UCDAVIS

Is Perfect Quantum State Transfer Possible?

- The spreading of wave packets
- Lattice Models
- Theory of Perfect Quantum State Transfer
  (vs. The Real World)
- Recovering high fidelity with Monte Carlo
- Superconducting Qubit Array Results
- Conclusions

Funding:

U.S. DEPARTMENT OF ENERGY | Office of Science

Cast of Characters

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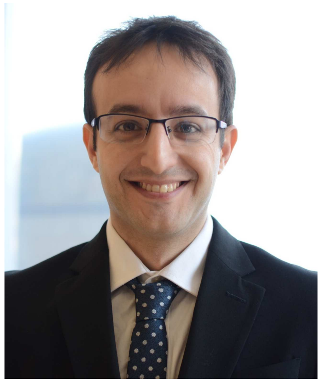

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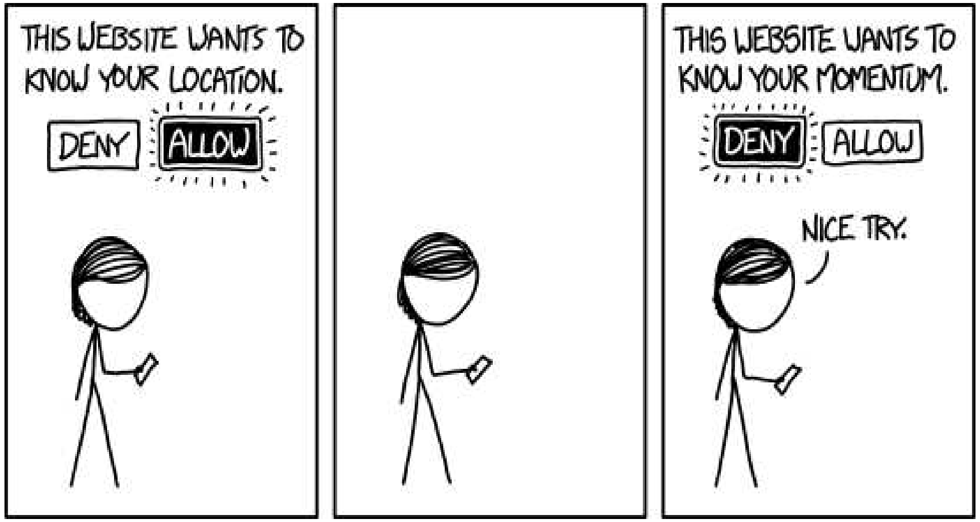
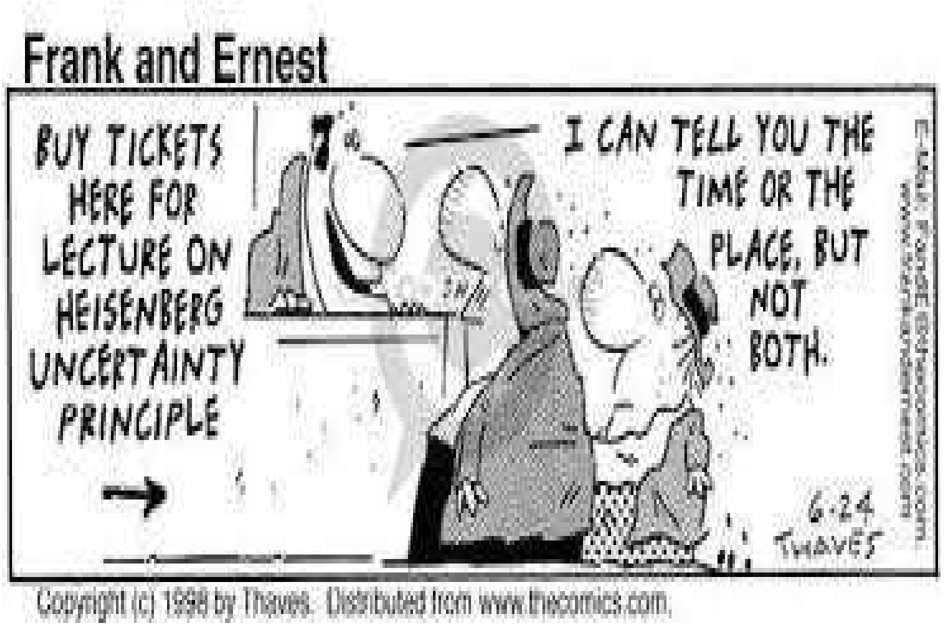
1. The Spreading of Wave Packets

Heisenberg uncertainty principle has become part of popular culture.



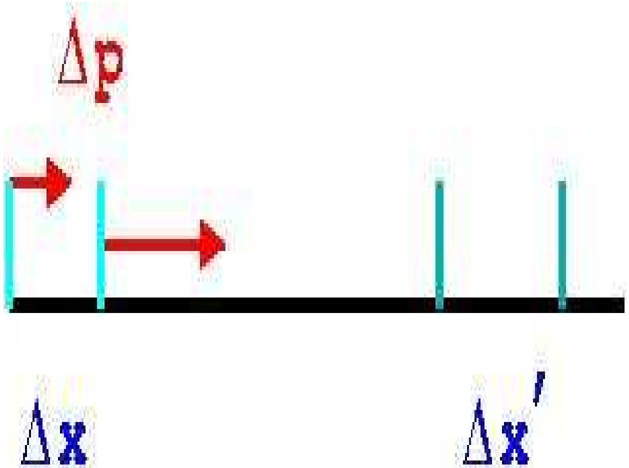
Heisenberg could have prevented your attendence of this talk · · ·

Cartoons can get it wrong.



$\Delta x \, \Delta t > ?!?!$

Uncertainty $\Delta x \Delta p > \frac{\hbar}{2}$, of static $\Psi(x)$: wavefunction also spreads in time.



This picture useful qualitatively,
but, like cartoon, also wrong.
Would imply linear in t growth of Δx.

Doing it right: free-particle Schroedinger Equation

$$i\hbar \frac{\partial \psi(x,t)}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi(x,t)}{\partial x^2}$$

'Imaginary time' diffusion equation

$$\frac{\partial \rho(x,t)}{\partial t} = D \frac{\partial^2 \rho(x,t)}{\partial x^2}$$

QM probability density spreads as $\sqrt{t}$.

(This analogy underlies a powerful computational approach to the solution of the Schroedinger equation: "diffusion Monte Carlo")

External potential can control spreading: e.g. Hydrogen atom

$$\frac{\hbar}{i} \frac{\partial \psi(x,t)}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi(x,t)}{\partial x^2} + V(x)\, \psi(x,t)$$

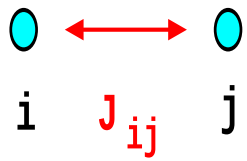
But in the absence of $V(x)$ we expect spreading.

2. Lattice Models

Diffusion, localization and quantum state transfer often formulated on a lattice.

$\sigma_i^{\pm}$ raise/lower a qubit state on site i

$$\mathcal{H} = \sum_{\langle ij \rangle} J_{ij} \left(\sigma_i^+ \sigma_j^- + \sigma_j^+ \sigma_i^- \right)$$



Discretized version of second derivative in diffusion equation:

$$\frac{d^2 f}{dx^2} = \frac{f(x+dx) - 2f(x) + f(x-dx)}{dx^2}$$

$$\frac{d^2}{dx^2} \equiv \begin{pmatrix} -2 & 1 & 0 & 0 & 0 \\ 1 & -2 & 1 & 0 & 0 \\ 0 & 1 & -2 & 1 & 0 \\ 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 1 & -2 \end{pmatrix}$$

Precisely the matrix for $\mathcal{H}$ for uniform $J_{ij} = 1$ acting on single excited qubit states.

