Theoretical Landscape Behind the Nab Experiment

Nab Collaboration Meeting, UTK

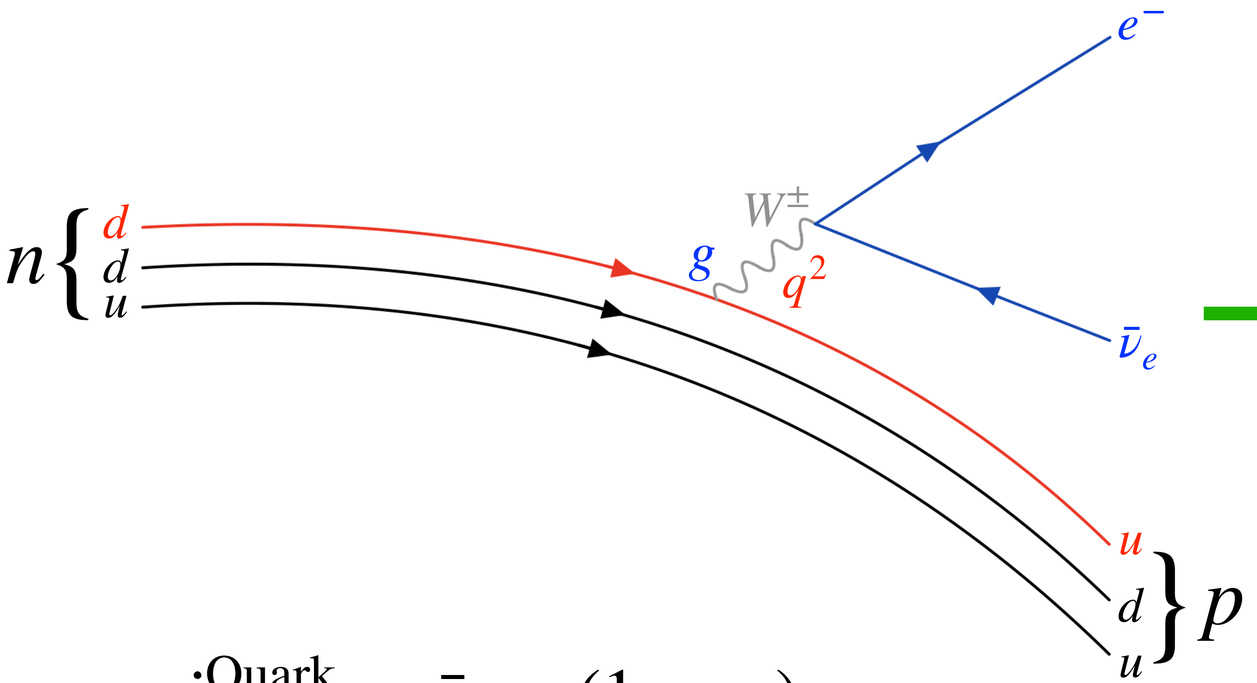
December 15, 2025

Kaushik Borah

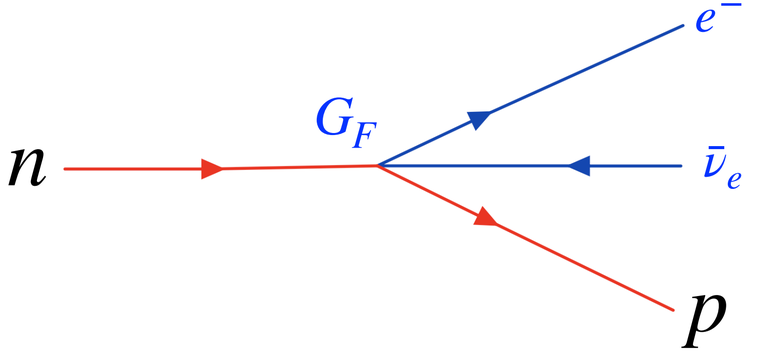
Motivation

Systematic Error Propagation

From Effective Coupling λ to Fundamental CKM Parameter V_{ud}



$q^2 \ll M_W^2$



$$j_\mu^{\text{Quark}} \propto \bar{\psi}_u \, \gamma_\mu (1 - \gamma_5) \psi_d$$

