HFOFO Project Updates

Weeks of August 31 - September 13, 2025

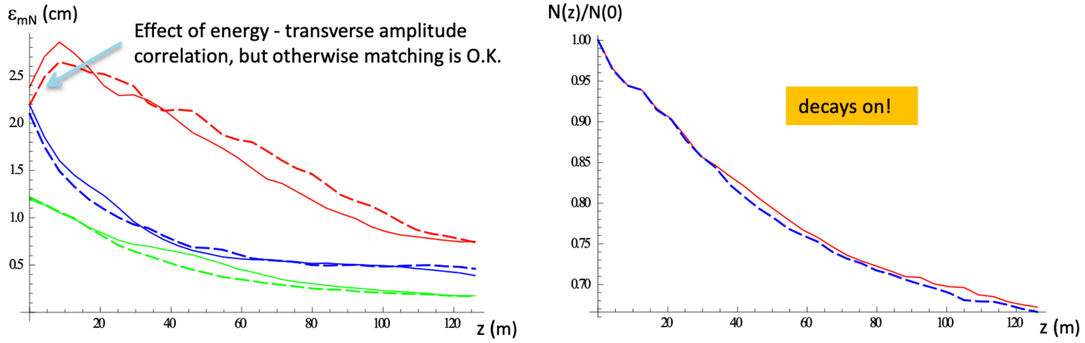
https://github.com/criggall/muon-cooling/tree/main

Progress since last meeting

- ☑ Recompute transmission rate with momentum cuts
- ☑ Dissect the multipole expansion script
- ☑ Add monopole term to multipole expansion
- ☑ Subtract non-rotated field from HFOFO field, then compute multipole coefficients
- ☐ Use particle tracking to determine field components
- ☐ Study effects of field components on particle dynamics

Revisiting the transmission rate

Cooling & Transmission (G4BL)



Normalized emittances (cm) from Gaussian fit: μ^+ - solid lines, μ^- - dashed lines.

Transmission as a ratio of the number of muons in the Gaussian core: red solid line - μ^+, blue dashed line - μ^-.

Final/Initial values (Gaussian fit):

	$N^{(total)}$	$N^{(150<p<360)}$	$N^{(core)}$	$p^{(cnt)}$, MeV/c	ε_{mN}, cm			ε_{6D}, cm^3
μ^+	5378/11755	5167/7998	5010/7329	208.2/248.0	0.19/1.19	0.36/2.19	0.76/2.38	0.051/6.22
μ^-	5896/12396	5743/9020	5499/8248	207.7/248.8	0.16/1.22	0.46/2.10	0.72/2.19	0.051/5.59
	48%	64%	67%					

Fermilab

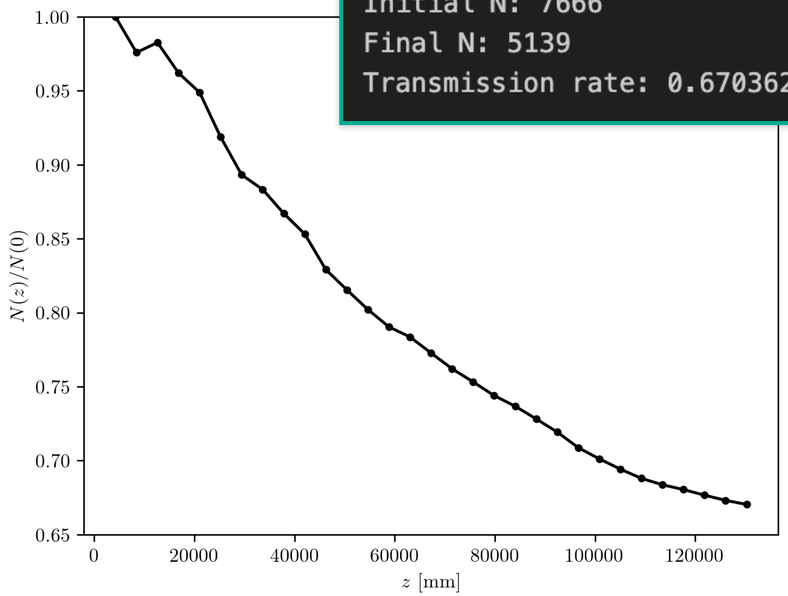
9 Y.Alexahin | HFOFO Snake Status 05/21/2015

Total

```
Initial N: 11452
Final N: 5348
Transmission rate: 0.4669926650366748
```

$150 < p < 350$ MeV/c

```
Initial N: 7666
Final N: 5139
Transmission rate: 0.6703626402295851
```

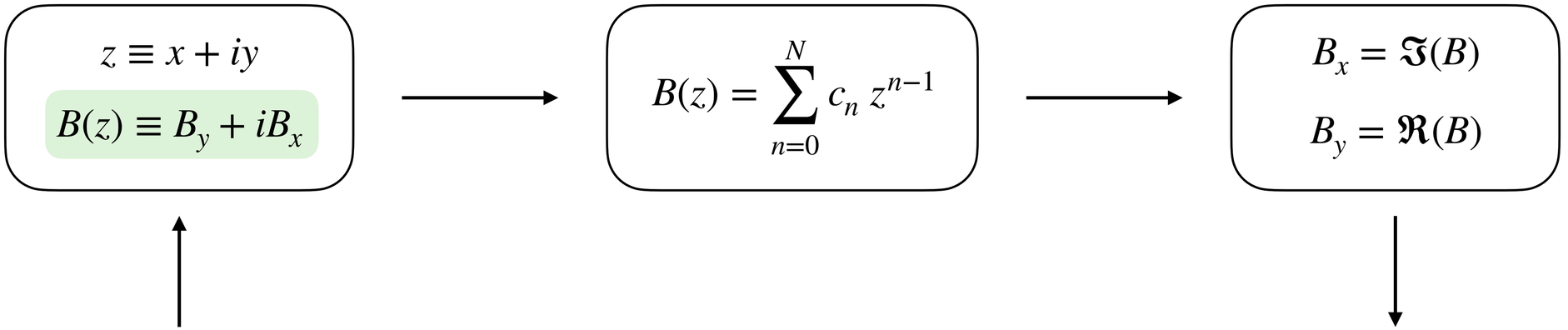


Multipole expansion

Performing a dissection

Updates to the script

- Fixed a mistake! Previously had B_x and B_y swapped
- Added monopole term (though it had negligible effect)

