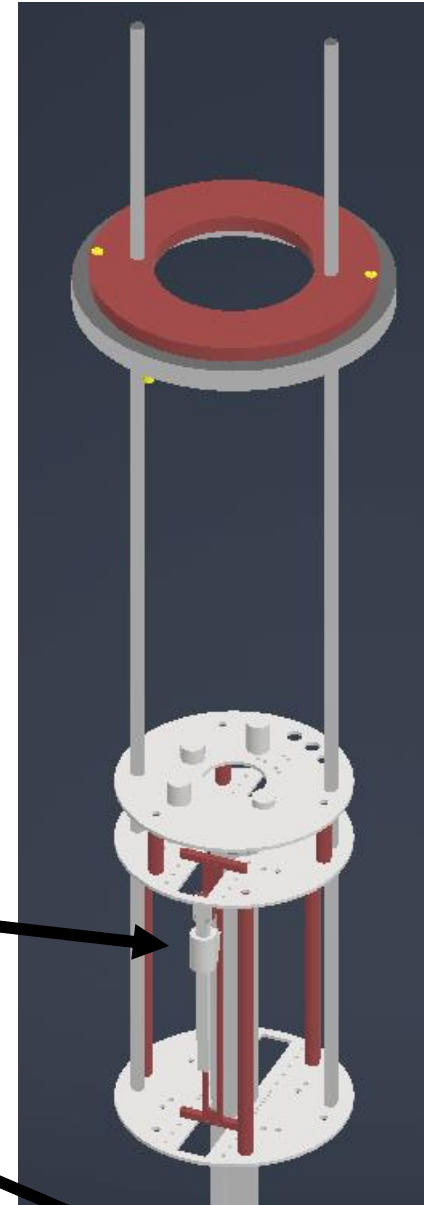
Motivation

- Precisely place the Hall Probe through ~ 7 m of the Nab spectrometer in Z, R, ϕ
- Rigid
- non-magnetic materials
- Capability to hold two 3D Hall Probes

General Picture

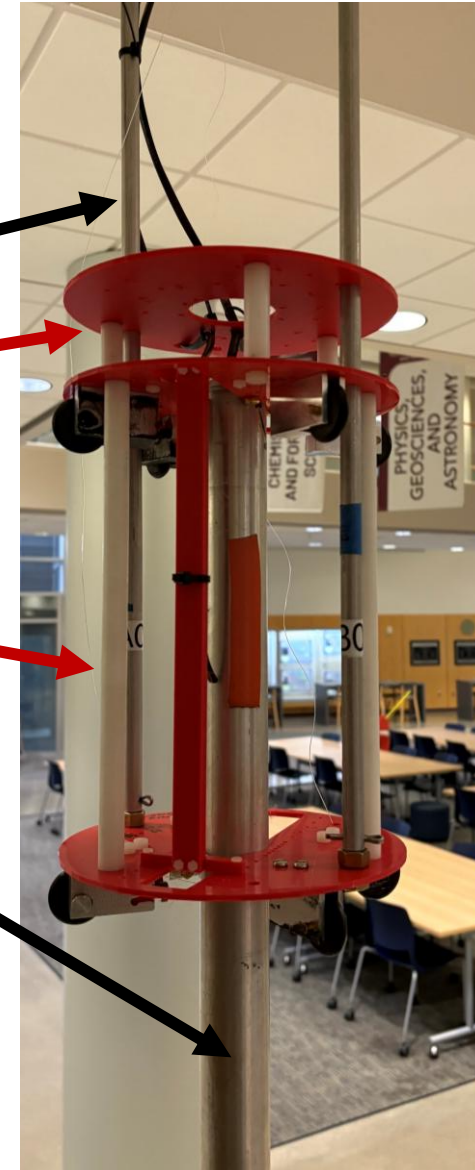
- Stand (not pictured)
- 3-story trolley
- Rigid
- Non-magnetic
- Removable nose

Probe 1
Probe 2
(~ 1.2 meters lower)