$$j_\mu^{\text{Hadron}} \propto \bar{\psi}_p \, \gamma_\mu (g_V + g_A \gamma_5) \psi_n$$

$$\lambda = \frac{g_A}{g_V}$$

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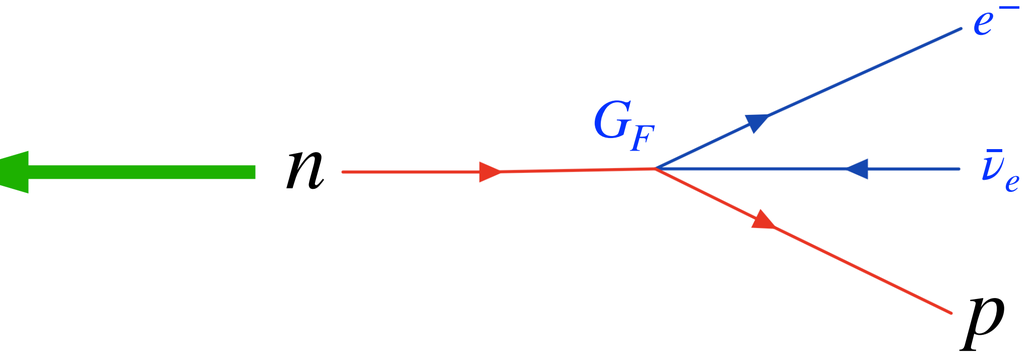
Systematic Error Propagation

From Effective Coupling λ to Fundamental CKM Parameter V_{ud}

$$|V_{ud}|^2 = \frac{2\pi^3/m_e^5}{\tau_n \left(1+3\lambda^2\right) \; G_F^2 \; g_V^2 \; \left(1+\Delta_R^V\right) \; f \; \left(1+\delta_R\right)}$$



$$j_\mu^{\text{Hadron}} \propto \bar{\psi}_p \, \gamma_\mu (g_V + g_A \gamma_5) \psi_n$$

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Motivation

Systematic Error Propagation

From Effective Coupling λ to Fundamental CKM Parameter V_{ud}

Free Neutron Lifetime

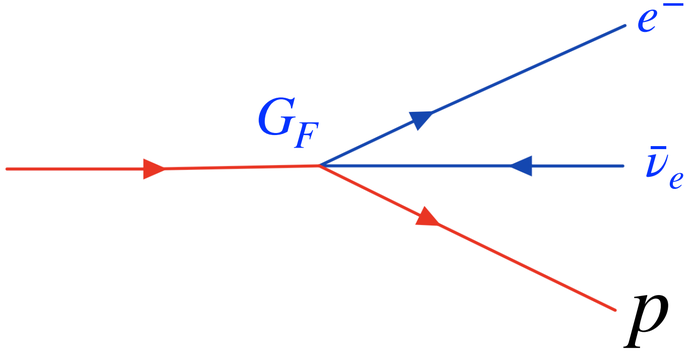
Radiative Corrections

$$|V_{ud}|^2 = \frac{2\pi^3/m_e^5}{\tau_n \left(1+3\lambda^2\right) \; G_F^2 \; g_V^2 \; \left(1+\Delta_R^V\right) \; \boxed{f} \; \left(1+\delta_R\right)}$$

Phase Space Factor

$$\lambda = g_A/g_V$$

Ratio of Axial to Vector Couplings



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Neutron Lifetime from Experiments

UCNτ [Phys. Rev. Lett. 127, 162501 (2021)]

$\tau_n^{\mathrm{UCN}\tau} = 877.75(34)$ s

NIST [Phys. Rev. Lett. 111, 222501 (2013)]