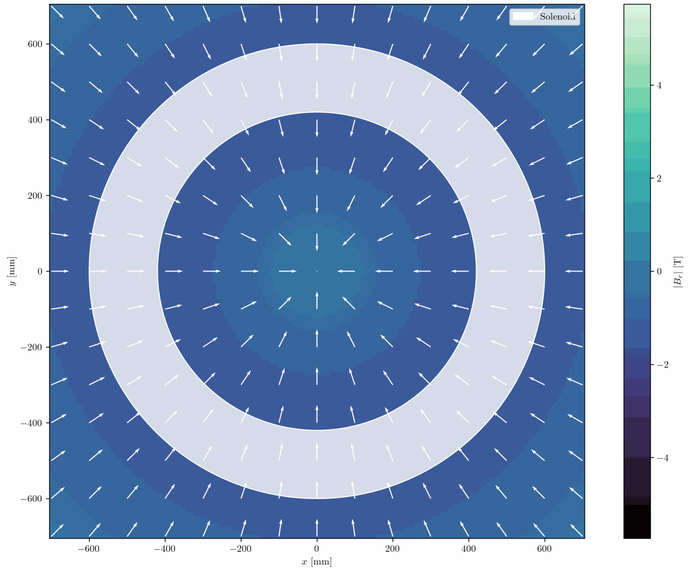


Input B_x and B_y values on some x, y grid from G4beamline

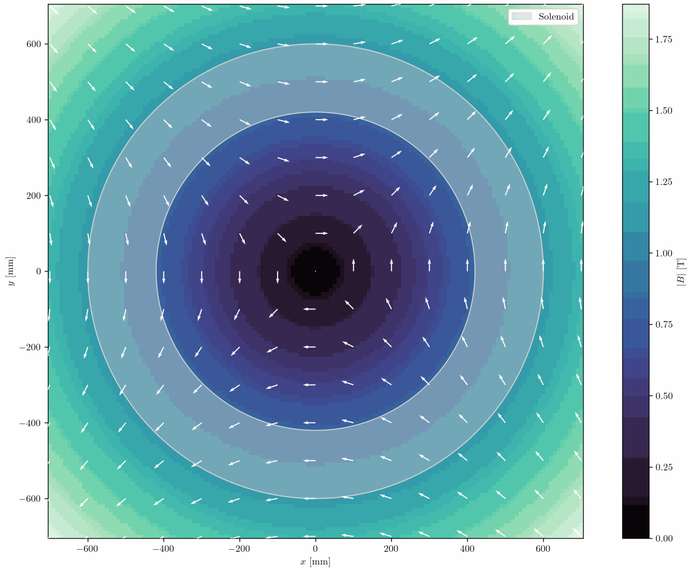
Output B_x and B_y values on some x, y grid by summing with c_n from fit

Tests with single solenoid (no rotations)

Expectation from G4beamline:

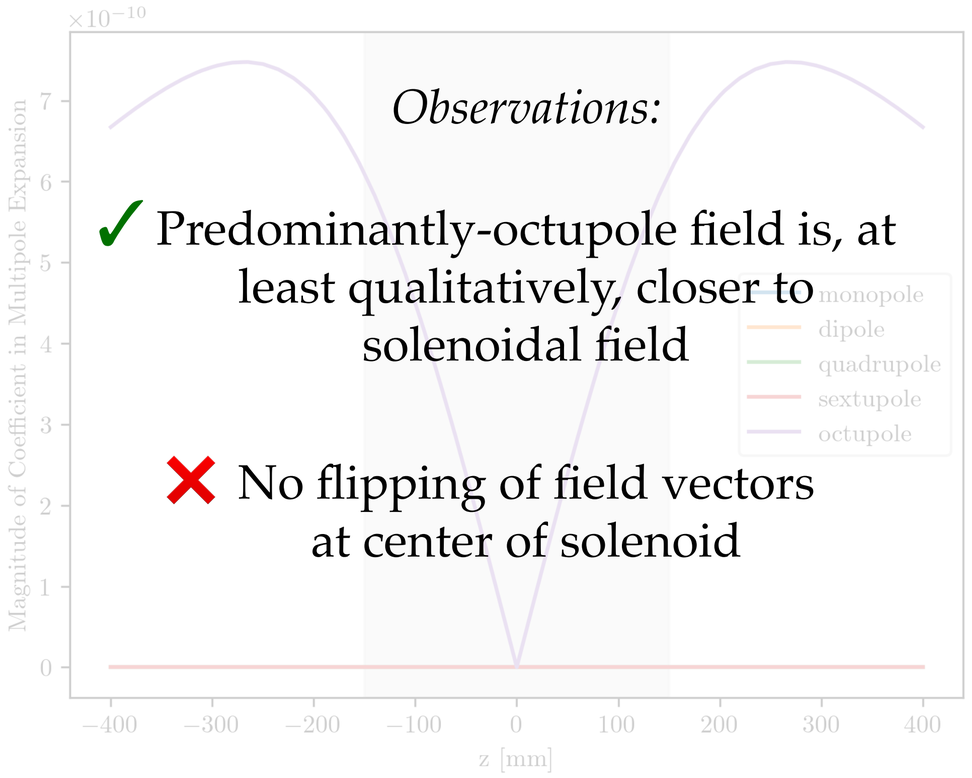


Results from last week, with presumed error in script:



Tests with single solenoid (no rotations)

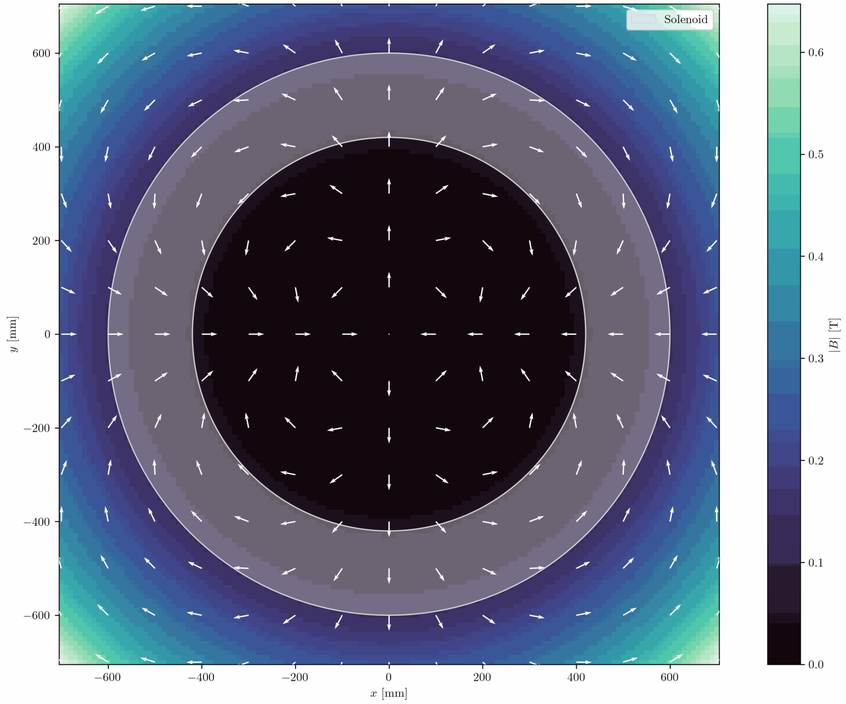
Results with amended script:



Observations:

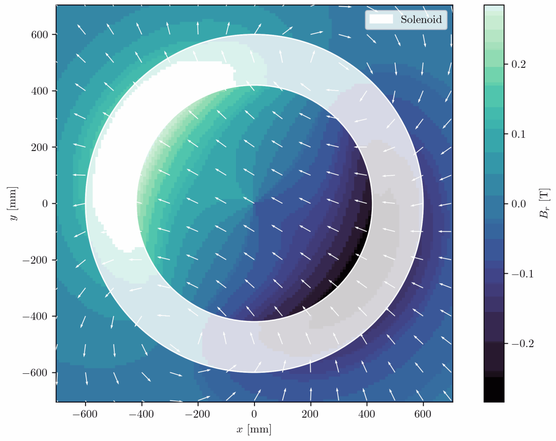
✓ Predominantly-octupole field is, at least qualitatively, closer to solenoidal field

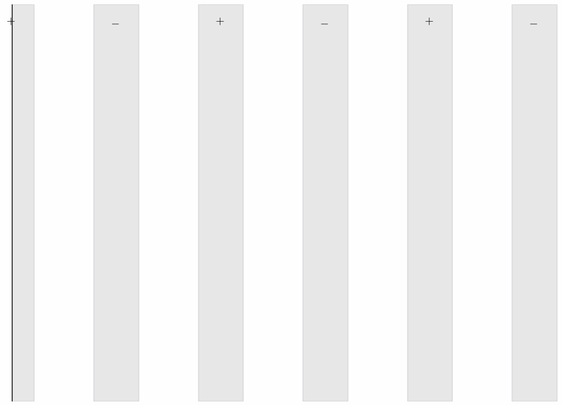
✗ No flipping of field vectors at center of solenoid



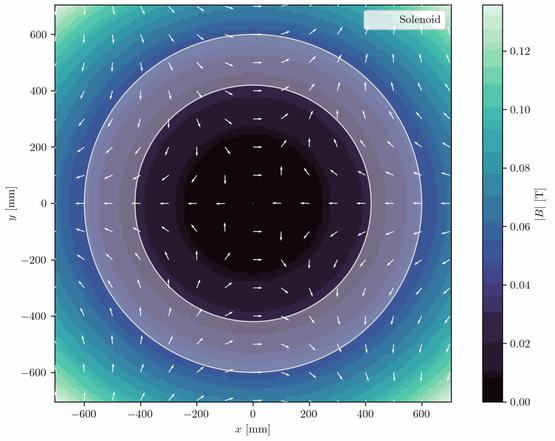
Tests with HFOFO

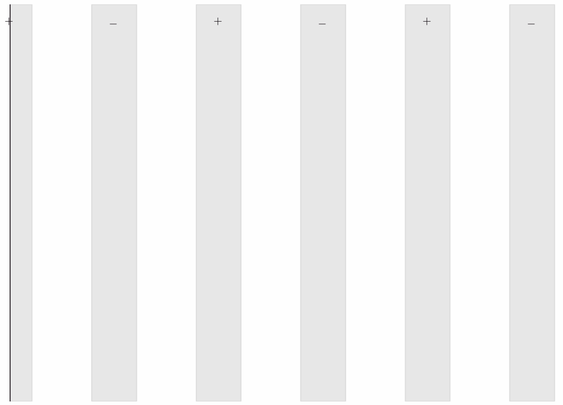
Expectation from G4beamline:





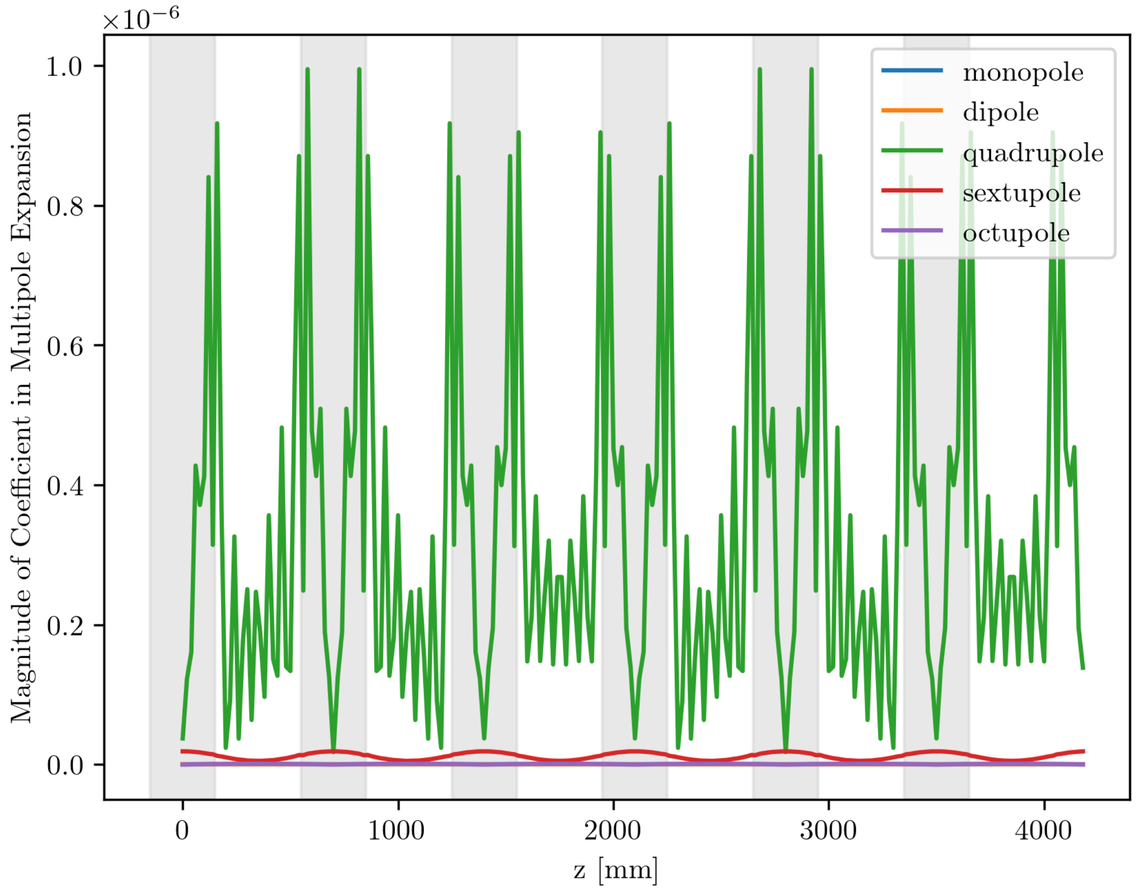
Results from last week, with presumed error in script:

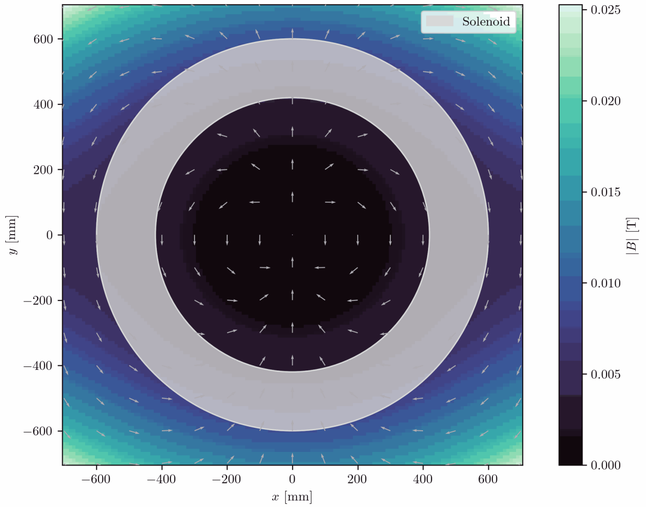




Tests with HFOFO

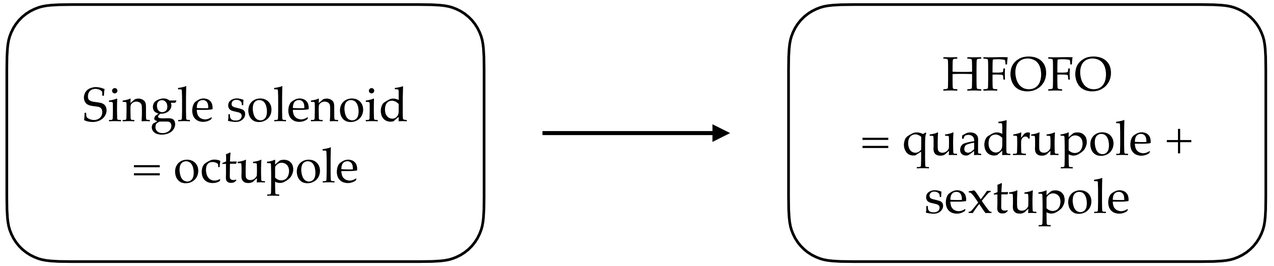
Results with amended script:





Conclusions

Observation:



Questions:

What happens with a FOFO lattice? i.e., is the shift from octupole to quadrupole+sextupole due to the rotations or to superposition of field effects from having solenoids in a lattice?

Can we imagine a solenoid field arising in the limit as $n \to \infty$?

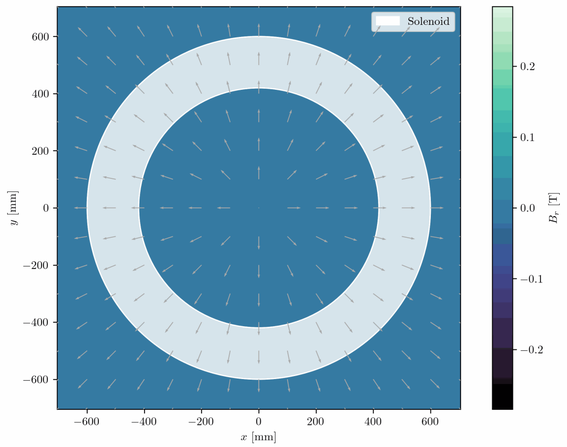
What maximal order is reasonable to set, normalized to the fact that we require a physically-realizable design?

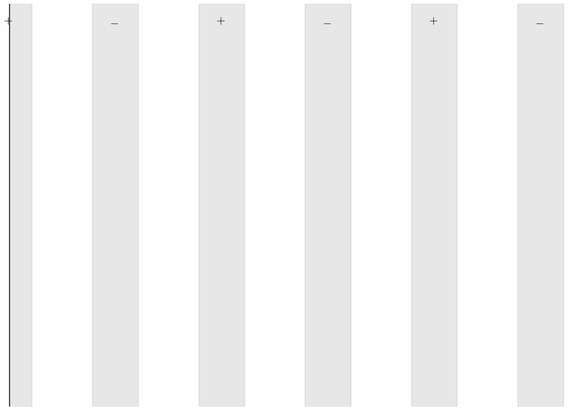
Multipole expansion

Isolating field components from solenoid rotations

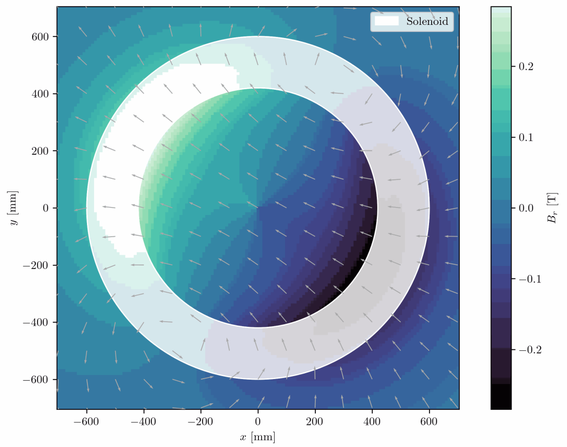
Isolating contributions from solenoid rotations