(apart from irrelevant constant on diagonal).

Action of $\mathcal{H} \leftrightarrow$ diffusion operator

Usual uniform hopping tight binding Hamiltonians $\rightarrow$ diffusive behavior.

Diagonalize Hamiltonian: eigenvectors $\phi_\alpha(j)$ eigenvalues λ_α

$$\psi(j,t=0) = \sum_\alpha c_\alpha \, \phi_\alpha(j)$$

$$\psi(j,t) = \sum_\alpha c_\alpha \, e^{-i\lambda_\alpha t/\hbar} \, \phi_\alpha(j)$$

Preceding argument shows wave packets will propagate and spread (diffuse).

Analog of nuclear potential confining e^- in an atom $\Rightarrow$ Anderson localization:

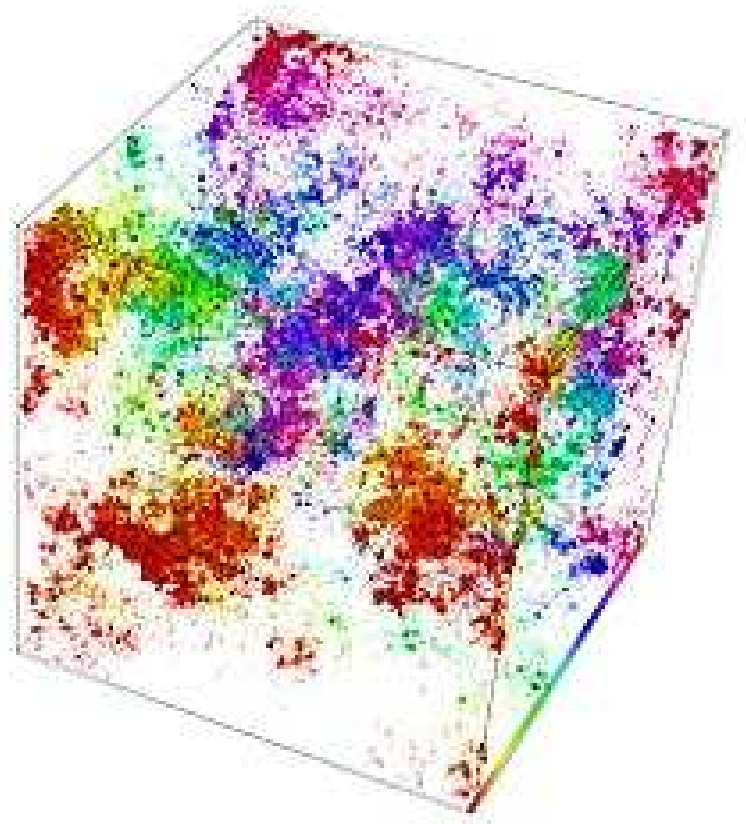
Random site energies μ_i localize quantum particles on lattice sites i of lowest μ_i.

$$\mathcal{H} = -t \sum_{\langle ij \rangle} \left(\sigma_i^+ \sigma_j^- + \sigma_j^+ \sigma_i^- \right) + \sum_i \mu_i \, \sigma_i^+ \sigma_i^-$$

Quantify 'size' of ψ via inverse participation ratio: $\mathcal{P}^{-1} \equiv \sum_j |\psi_j|^4$

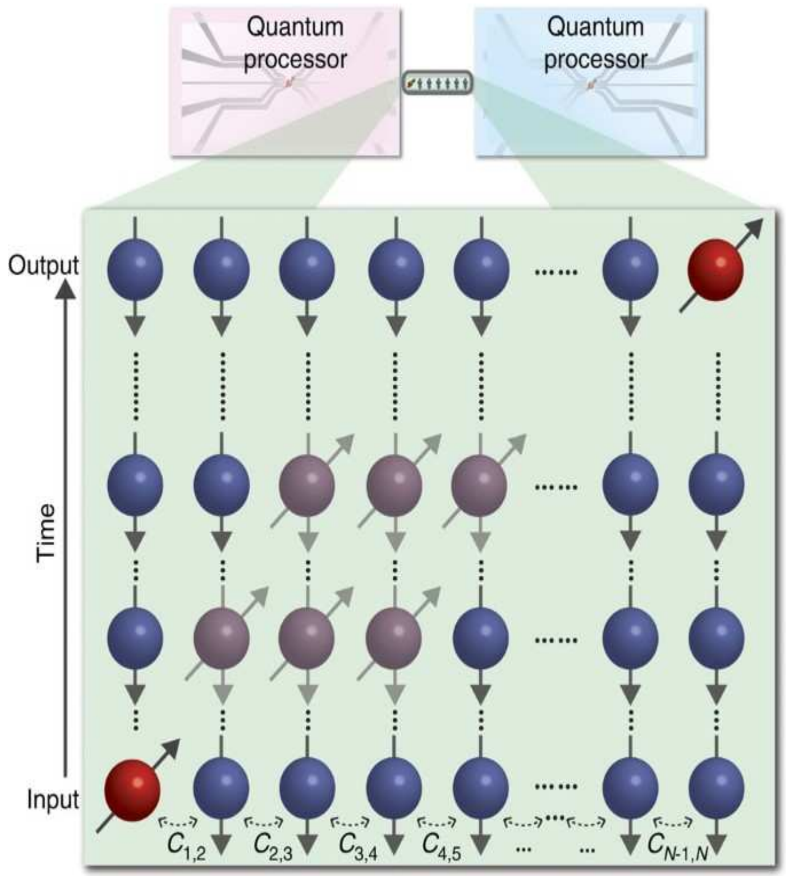
$$\psi(j) = \delta_{j,j_0} \Rightarrow \mathcal{P}^{-1} = 1 \qquad \psi(j) = \frac{1}{\sqrt{N}} \Rightarrow \mathcal{P}^{-1} = \frac{1}{N}$$

$\mathcal{P}^{-1}$ is inverse of number of sites 'participating' in wave function ψ.



Some eigenstates of the Anderson model in 3D.

3. Perfect Quantum State Transfer



In designing a quantum computer,
or other quantum information
applications, spreading is very bad news.
Would like instead to be able
to transport a quantum state
precisely from one location to another.

This goal is at variance with our intuition concerning the Schroedinger equation!

After all, imaginary time *diffusion* equation.

Can we engineer a lattice Hamiltonian exhibiting perfect quantum state transfer?

Revisit:

$$\mathcal{H} = -\sum_{\langle ij \rangle} J_{ij} \left(c_i^\dagger c_j + c_j^\dagger c_i \right) + \sum_i \mu_i \, c_i^\dagger c_i$$

Tune $\{ J_{ij}, \mu_i \}$ to engineer eigenstates ϕ_α and eigenenergies λ_α of $\mathcal{H}$.

Goal: At some passage time t_p

$$\psi(j, t = 0) = \sum_\alpha c_\alpha \, \phi_\alpha(j) = \delta_{j,1} \qquad \Rightarrow \qquad \psi(j, t_p) = \sum_\alpha c_\alpha \, e^{-i\lambda_\alpha t_p/\hbar} \, \phi_\alpha(j) = \delta_{j,N}$$

Is this possible?!

Intuition: Eigen-energies λ_α must allow ψ to be 'in phase' at later time t.

$\lambda_\alpha - \lambda_\beta$ related as *rational fractions*. Simplest scenario: $\lambda_\alpha - \lambda_\beta = c$.

Do we know any quantum mechanical system with equi-spaced eigenenergies?