$\tau_n^{\mathrm{beam}} = 887.7(2.2)$ s

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Correlation Coefficients from Experiments

[Phys. Rev. Lett. 132, 102501 (2024)]

$\lambda = -1.2668(27)$ *a*SPECT

[Phys. Rev. C 110, 015502 (2024)]

$\lambda = -1.2712(61)$ *a*CORN

[Phys. Rev. Lett. 122, 242501 (2019)]

$\lambda = -1.27641(56)$ PERKEO III

$$|V_{ud}|^2 = \frac{2\pi^3/m_e^5}{\tau_n \left(1+3\lambda^2\right) \underbrace{G_F^2\, g_V^2 \left(1+\Delta_R^V\right)}_{\text{Renormalize Effective Couplings}} \underbrace{f \left(1+\delta_R\right)}_{\text{Change Phase Space Kinematics and Shape of Spectra}}}$$

Radiative Corrections (RC)

Neutron Lifetime from Experiments

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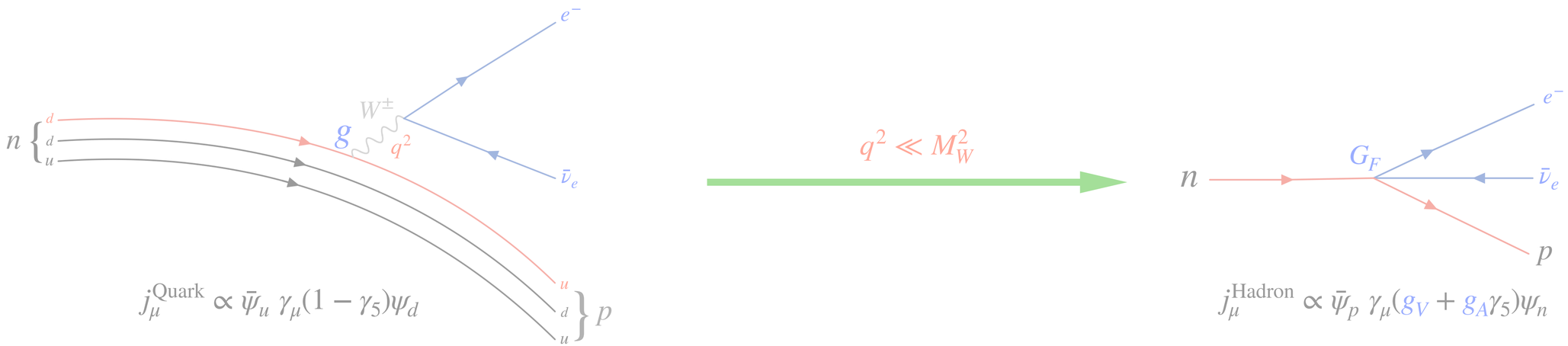

$$\langle p\,|\,\bar{\psi}_u\ \gamma_\mu\left(1-\gamma_5\right)\psi_d\,|\,n\rangle = \bar{u}^{(p)}\left[\gamma_\mu g_V + \gamma_\mu\gamma_5 g_A\right]u^{(n)}$$