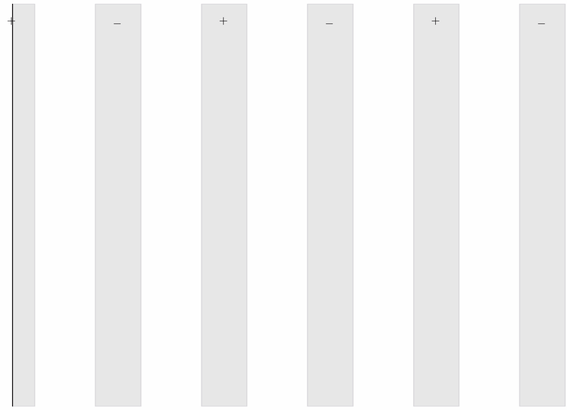
FOFO



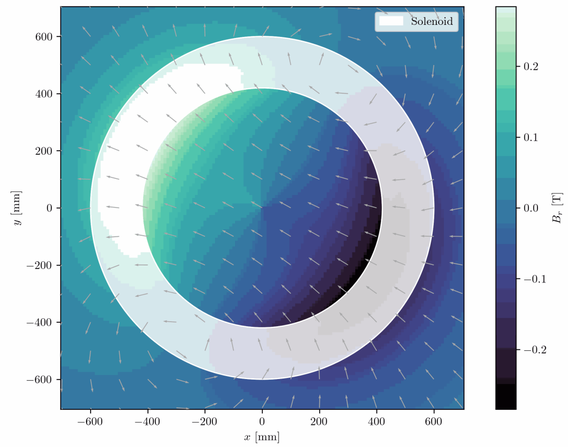


HFOFO



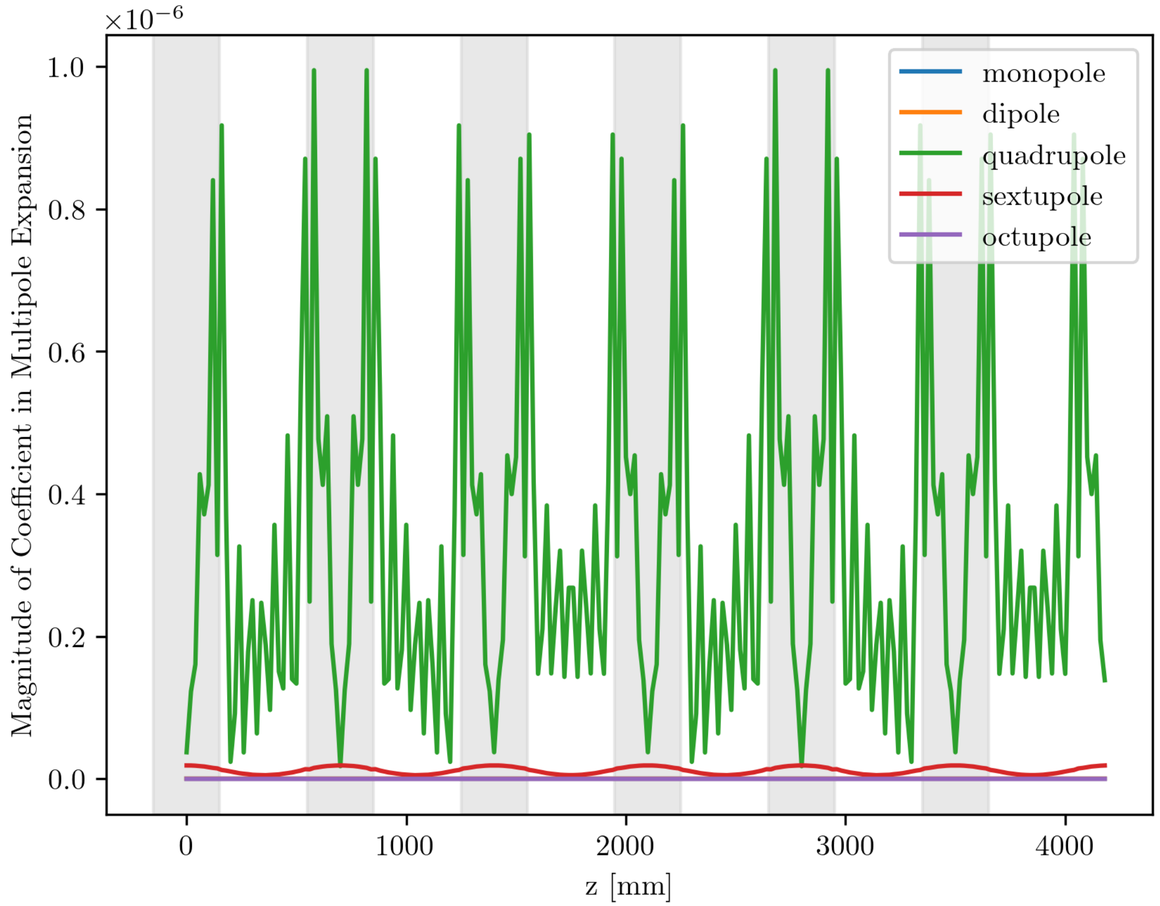


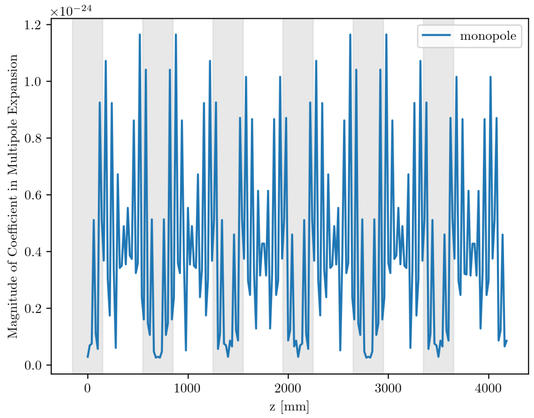
Difference

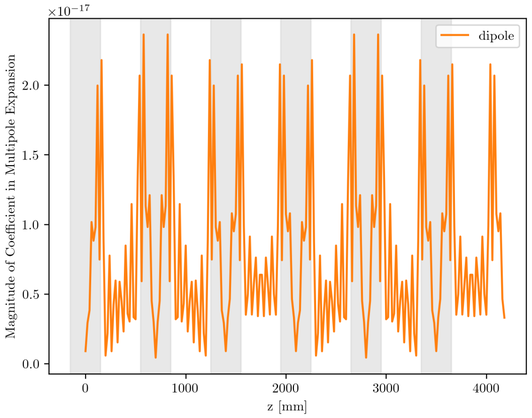


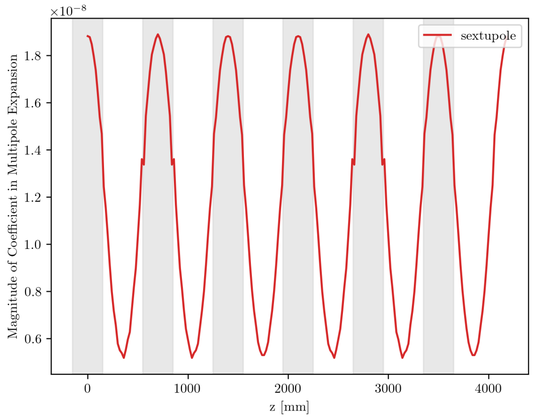
Multipole expansion of the difference

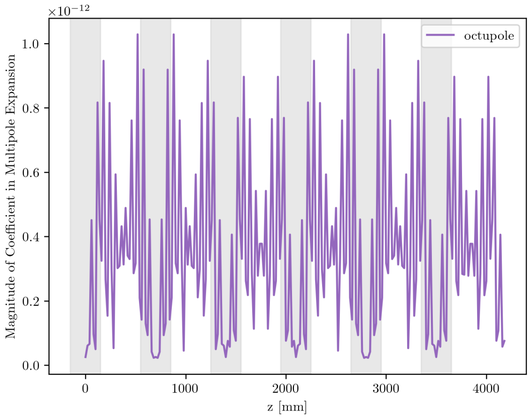
Looks very similar to the results without taking the difference…
Dominated by quadrupole terms → not terribly promising