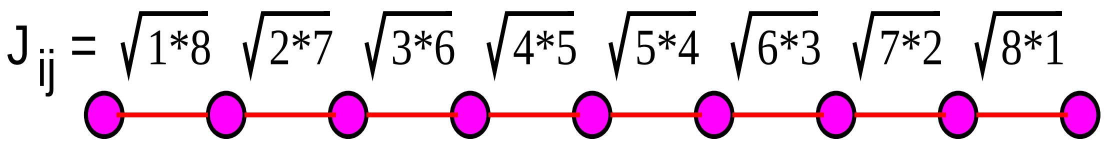
We sure do! Quantum harmonic oscillator.

Crud! That's a infinite collection $\Rightarrow$ infinite length chain.

Ah-ha. Angular momentum J has $J_z = m = \hbar(-j, -j+1, \cdots j)$

$$J_+ \, |j, m\rangle = \sqrt{j(j+1) - m(m+1)} \, |j, m+1\rangle$$

$j = 4$ has nine $m = -4, -3, -2, -1, 0, 1, 2, 3, 4$.



Spin Chain: 'engineered' hoppings (for $N = 9$) which will give perfect QST!

Passage time: $t_p = \frac{\pi}{2}$.

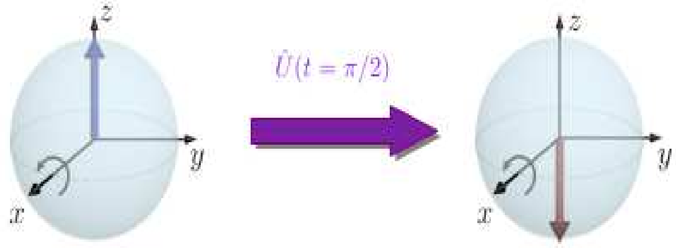
Symmetry $t_i = t_{N-i}$ will be important. Notice too: No μ_i (as yet).

More precisely Christandl says:

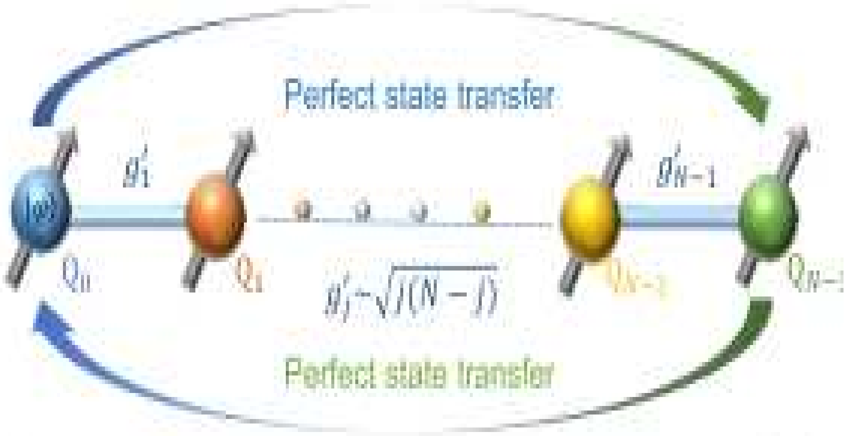
$$\mathcal{H} = \begin{pmatrix} 0 & \sqrt{(N-1)\cdot 1} & 0 & \dots & 0 \\ \sqrt{(N-1)\cdot 1} & 0 & \sqrt{(N-2)\cdot 2} & \dots & 0 \\ 0 & \sqrt{(N-2)\cdot 2} & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \sqrt{1\cdot(N-1)} \\ 0 & 0 & 0 & \sqrt{1\cdot(N-1)} & 0 \end{pmatrix}$$

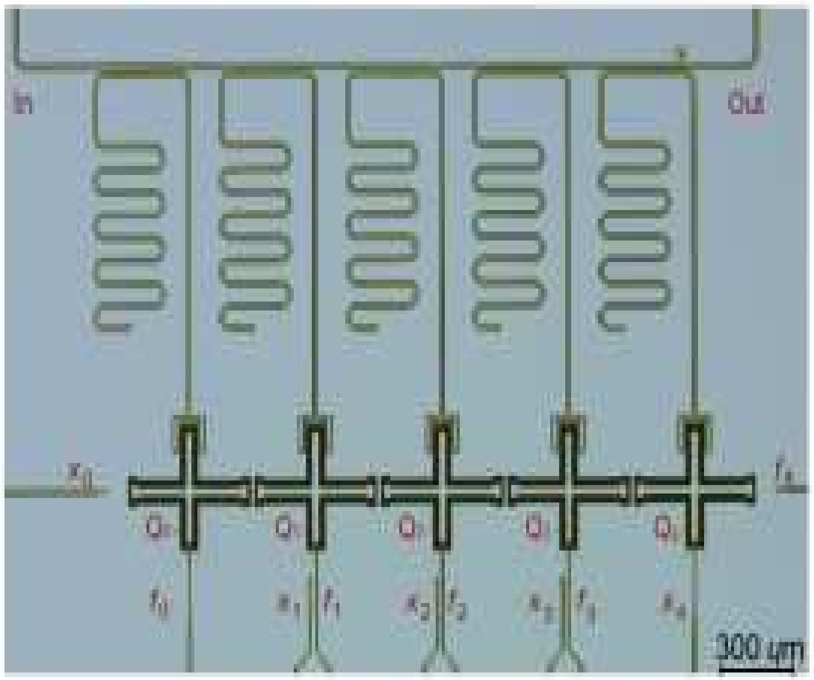
$$\hat{\mathcal{H}} = 2\,J\,\hat{S}_x$$

Time evolution corresponds to rotation of wave function about $\hat{x}$-axis.



These ‘quantum spin chain’ perfect state transfer systems are being built!





Well-studied problem.

“Perfect transfer of arbitrary states in quantum spin networks”,
M. Christandl etal,
Phys. Rev. A71 032312 (2005).

$\Rightarrow$

“Perfect quantum state transfer in a superconducting qubit chain with parametrically tunable couplings”,
X. Li, *etal*,
Phys. Rev. Applied 10, 054009 (2018).

Five Qubits.

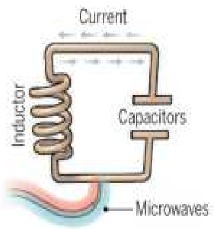
We will be interested in more complex geometries.

3. ⇒ 3'. Real World

[Science, **354** 6316]

Existing **qubit** platforms

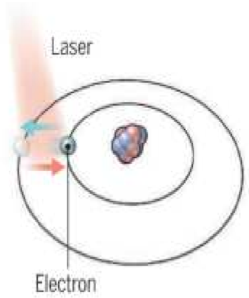
Superconducting loops



- Resistance-free current oscillates back and forth around a circuit loop
- Injected microwave signal excites the current into superposition states
- Emulates a quantum anharmonic oscillator

Google, IBM, ...
ZJU, UESTC, ...

Trapped ions



- Ions, have quantum energies that depend on the location of electrons.
- Tuned lasers cool and trap the ions, and put them in superposition states.

IonQ, Honeywell
Maryland, ...

Silicon quantum dots

- "Artificial atoms" made by adding an electron to a small piece of pure silicon.
- Microwaves control the electron's quantum state.

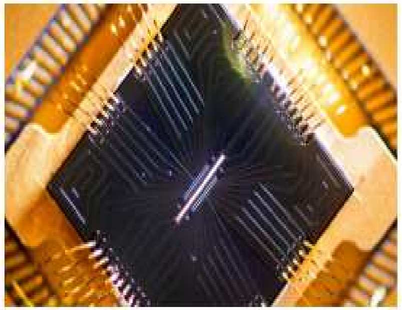
Intel, HRL, QuTech
UNSW, Delft, RIKEN, ...