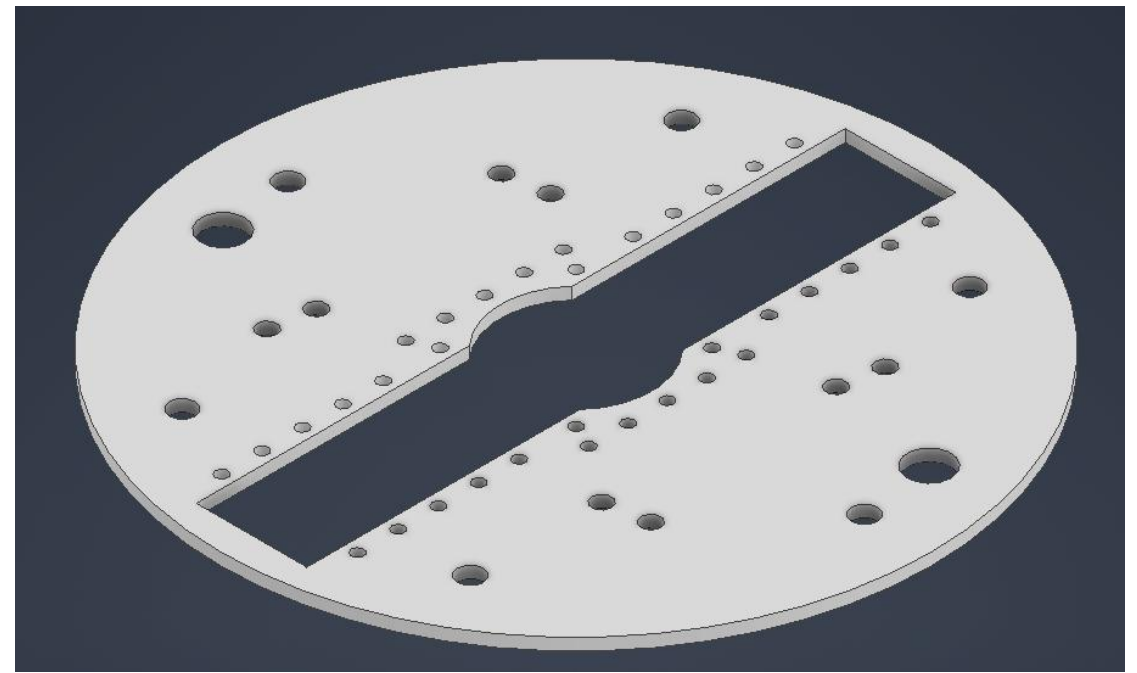
Material List (non-magnetic)

- Stand & Rotary Stage (not pictured)
 - Aluminum
 - 80/20 structure
- Trolley
 - Aluminum
 - Plastics
 - Machined G10
 - 3D printed backups
- Hardware
 - Aluminum
 - Plastic
 - Copper (stand only)
 - Brass (stand only)
 - Titanium