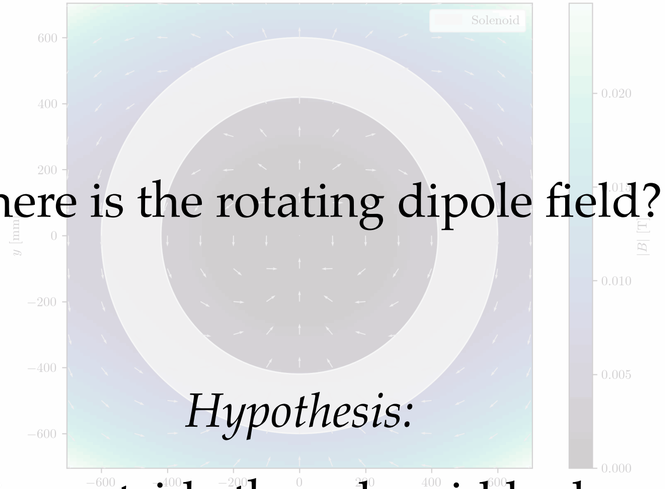








Multipole expansion of the difference



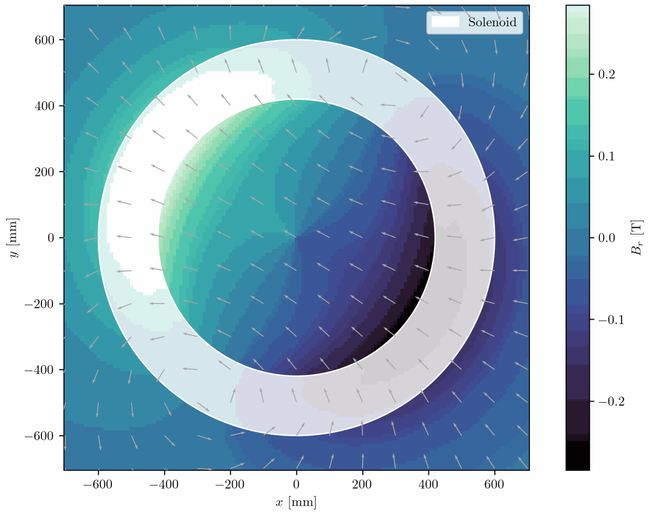
Where is the rotating dipole field?

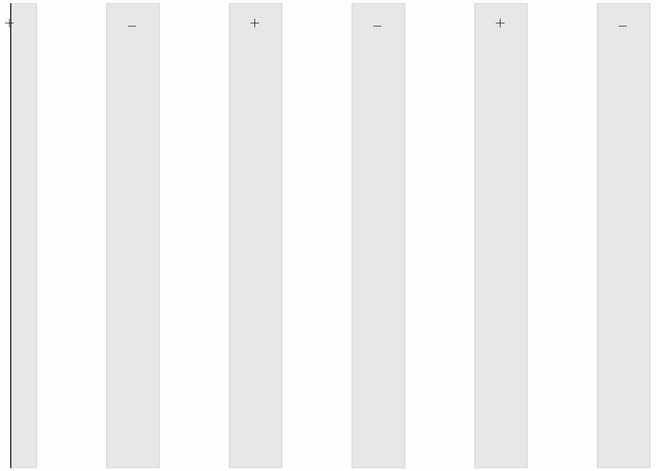
Hypothesis:

Behavior outside the solenoid looks more non-linear — maybe this is biasing the fit?

Idea:

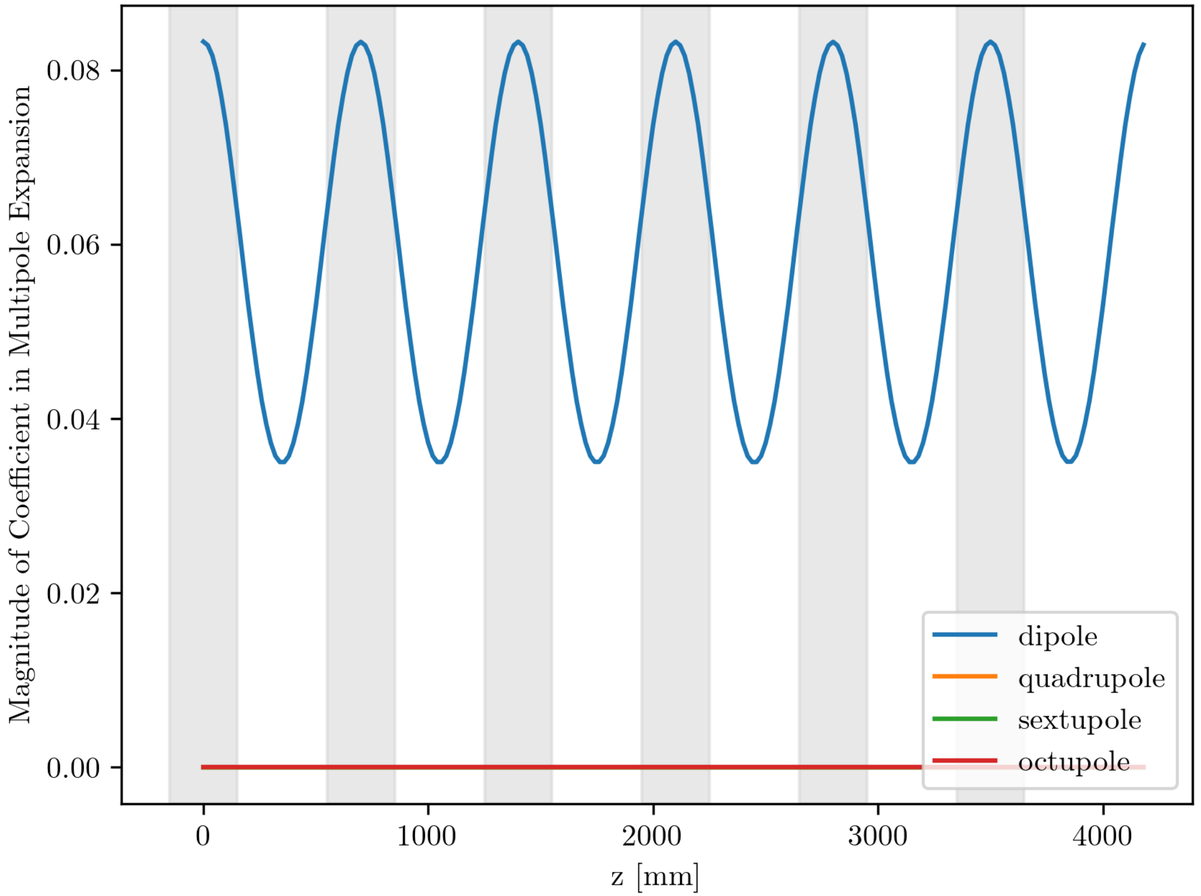
Isolate the region inside the solenoid

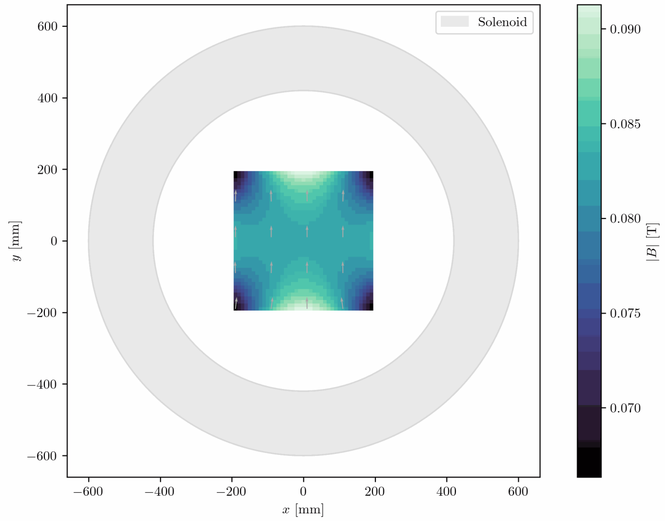


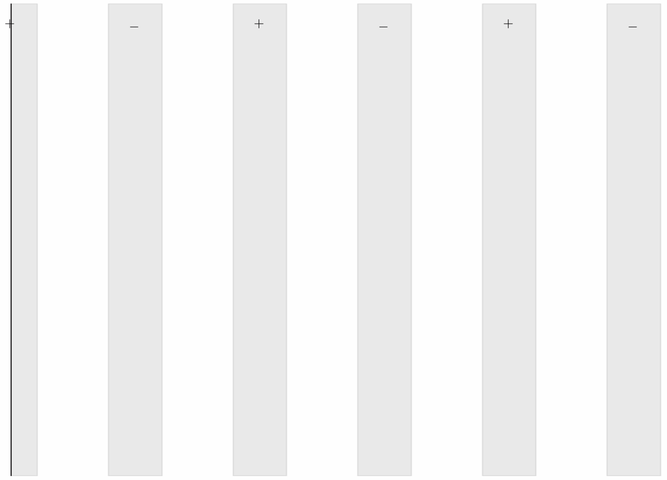


Multipole expansion of the difference

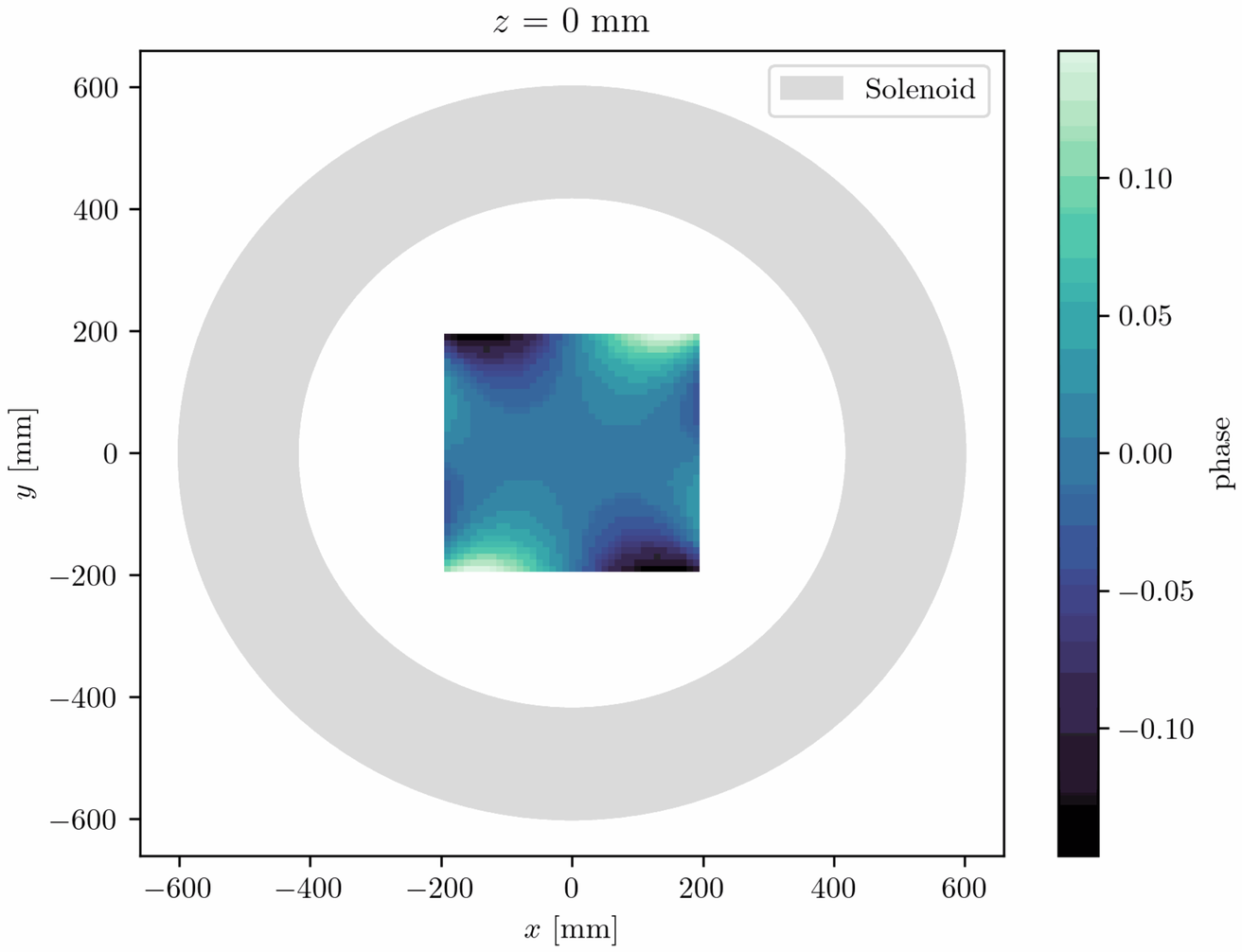
Limit to -200 < x, y < 200 mm → only dipole contribution!