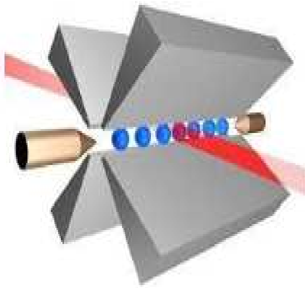
Building many of them – **Noisy intermediate quantum devices**

Superconducting quantum circuits

[IOP, ZJU]

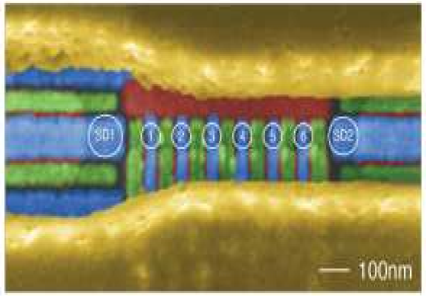
Trapped ions





[Maryland, IonQ]

Silicon quantum dots

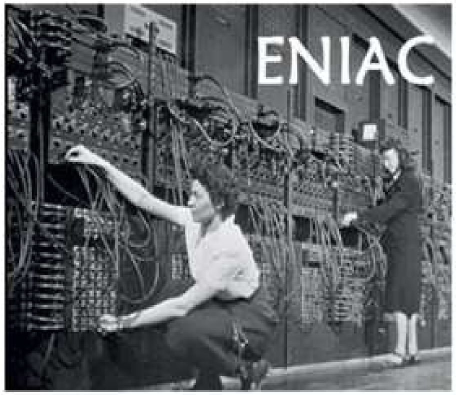


[Qutech 2022]

These are not laptop computers or cell phones $\cdots$

The more things change, the more they stay the same...

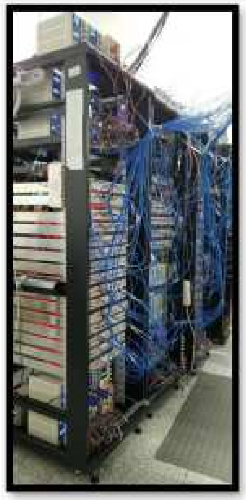
First general-purpose digital computer



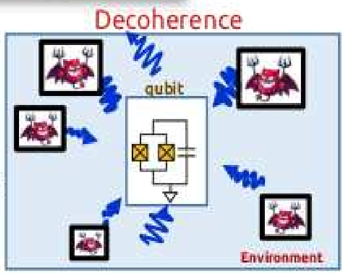
- 30 tons
- 18,000 vacuum tubes
- 1,500 relays
- +100,000 of resistors, capacitors and inductors,

= add or subtract 5,000 times per second!

SC quantum circuit @ZJU



- 36-qubits (121 available)
- Fully programmable
- Emulates dynamics of $\hat{H}$ with dim = 9B states
- Operates at 20mK...

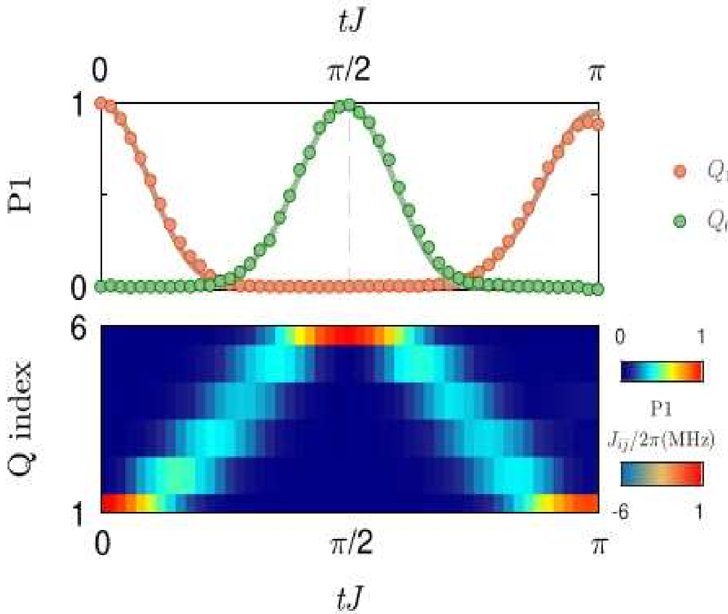


It really works in 1D:

"Enhanced quantum state transfer: Circumventing quantum chaotic behavior"
Nature Communications **15**, 4918 (2024)
Liang Xiang, RTS, *etal*

Quantum state transfer: Experimental results

1d chain of qubits:



Emulated Hamiltonian (NN couplings are tunable)

$$\hat{H} = \sum_{\langle i,j \rangle} J_{ij}[\hat{\sigma}_i^+ \hat{\sigma}_j^- + \hat{\sigma}_i^- \sigma_j^+]$$

$$J_{n,n+1} = J\sqrt{n(6-n)}$$

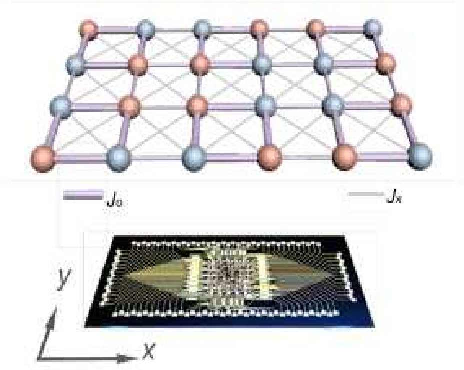
$$J/2\pi = -1 \text{ MHz}$$

- Transfer of one-excitation states with remarkable fidelity

How about different geometries?

3'. Real World Problems

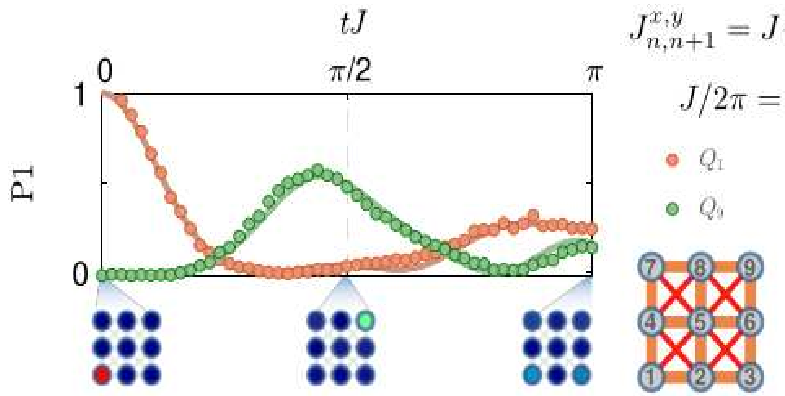
2d quantum state transfer - 3x3



Fabricated @ ZJU

- Parasitic cross couplings $J_\times$ naturally occur in current devices

$$
\begin{aligned}
\hat{H}_{\mathrm{tot}} &= 2J(\hat{S}_{1,x} + \hat{S}_{2,x}) + \\
&\quad J_\times(\hat{S}_{1-}\hat{S}_{2+} + \hat{S}_{1+}\hat{S}_{2-} + \hat{S}_{1+}\hat{S}_{2+} + \hat{S}_{1-}\hat{S}_{2-}) \\
&= 2J(\hat{S}_{1,x} + \hat{S}_{2,x}) + 4J_\times \hat{S}_{1x}\hat{S}_{2x}
\end{aligned}
$$



→ spoils "Christandl's prescription"

4. Monte Carlo and the "QST Inverse Problem"

Proceed via Monte Carlo.