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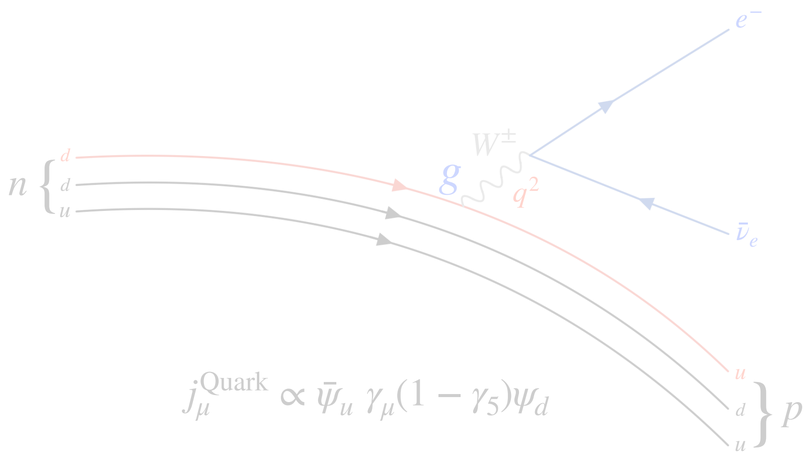
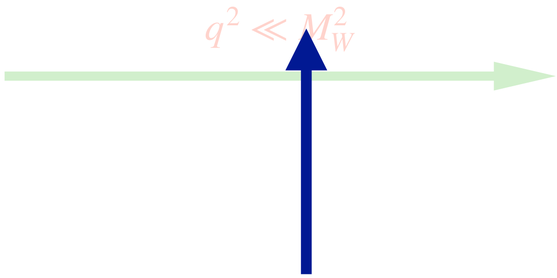
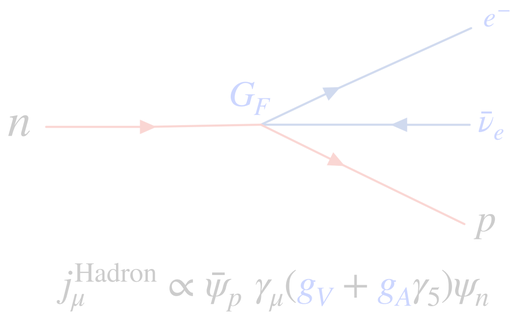
[Phys. Rev. 106, 517 (1957)]

$$\frac{d^5\Gamma}{dE_e d\Omega_e d\Omega_\nu} = \frac{G_F^2|V_{ud}|^2}{(2\pi)^5} F\left(E_e\right)|\vec{p}_e|E_e\left(E_0-E_e\right)^2\left(1+3\lambda^2\right)g_V^2\left[1+a\frac{\vec{p}_e\cdot\vec{p}_\nu}{E_e E_\nu}+\cdots\right]$$

$$a = \frac{1-\lambda^2}{1+3\lambda^2}$$







$$\langle p|\bar{\psi}_u\,\gamma_\mu\left(1-\gamma_5\right)\psi_d|n\rangle = \bar{u}^{(p)}\left[\gamma_\mu g_V+\gamma_\mu\gamma_5 g_A\right]u^{(n)}$$

$$|V_{ud}|^2 = \frac{2\pi^3/m_e^5}{\tau_n \left(1+3\lambda^2\right) \; G_F^2 \; g_V^2 \; \left(1+\Delta_R^V\right) \; f \; \left(1+\delta_R\right)}$$

$$\Gamma = \frac{1}{\tau_n} = \int dE_e d\Omega_e d\Omega_\nu \, [\cdots]$$

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$$\langle p | \bar{\psi}_u \, \gamma_\mu \left(1-\gamma_5\right) \psi_d | n \rangle = \bar{u}^{(p)} \left[\gamma_\mu g_V + \gamma_\mu \gamma_5 g_A\right] u^{(n)}$$

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E_e independent corrections

- Virtual W/Z bosons loop corrections
- Does not change energy spectrum shapes
- Can be absorbed into weak couplings

[Rev. Mod. Phys. 50, 573 (1978)]

$$\langle p\,|\,\bar{\psi}_u\ \gamma_\mu\left(1-\gamma_5\right)\psi_d\,|\,n\rangle = \bar{u}^{(p)}\left[\gamma_\mu g_V + \gamma_\mu\gamma_5 g_A\right]u^{(n)}$$

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E_e independent corrections

- Virtual W/Z bosons loop corrections
- Does not change energy spectrum shapes
- Can be absorbed into weak couplings

[Rev. Mod. Phys. 50, 573 (1978)]