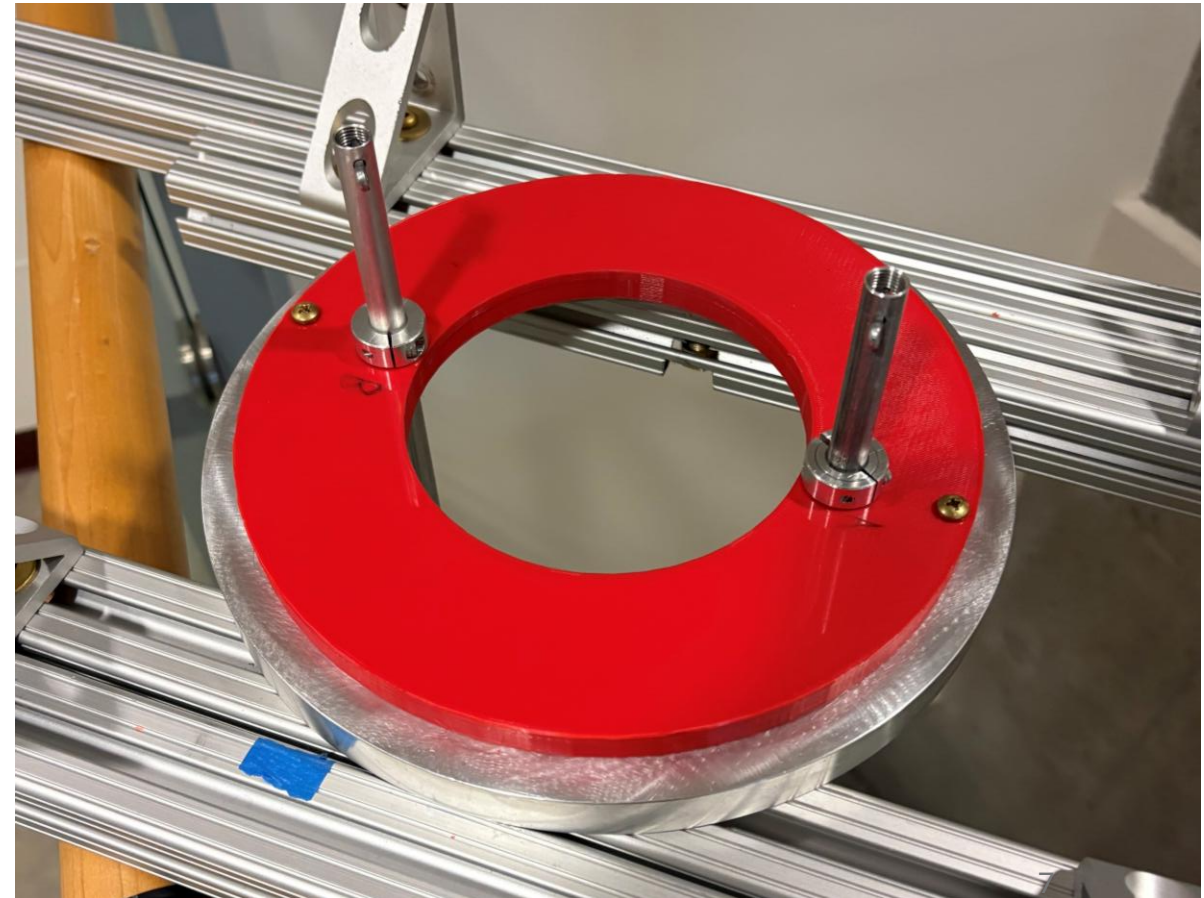
Radial Placement

- Trolley Body (Nose Attached)
 - Maximum Radius: 3.097" = 7.86 cm
 - Minimum Radius: 2.257" = 5.73 cm
(reversed: 1.183" = 3 cm)
- Trolley Nose
 - Maximum Radius: 0.382" = 0.97 cm
(subject to change, modelling bigger nose)
 - Minimum Radius: 0.007" = 0.018 cm



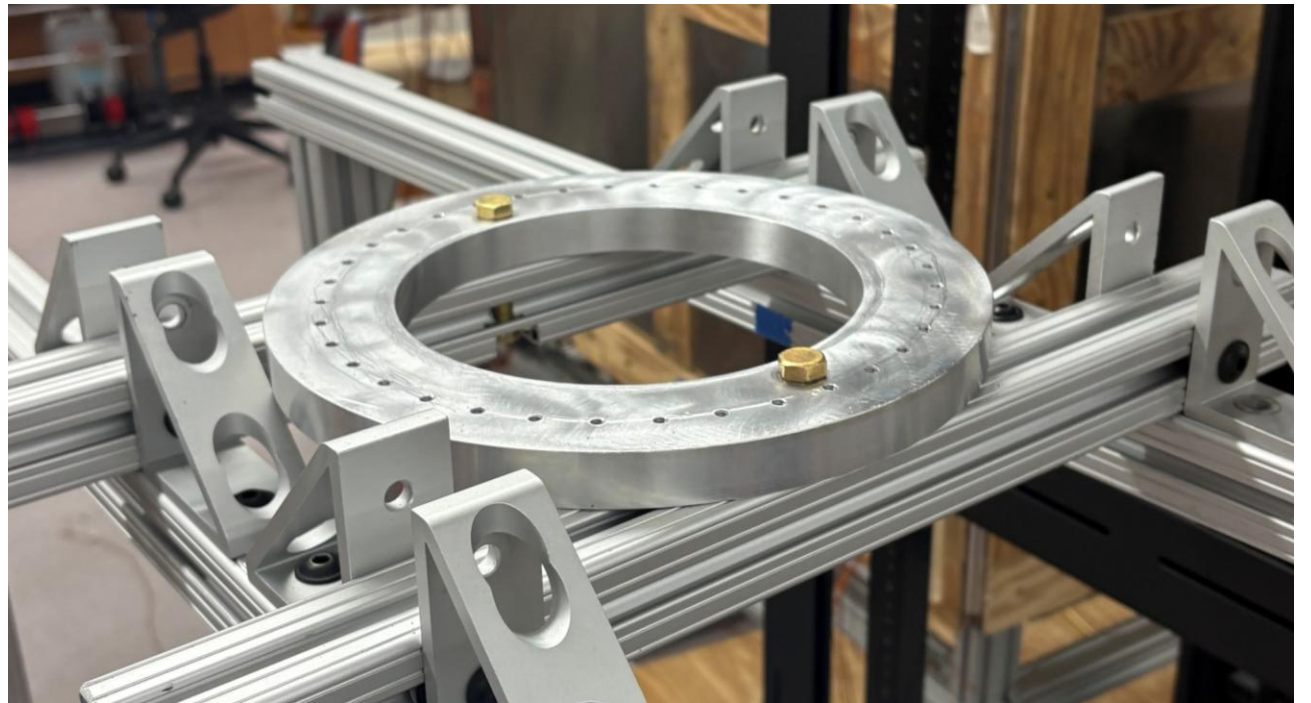
Linear Motion

- Threaded rod sections ~30" long
- Adjustable shaft collars set the depth into the spectrometer
- Talks ongoing of adding permanent positions at points of interest



Azimuthal Motion

- Aluminum rotary stage (machined @ UKY)
- 36 azimuthal positions



Initial Tests @ EKU

- Extended the trolley by 5 modular rods (~ 4.4 meters)



Summary / Work In Progress

- Second magnetometry campaign trolley has been designed & tested
- Meeting with S&A this week @ ORNL
- Final G10 trolley plates being machined @ UKY
- Order professionally 3D printed materials (backup)
- Wrap up 3D printing @ UKY (backup to the backup)

Thanks to...