Engineer $\{ J_{ij} \}$ to achieve 'Target' time evolution operator

$$\mathcal{U}^* = e^{-i\mathcal{H}^* t}$$

Define an action:

$$\mathcal{S} = \sum_{i,j} (\mathcal{U}_{ij} - \mathcal{U}^*_{ij})^2$$

Begin with a random set of $\{ J_{i,j} \}$.

Propose 'moves' which change $\{ J_{i,j} \}$.

Accept with the 'heat bath' probability $e^{-\beta \Delta \mathcal{S}} \left(1 + e^{-\beta \Delta \mathcal{S}}\right)^{-1}$.

$\Delta \mathcal{S} \equiv$ the change in action from Monte Carlo move.

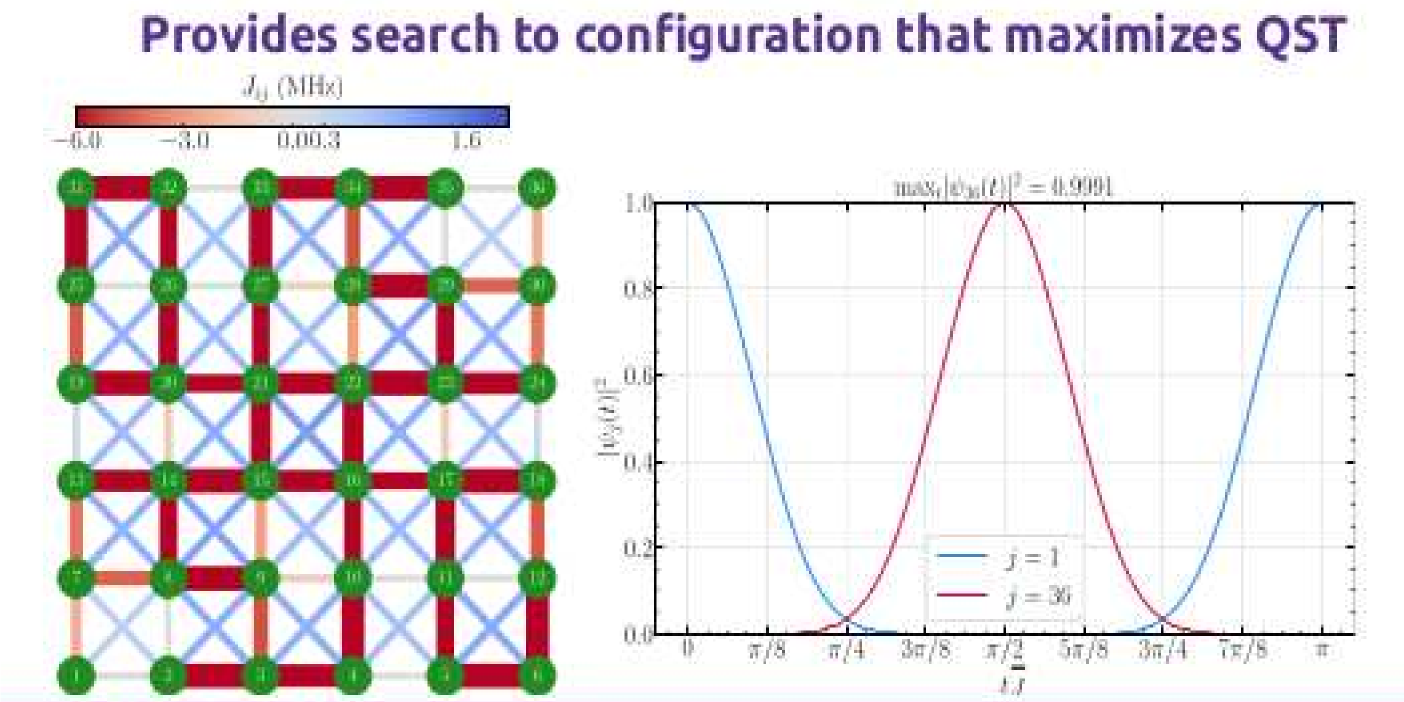
‘Annealing:’ β starts at a small value (e.g. $\beta_{\rm i} \sim 0.1$).

Do Monte Carlo, then increase β. After K steps $\beta_{\rm f} = \alpha^K \beta_{\rm i}$ (typical $\beta_{\rm f} = 10^4$.)

Statistical mechanics language: $\beta = 1/T$ is the inverse temperature.

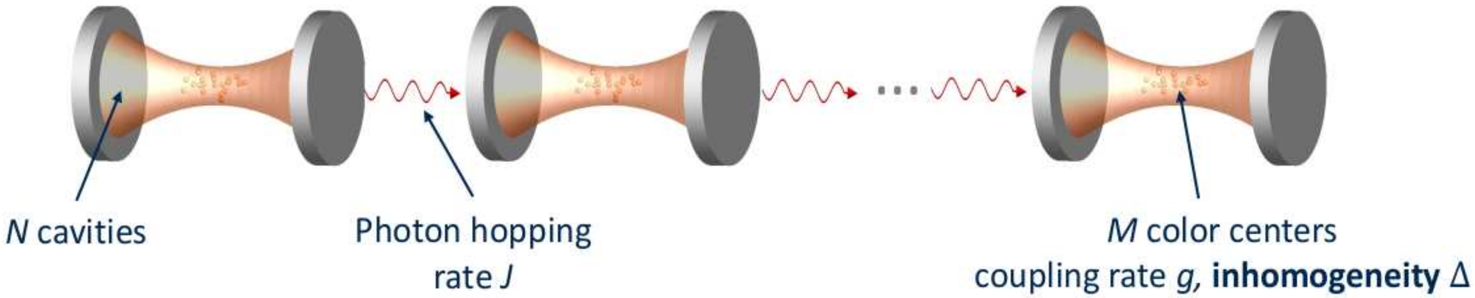
$\beta_{\rm i} = 0.1$: high temperature. $\beta_{\rm f} = 10^4$: low temperature. Escape metastable states.

$\{J_i, g_i\}$ give target $\mathcal{U}^*$ high accuracy.



Similar protocol for coupled cavity-emitter arrays (Radulaski group).

Phys. Rev. B105, 195429 (2022).