Multipole expansion of the difference



…But still fail to capture the *rotation* of the field vectors

Sanity check by looking at the phase
→ no rotation confirmed

Next steps?

Proposed next steps

Let us try to replicate the rotating dipole field with physical dipole elements!

1. Obtain magnitude and orientation of field vectors from difference field map from G4beamline
2. Construct dipole lattice in G4beamline to reproduce this field
3. Add solenoids
4. Particle tracking → orbit, dispersion, beta function

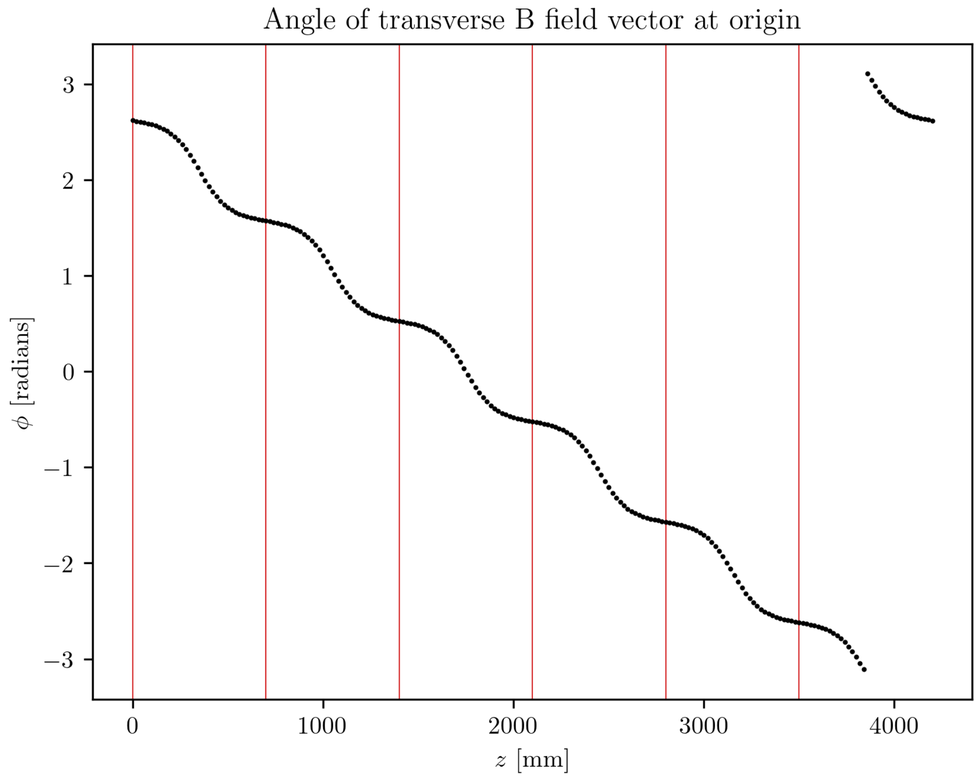
End goal: reproduce HFOFO field with dipoles + non-rotated solenoids

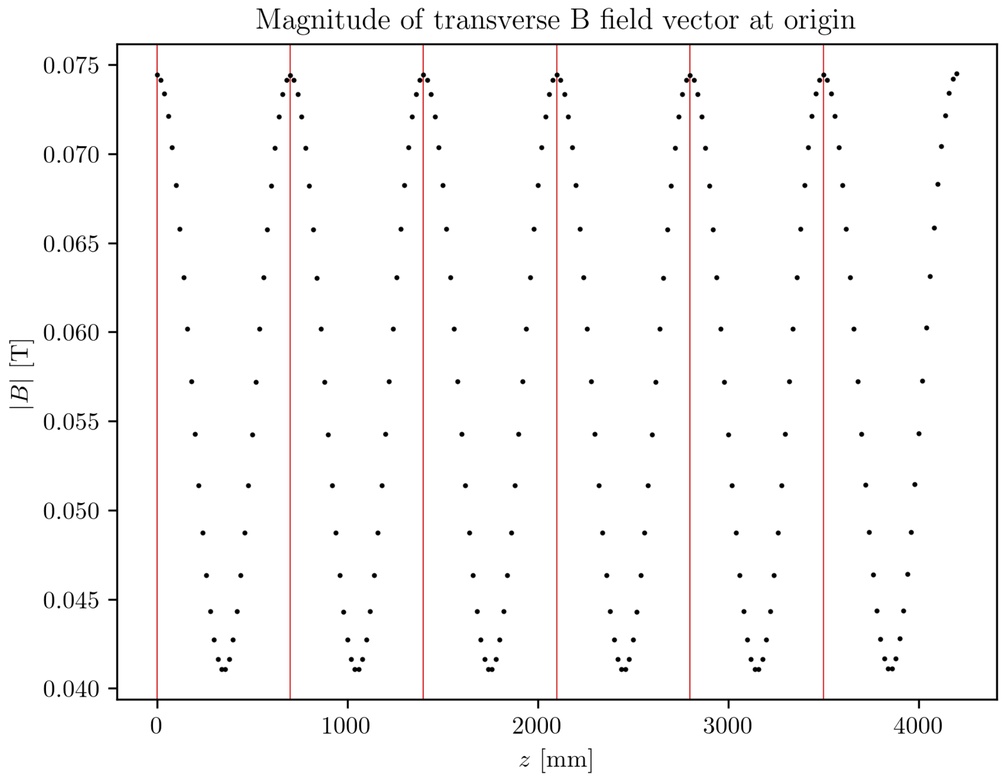
Building a rotating dipole field

Using results of previous studies

Field requirements

From the difference field maps, we can define the requirements for the rotating dipole field:





What if we place dipole magnets here?

Status of investigation

Question: How should the extent of the field be defined in order to replicate the observed sinusoidal behavior?

I cannot seem to get the dipole field to work in G4beamline…

```
multipole Dipole dipole=1 fieldLength=100
```

```
do i 0 6
    place Dipole z=$i*$period_len/6
enddo
```

Any suggestions?