E_e dependent corrections

- Low-energy virtual and bremsstrahlung photons
- Changes spectrum shapes and asymmetries
- Sensitive to experimental details

[Phys. Rev. 164, 1767 (1967)]

$$\langle p\,|\,\bar{\psi}_u\ \gamma_\mu\left(1-\gamma_5\right)\psi_d\,|\,n\rangle = \bar{u}^{(p)}\left[\gamma_\mu g_V+\gamma_\mu\gamma_5 g_A\right]u^{(n)}$$

$$|V_{ud}|^2 = \frac{2\pi^3/m_e^5}{\tau_n \left(1+3\lambda^2\right)\ G_F^2\ g_V^2\ \left(1+\Delta_R^V\right)\ \ f\ \left(1+\delta_R\right)}$$

What *theoretical corrections* required to extract λ from experiments ?

$$\langle p\,|\,\bar{\psi}_u\ \gamma_\mu\left(1-\gamma_5\right)\psi_d\,|\,n\rangle = \bar{u}^{(p)}\left[\gamma_\mu g_V + \gamma_\mu\gamma_5 g_A\right]u^{(n)}$$

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$$\langle p\,|\,\bar{\psi}_u\ \gamma_\mu\left(1-\gamma_5\right)\psi_d\,|\,n\rangle = \bar{u}^{(p)}\left[\gamma_\mu F_1^V\left(q^2\right) + \frac{i\sigma^{\mu\nu}q_\nu}{2M}F_2^V\left(q^2\right) + \gamma_\mu\gamma_5 F_A\left(q^2\right) + \frac{q_\mu}{M}\gamma_5 F_P\left(q^2\right)\right]u^{(n)}$$

next leading order

$q^2 \longrightarrow 0$

$$F_1^V(0) = g_V \quad F_A(0) = g_A$$

$$\langle p\,|\,\bar{\psi}_u\ \gamma_\mu\left(1-\gamma_5\right)\psi_d\,|\,n\rangle = \bar{u}^{(p)}\left[\gamma_\mu g_V + \gamma_\mu\gamma_5 g_A\right]u^{(n)}$$

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[Universe, vol. 9, no. 9, (2023)]

$$\frac{d^5\Gamma}{dE_e d\Omega_e d\Omega_\nu} = \frac{G_F^2|V_{ud}|^2}{(2\pi)^5} F\left(E_e\right)|\vec{p}_e|E_e\left(E_0-E_e\right)^2\left(1+3\lambda^2\right)\ g_V^2\left(1+\frac{\alpha}{2\pi}\delta^{(1)}\left(E_e\right)\right)\left[1+\left(a+\delta a_r\right)\left(1+\frac{\alpha}{2\pi}\delta^{(2)}\left(E_e\right)\right)\frac{\vec{p}_e\cdot\vec{p}_\nu}{E_e E_\nu}+\cdots\right]$$

$$\langle p|\bar{\psi}_u\ \gamma_\mu\left(1-\gamma_5\right)\psi_d|n\rangle = \bar{u}^{(p)}\left[\gamma_\mu F_1^V\left(q^2\right)+\frac{i\sigma^{\mu\nu}q_\nu}{2M}F_2^V\left(q^2\right)+\gamma_\mu\gamma_5 F_A\left(q^2\right)+\frac{q_\mu}{M}\gamma_5 F_P\left(q^2\right)\right]u^{(n)}$$

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?