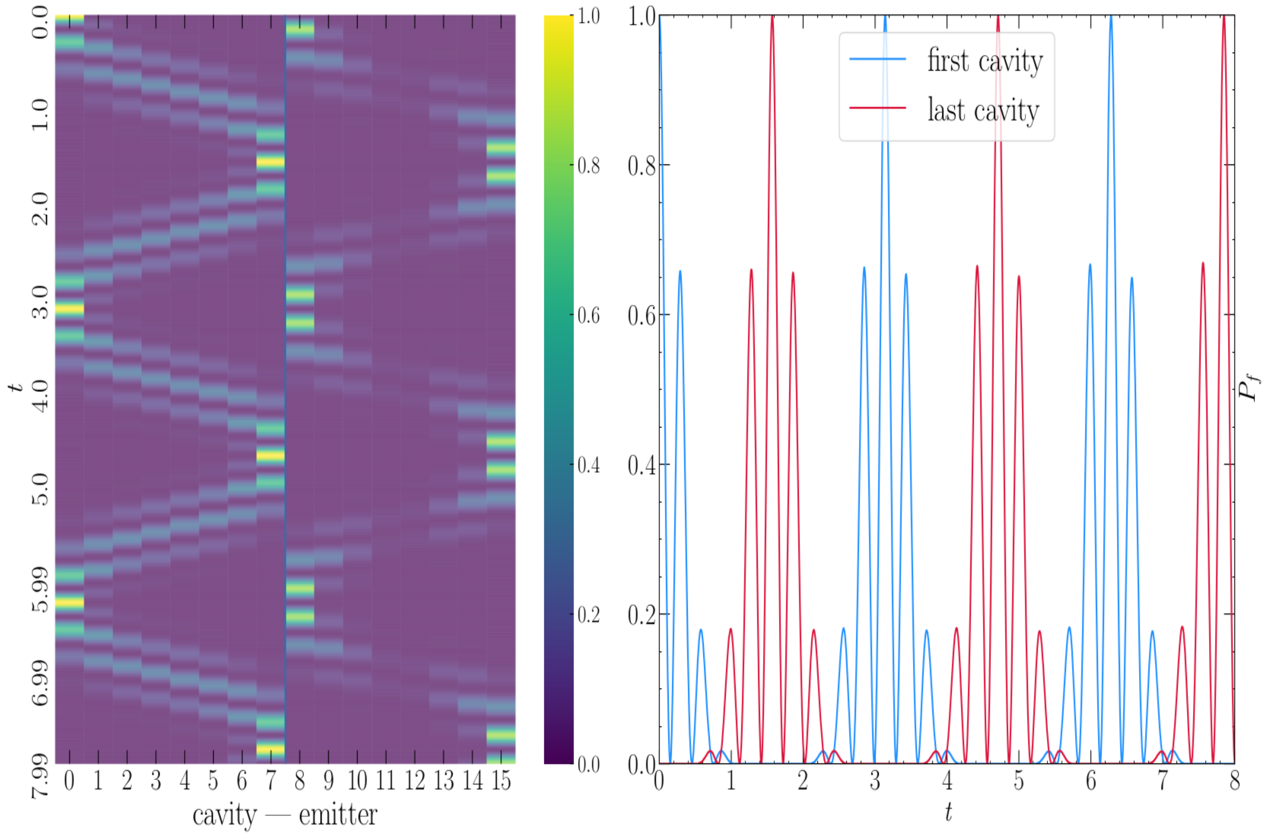


$$\mathcal{H} = \begin{pmatrix} \Omega_1 & J_{1,2} & 0 & 0 & g_1 & 0 & 0 & 0 \\ J_{1,2} & \Omega_2 & J_{2,3} & 0 & 0 & g_2 & 0 & 0 \\ 0 & J_{2,3} & \Omega_3 & J_{3,4} & 0 & 0 & g_3 & 0 \\ 0 & 0 & J_{3,4} & \Omega_4 & 0 & 0 & 0 & g_4 \\ g_1 & 0 & 0 & 0 & \omega_1 & 0 & 0 & 0 \\ 0 & g_2 & 0 & 0 & 0 & \omega_2 & 0 & 0 \\ 0 & 0 & g_3 & 0 & 0 & 0 & \omega_3 & 0 \\ 0 & 0 & 0 & g_4 & 0 & 0 & 0 & \omega_4 \end{pmatrix}$$

Explored 'imperfections' about optimized $\mathcal{H}$.)

- Randomness in J_{ij}, g_i
- Randomness in Ω_i

Perfect Quantum State Transfer for the CCA geometry:



Monte Carlo works! Transfer with perfect fidelity from site $i = 1$ to site $i = N$.

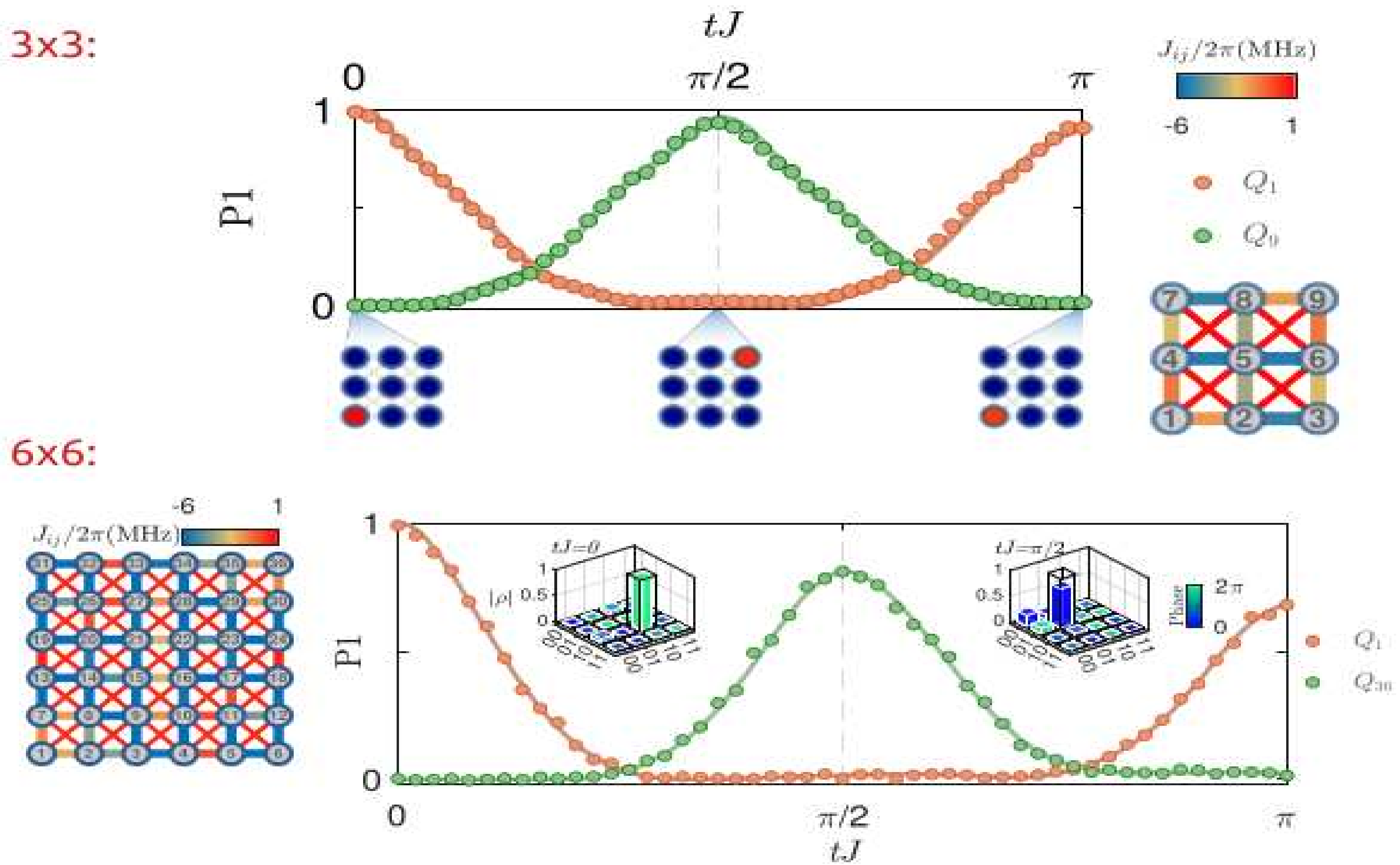
Small/negligible deviation from fidelity $f = 1$ due to finite MC simulation time.

Can achieve arbitrary accuracy by lengthening run.

"Effect of Emitters on Quantum State Transfer in Coupled Cavity Arrays," E. Baum, A. Broman, T. Clarke, N.C. Costa, J. Mucciaccio, A. Yue, Y. Zhang, V. Norman, J. Patton, M. Radulaski, and RTS, Phys. Rev. B105, 195429 (2022).

Can rectify the "real world problems" cross coupling (and defective coupler).