- Karl Belin UKY machinist
- Harvey Beber UKY gas handler
- Magnetometry Team

Magnetometry Campaign: Proposed Magnetic Field Mapping Strategy and Execution Timeline

Sharia Sharmin

Advisor: Dr. Brad Plaster

University of Kentucky

Nab Collaboration Meeting | University of Tennessee, Knoxville

Experiment Site: Oak Ridge National Laboratory (ORNL)

December 2025

Strawperson Mapping Plan

Testing

Days 1-2

Objective: To test retro reflector system, tilt control, proper cable management, and calibrate z-positions prior to final mapping



On-Axis

Days 3-6

Objective: To acquire high-precision on-axis magnetic field data to enable meaningful comparison with simulations and previous mapping campaigns



Off-Axis

With 1 probe-Days 7-35
With 2 probes-Days 7-25

Objective: To document off-axis field structure through dense axial (z) scans at >6 radial positions with at least 3 azimuthal (ϕ) scans for further analysis

Testing

Days 1-2

Objective: To ensure accurate probe positioning, tilt control, proper cable management, and calibrate z-positions prior to final mapping



On-Axis

Days 3-6

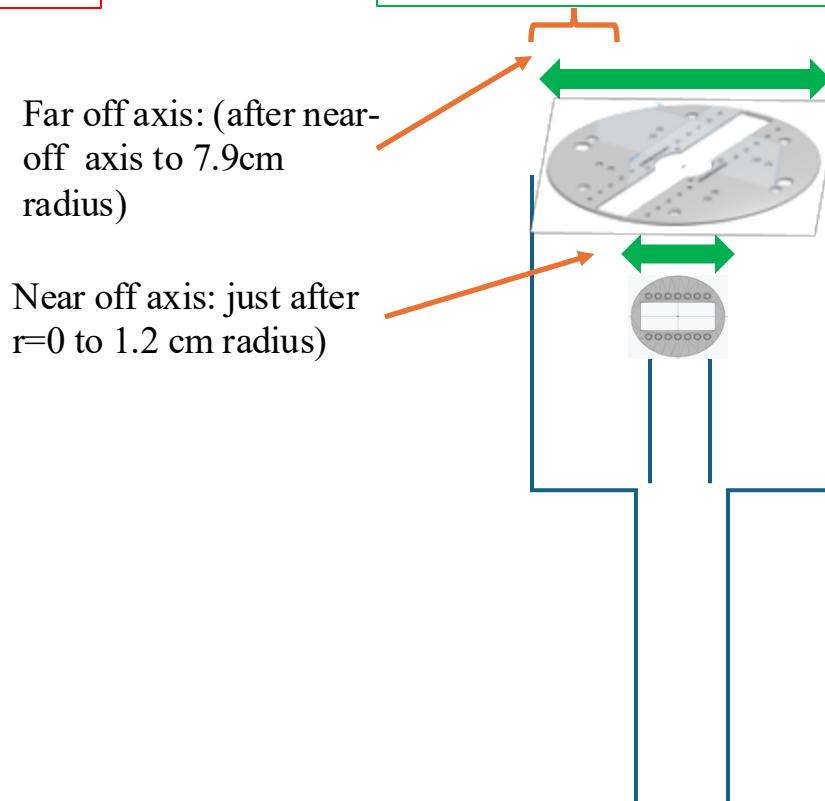
Objective: To acquire high-precision on-axis magnetic field data to enable meaningful comparison with simulations and previous mapping campaigns



Off-Axis

With 1 probe-Days 7-35
With 2 probes-Days 7-25

Objective: To document off-axis field structure through dense axial (z) scans at 6–7 radial positions with multiple azimuthal (ϕ) scans for further analysis



Dewar diameter: 19cm
Big disk diameter: 18.7cm
Small disk diameter: 3.0cm

Near off axis: in that region we will be putting our small nose
Far off axis: we will be putting our big disk with outer slots