[Universe, vol. 9, no. 9, (2023)]

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Modern Classification of Theoretical Corrections

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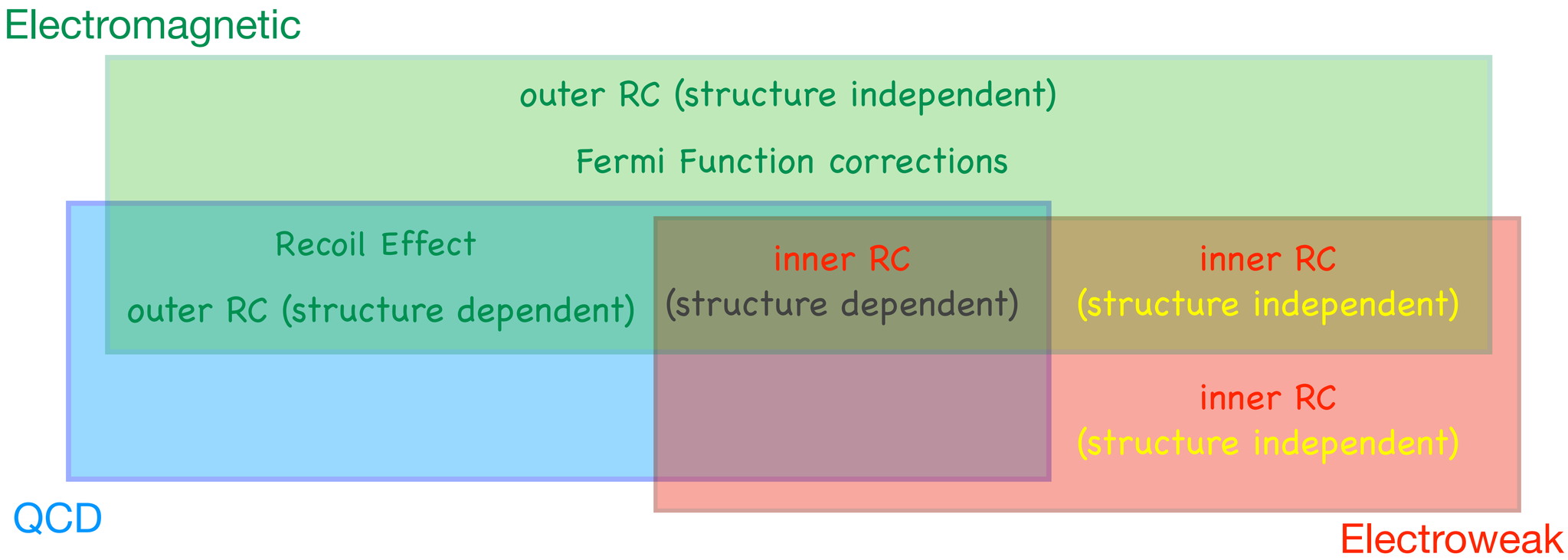
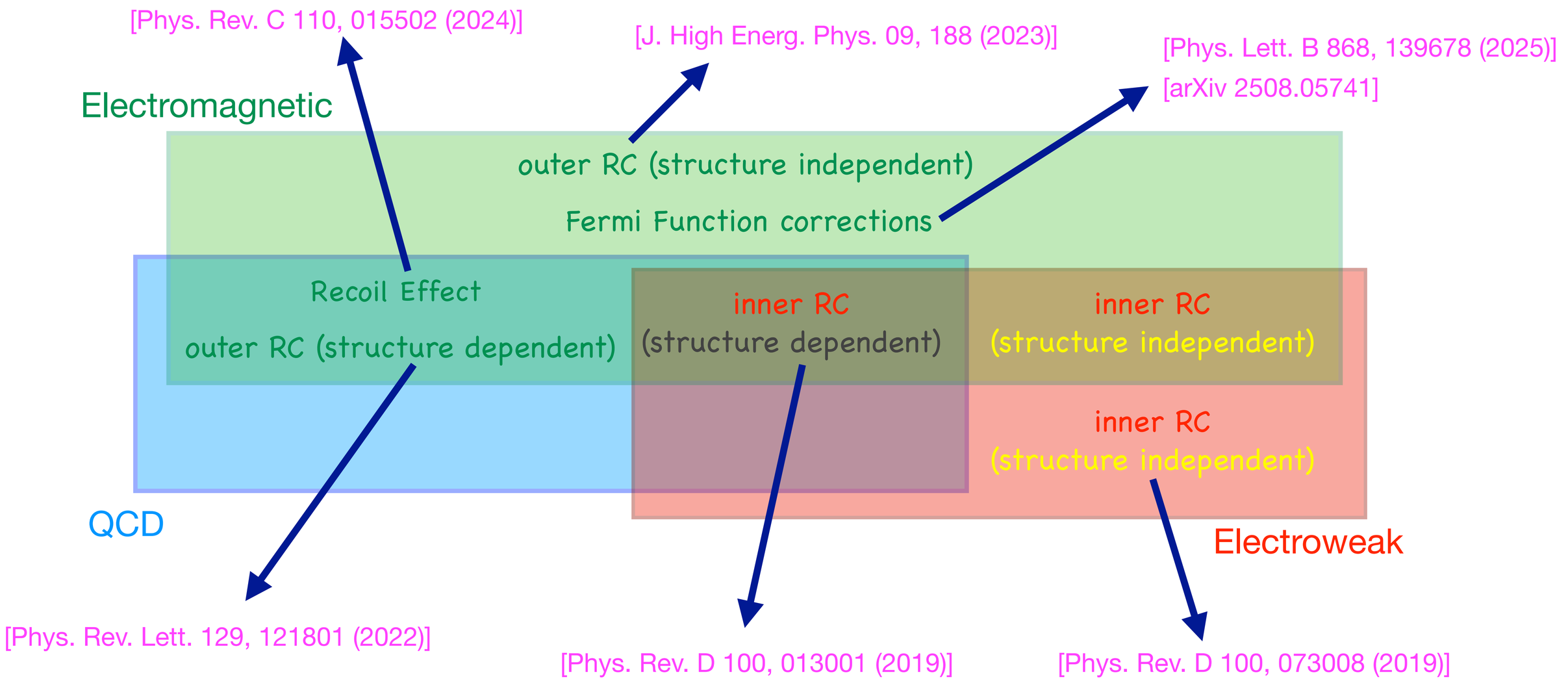