And providing guide to experiments!



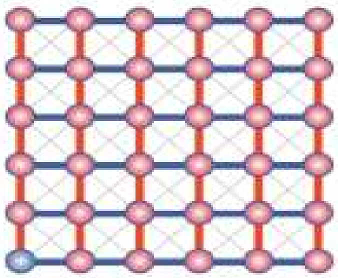
"Enhanced quantum state transfer: Circumventing quantum chaotic behavior", Liang Xiang, RTS, *etal*, Nature Communications **15**, 4918 (2024)

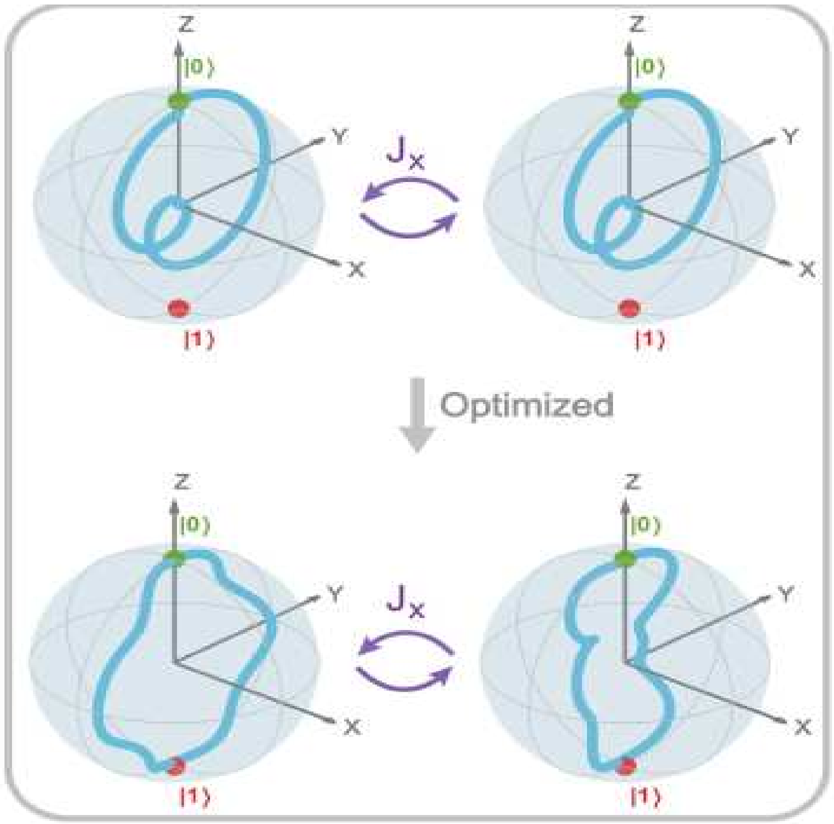
Generalization of Christandl to 2D:

$\hat{S}_x$ and $\hat{S}_y$.

Christandl prescription (top) misses propagation to target qubit.

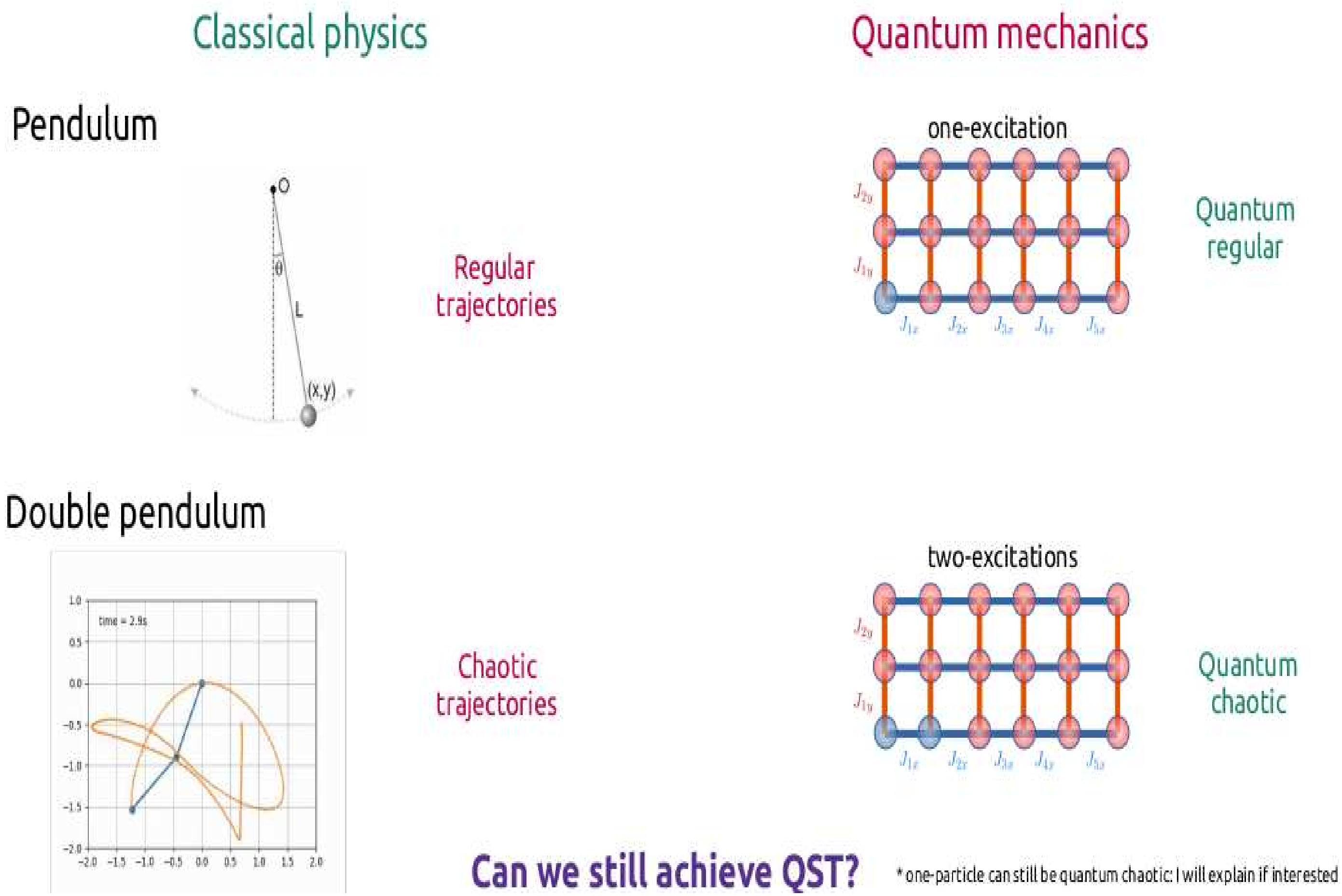
Monte Carlo optimized $\mathcal{H}$ recovers high fidelity (through intricate path).





Preceding: Can compensate for cross-couplings and defective coupler.

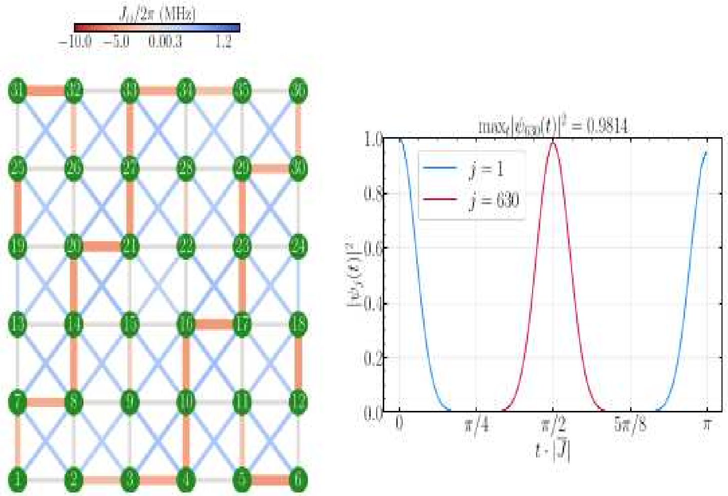
What about multiple excitations?