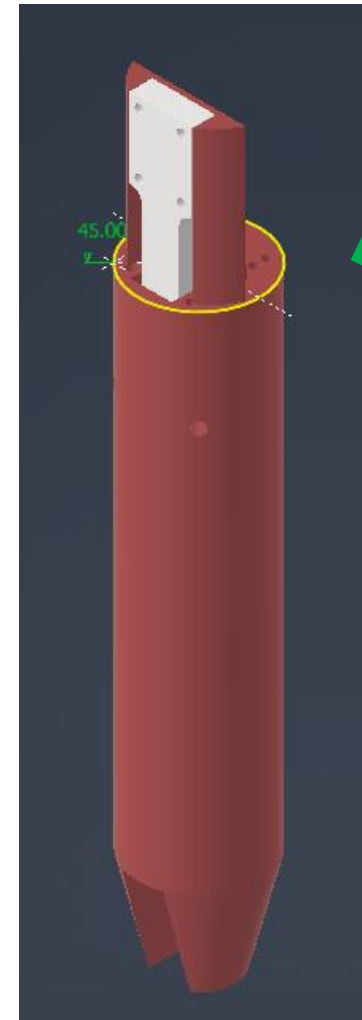
Mapping strategy(On axis)

On axis($r = 0$, & Total 342 positions in z)

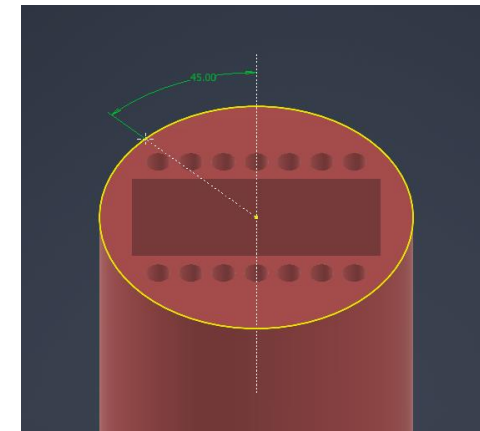
Goal is to:

- Collect dense scan data along on axis
- Maintain the scan parameters to ensure consistency across runs in different regions
- Provide a stable on-axis dataset to serve as a baseline for off-axis scans
- Ensure compatibility with simulation geometry and prior mapping datasets

Z(cm)	Description	Δz (cm)	Number of points
-120 \rightarrow -100	LDet	1	20
-100 \rightarrow -50	LDet Drift	5	10
-50 \rightarrow -10	Below Filter	1	40
-10 \rightarrow +20	Filter	0.5	60
+20 \rightarrow +50	Above Filter	1	30
+50 \rightarrow 450	TOF Drift	5	82
450 \rightarrow 500	UDet	0.5	100



**With the nose: On axis
and near off axis**

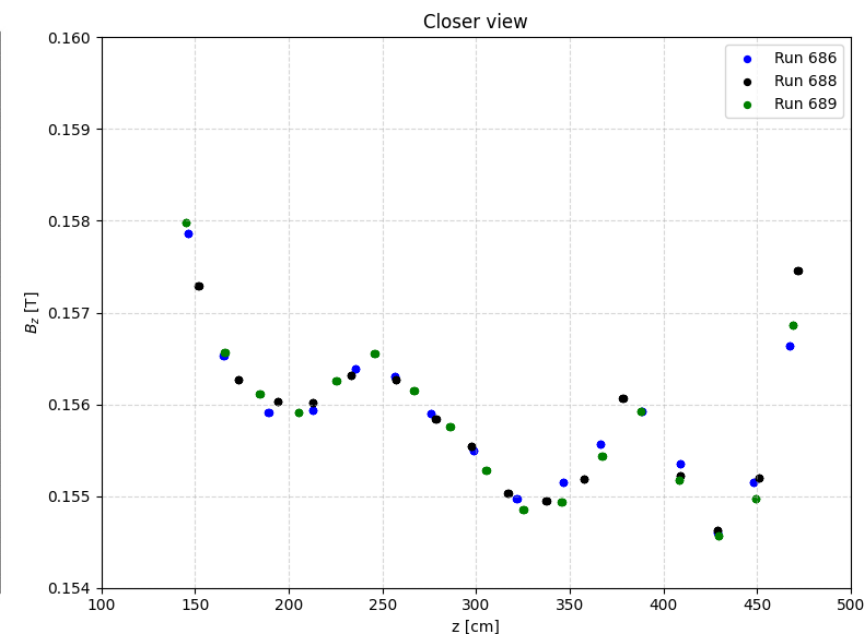
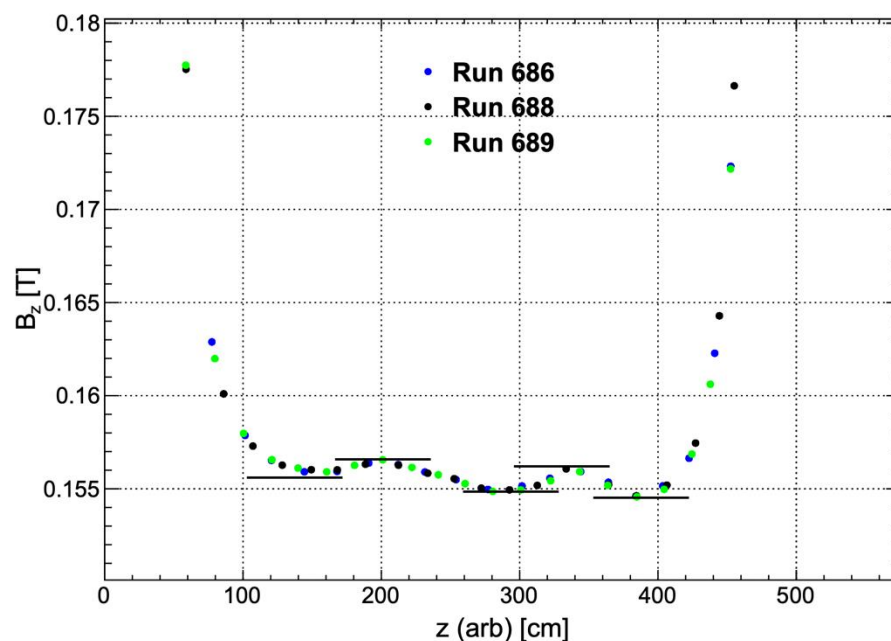