Modern Classification of Theoretical Corrections



Modern Classification of Theoretical Corrections

outer RC (structure independent)

$$\frac{d^5\Gamma}{dE_e d\Omega_e d\Omega_\nu} = \frac{G_F^2 |V_{ud}|^2}{(2\pi)^5} F(E_e) |\vec{p}_e| E_e (E_0 - E_e)^2 (1+3\lambda^2) \; g_V^2 \left(1 + \frac{\alpha}{2\pi}\delta^{(1)}(E_e)\right) \left[1 + (a+\delta a_r)\left(1 + \frac{\alpha}{2\pi}\delta^{(2)}(E_e)\right) \frac{\vec{p}_e \cdot \vec{p}_\nu}{E_e E_\nu} + \cdots\right]$$

Recoil Effect

*a*CORN Experiment

[Phys. Rev. C 110, 015502 (2024)]

$$a(E_e, \theta_{e\nu}, \lambda) = \frac{1-\lambda^2}{1+3\lambda^2} + \frac{1}{(1+3\lambda^2)^2} \left[\frac{4E_e^{\max}}{M_n}(1+\lambda^2)\left(\lambda^2 + |\lambda| + 2|\lambda| \frac{F_2^V(0)}{F_1^V(0)}\right) - \frac{3E_e}{M_n}(1-\lambda^2)(1+3\lambda^2)\sqrt{1-\left(\frac{m_e}{E_e}\right)^2} \, Cos\ \theta_{e\nu} + \cdots\right]$$

Conclusions

- Significant recent theoretical progress on radiative corrections, recoil effects, and other systematics is highly relevant for precision analysis in Nab
- Next step: develop and integrate a modular theoretical correction template into the Nab experiment's systematic uncertainty framework
- Investigate *data-blinding* methodologies based on different theoretical templates within controlled uncertainty envelopes