Can get high fidelity QST in theory (left).

But experimental implementation of theory-guided J_{ij} not quite there yet.

Numerical solutions

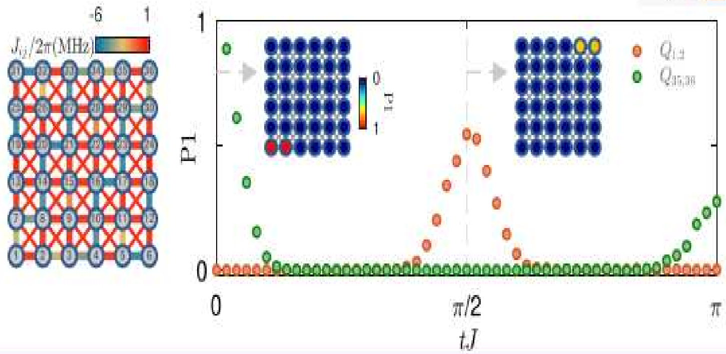


→ Problem is exponentially harder:

$$\mathcal{D}_H = \binom{36}{2} = 630$$

Preliminary experimental results

Sensitivity of fine-tuning of the qubit couplings ←



This all seems a black box!

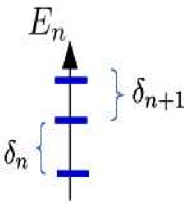
Adjust $\mathcal{H}$ in some (strange) way to get good QST.

Is any insight possible into what's happening?

How? By "curing" quantum chaos!

- Adjacent gap analysis (eigenenergy repulsion):

$$r_n = \frac{\min(\delta_n, \delta_{n+1})}{\max(\delta_n, \delta_{n+1})} \quad \text{where} \quad \delta_n = E_{n+1} - E_n$$

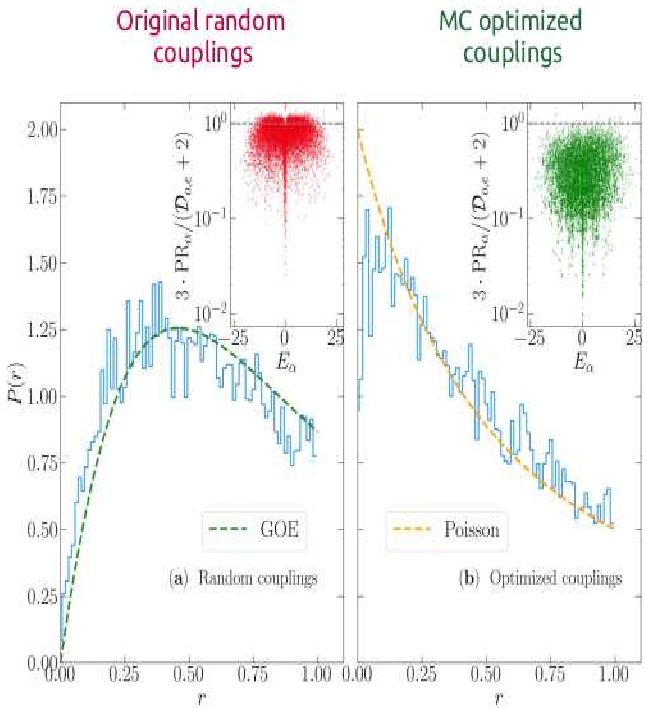


$$P_{GOE} = \frac{27}{4}\frac{r + r^2}{(1 + r + r^2)^{5/2}}\Theta(1 - r)$$

$$P_P = \frac{2}{1 + r^2}\Theta(1 - r)$$

- Participation ratio (eigenstate spread in the basis)

$$\mathrm{PR}_\alpha = \frac{1}{\sum_{n=1}^{\mathcal{D}_{o,e}} |c_\alpha^n|^4} \qquad \mathrm{PR}^{\mathrm{GOE}} = \frac{\mathcal{D} + 2}{3}$$



- The key is that the system with two excitations is weakly chaotic… and can be "fixed" → But with a large number of excitations quantum chaos kicks in!

"Enhanced quantum state transfer: Circumventing quantum chaotic behavior", Liang Xiang, RTS, *etal*, Nature Communications **15**, 4918 (2024)

6. Conclusions

- Usual diffusion of wave function can be circumvented by ‘engineering’.
- Monte Carlo method used in achieving target time evolution operator.
- Generalize Christandl prescription in 1D.
- High fidelity quantum state transfer achievable.

  Cavity-Emitter Arrays (with disorder).

  2D with ‘real world’ effects (cross coupling, dead coupler).

  Multiple excitation (physical insight into where $\mathcal{H}$ evolves.

- “Enhanced quantum state transfer: Circumventing quantum chaotic behavior”, Liang Xiang, RTS, *etal*, Nature Communications **15**, 4918 (2024)
- “Effect of Emitters on Quantum State Transfer in Coupled Cavity Arrays,” E. Baum, A. Broman, T. Clarke, N.C. Costa, J. Mucciaccio, A. Yue, Y. Zhang, V. Norman, J. Patton, M. Radulaski, and RTS, Phys. Rev. B105, 195429 (2022).
- “Quantum State Transfer in Interacting, Multiple-Excitation Systems,” Alexander Yue, Rubem Mondaini, Qiujiang Guo, and RTS, Phys. Rev. B110, 195410, (2024), Editor’s suggestion.

Previous Experimental Efforts: 01

Kandel, "*Adiabatic quantum state transfer in a semiconductor quantum dot spin chain*", Nat. Comm. 10.1038/s41467-021-22416-5 (2021).

Quantum device is GaAs/AlGaAs heterostructure with overlapping gates.

Qubits are electron spins on quantum dot.

Three electron spins. Transfer from spin 1 to spin 3.

Transfer one and two spin states in 127 ns.

Limiting factor is nuclear hyperfine noise in GaAs/AlGaAs heterostructure.

More robust to transfer a singlet state of two adjacent spins than a single spin up.

95% fidelity.

Read out of electron state: like NMR, coupling to and sensing of nuclear spin.

Previous Experimental Efforts: 02

X. Li, "*Perfect Quantum State Transfer in a Superconducting Qubit Chain with Parametrically Tunable Parameters*", PRA 10, 054009 (2018).

Aluminum film etched with electron beam lithography. 10 mK operating temperature.

Tune qubit coupling stength with ac magnetic flux applied to individual qubits.

Implemented the Christandl "angular momentum" scheme.

84 ns transfer with 99.2% fidelity across chain of four qubits.

Typical experimental CCA parameters:

$g_i \sim 5\,\mathrm{GHz}$	$0 < \Delta g_i < g_{\max}$
$J_i \sim 1\,\mathrm{GHz}$	can be ‘repaired‘
$\Omega_i \sim 200\,\mathrm{THz}$	can be ‘repaired‘

Systems that have been constructed:

- ~ 20 empty cavities (no emitters)
- One cavity with two emitters
- Two cavities with one emitter each

Origins of disorder:

- Δg_i position of emitter within cavity.
- $\Delta \omega_i$ variation of strain in material
- $\Delta J_i, \Delta\Omega_i$ can be ‘repaired’ (laborious) photo-oxidation of part of cavity.