Mapping strategy adapted from J. Fry, 2017 presentation
Image courtesy: Richard

Some specific regions of interest(in TOF)

- Data from the previous campaign show small, localized variations in the TOF region that appear consistently across multiple runs
- These observations motivate targeted, denser scans in selected TOF regions during the upcoming campaign to better resolve the observed structure

Image courtesy: Jason Fry



Positions	Start z(cm)	End z(cm)	Position(R) in cm	Phi position(ϕ)	Step size in z(cm)	Time(mins)	Total steps	Total time(minutes)
								0
On axis(different z positions) TOF	145	155	0	1	2	3	6	18
On axis(different z positions) TOF	175	210	0	1	2	3	8	24
On axis(different z positions) TOF	240	280	0	1	4	3	11	33
On axis(different z positions) TOF	315	350	0	1	5	3	8	24
On axis(different z positions) TOF	360	380	0	1	2	3	11	33
On axis(different z positions) TOF	420	460	0	1	4	3	11	33
								0
Off axis(different z positions) TOF	145	155	0.3	6	2	3	6	108
Off axis(different z positions) TOF	175	210	0.3	6	2	3	8	144
Off axis(different z positions) TOF	240	280	0.3	6	4	3	11	198
Off axis(different z positions) TOF	315	350	0.3	6	5	3	8	144
Off axis(different z positions) TOF	360	380	0.3	6	2	3	11	198
Off axis(different z positions) TOF	420	460	0.3	6	4	3	11	198

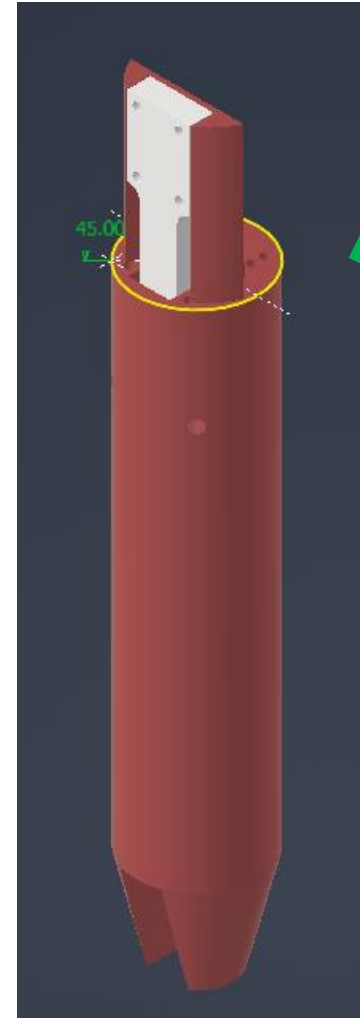
Mapping strategy(Near Off axis)

Near off axis(3 radii, at least 3 ϕ positions & Total 166 positions in z)

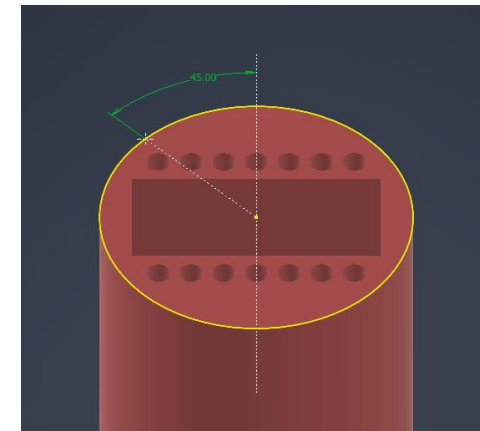
Goal is to:

- Perform dense axial (z) scans at radial positions spanning the accessible aperture
- Acquire multiple azimuthal (ϕ) scans at each radius to sample angular structure
- Utilize the 3-axis Hall probe to record all magnetic field components at each point
- Maintain consistent scan spacing and ordering across r, z, and ϕ coordinates

Z(cm)	Description	Δz (cm)	Number of points
-120 \rightarrow -100	LDet	2	10
-100 \rightarrow -50	LDet Drift	5	10
-50 \rightarrow -10	Below Filter	2	20
-10 \rightarrow +20	Filter	1	30
+20 \rightarrow +50	Above Filter	1	30
+50 \rightarrow 450	TOF Drift	10	41
450 \rightarrow 500	UDet	2	25



With the nose: On axis and near off axis



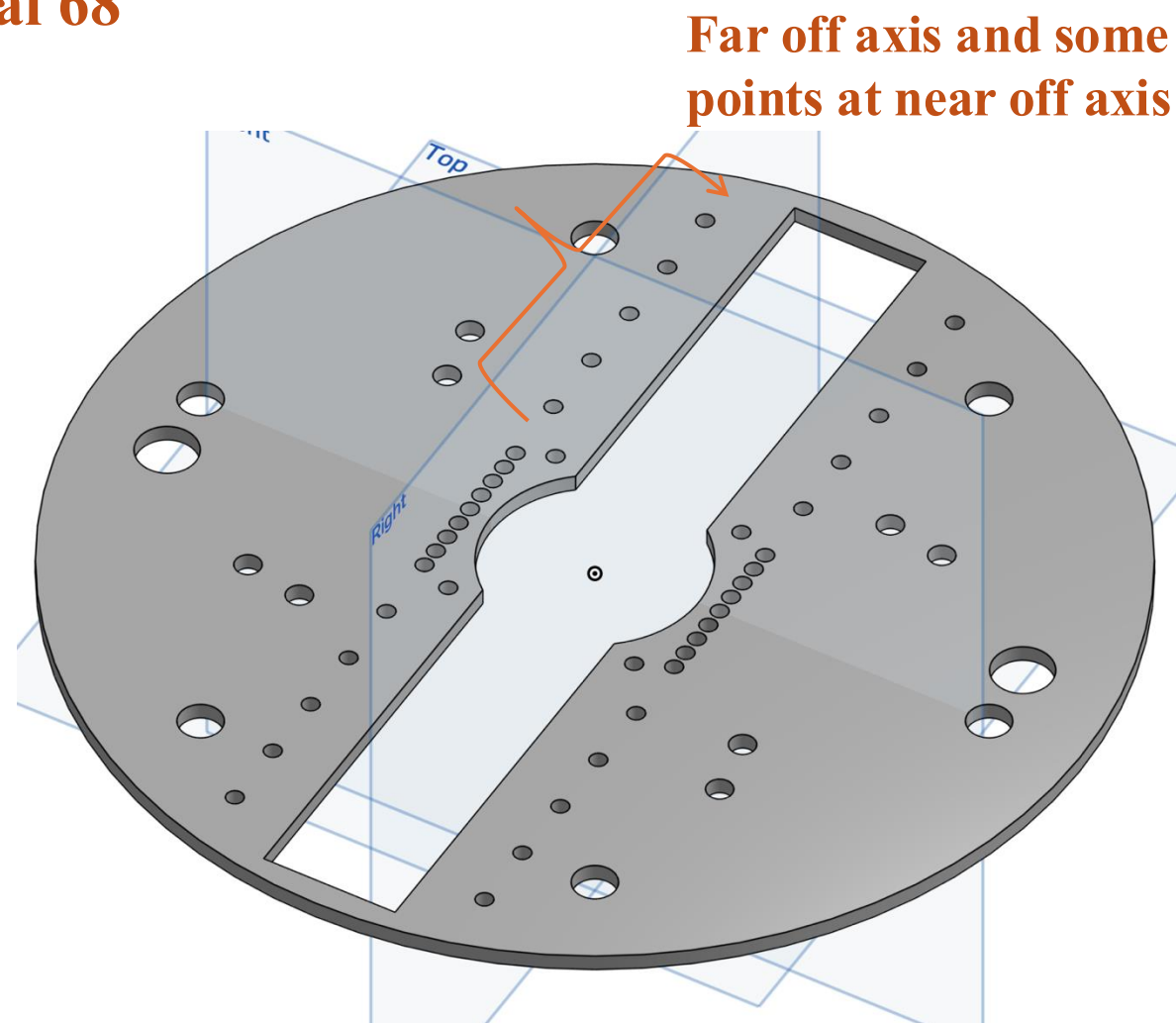
Mapping strategy(Far off axis)

Far off axis(5 radii, at least 3ϕ positions & Total 68 positions in z)

Z(cm)	Description	Δz (cm)	Number of points
450 \rightarrow 500	UDet	2	26
50 \rightarrow 450	TOF Drift	10	42

Far Off-Axis Mapping: Two-Probe Strategy

- A two-probe configuration in which one probe remains in the TOF region while the second probe scans other regions (Filter, LDET, etc.).
- Radial coverage without additional dedicated scan time.
- In the schedule, I pointed out locations where data can be collected in parallel with single-probe measurements, allowing the total time required for far off-axis scanning to be reduced to about 10 days.



Execution Strategy (with 1 probe)

Days	Positions	Start z(cm)	End z(cm)	Position(R) in cm	Phi position(ϕ)	Step size	Time(mins)	Total steps	Total time(minutes)	#VALUE!	
Day 1	Testing-									0	
	Goal is to coarse on axis measurement									0	
										0	
										0	
Day 2	Testing- (Take more data to make sure everything is working perfectly)									0	
										0	
										0	
										0	
Days	Positions	Start z(cm)	End z(cm)	Position(R) in cm	Phi position(ϕ)	Step size	Time(mins)	Total steps	Total time(minutes)	#VALUE!	
Day 3	On axis(different z positions) UDET	450	500	0	1	0.5	5	100		500	
Day 4	On axis(different z positions) TOF drift	50	450	0	1	5	5	81		405	
										0	
Day 5	On axis(different z positions) above Filter	20	50	0	1	1	5	30		150	
	On axis(different z positions) Filter	-10	20	0	1	0.5	5	60		300	
Day 6	On axis(different z positions) Below F	-50	-10	0	1	1	5	40		200	
	On axis(different z positions) LDET drift	-100	-50	0	1	5	5	10		50	
	On axis(different z positions) LDET	-120	-100	0	1	1	5	20		100	342 data points total in z
										0	
Day 7	Off axis(different z positions) UDET	450	500	0.3	6	2	3	25		450	
Day 8	Off axis(different z positions) TOF drift	50	450	0.3	6	10	3	41		738	
					6					0	
Day 9	Off axis(different z positions) above Filter	20	50	0.3	6	1	3	30		540	
Day 10	Off axis(different z positions) Filter	-10	20	0.3	6	1	3	30		540	
Day 11	Off axis(different z positions) Below F	-50	-10	0.3	6	2	3	20		360	
	Off axis(different z positions) LDET drift	-100	-50	0.3	6	5	3	10		180	
	Off axis(different z positions) LDET	-120	-100	0.3	6	2	3	10		180	166 data points total in z
										0	
Day 12	Off axis(different z positions) UDET	450	500	0.6	6	2	3	25		450	
Day 13	Off axis(different z positions) TOF drift	50	450	0.6	6	10	3	41		738	
										0	
Day 14	Off axis(different z positions) above Filter	20	50	0.6	6	1	3	30		540	
Day 15	Off axis(different z positions) Filter	-10	20	0.6	6	1	3	30		540	
Day 16	Off axis(different z positions) Below F	-50	-10	0.6	6	2	3	20		360	
	Off axis(different z positions) LDET drift	-100	-50	0.6	6	5	3	10		180	
	Off axis(different z positions) LDET	-120	-100	0.6	6	2	3	10		180	166 data points total in z

1 probe	2 probes	TOF specific z	1 probe max time	1 probe min time	+
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Backup slides

Measurement Logistics: Summary of Density of Points

On-axis:

z [m]	description	density of points [cm]	number of points
-1.2 \rightarrow -1.0	LDet	1	20
-1.0 \rightarrow -0.5	LDet drift	5	10
-0.5 \rightarrow -0.1	below F	1	40
-0.1 \rightarrow +0.2	F	0.5	60
+0.2 \rightarrow +0.5	above F	1	30
+0.5 \rightarrow +4.5	TOF drift	5-10	83
+4.5 \rightarrow +5	UDet	0.5-1	50-100
		total	293-343

Near off-axis:

- ▶ **150 (min), 1-2 radii?, 3 ϕ 's (min)**

External:

- ▶ **100 points in z (max), 1-3 radii, 3 ϕ 's**

