

Energy Loss Characterization and Stochastic Effects in G4beamline Simulations

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1 Introduction

The interaction of charged particles with matter is one of the central topics in accelerator and detector physics. As charged particles propagate through material media, they continuously lose energy through electromagnetic interactions with atomic electrons. Accurate modeling of these processes is essential in many areas of high-energy physics, including detector development, beam transport, radiation shielding, and accelerator optimization studies [10, 11, 3].

In the context of future Muon Collider facilities, understanding muon energy loss is especially important because proposed cooling channels rely heavily on ionization cooling. In ionization cooling, muons pass through low- Z absorber materials where they lose momentum through ionization processes. The longitudinal momentum can subsequently be restored using RF cavities, producing a net reduction in transverse beam emittance. Since cooling performance depends directly on both energy deposition and multiple scattering effects, accurate simulation of energy-loss mechanisms is required [9].

This work investigates the behavior of positive muons (μ^+) propagating through different absorber materials and geometries using the simulation framework `G4beamline`, which is built upon the Geant4 toolkit [4, 5, 8]. The study focuses on both average energy-loss behavior and the stochastic fluctuations arising from discrete ionization interactions. The primary goals of this work are:

1. To characterize the energy-loss distributions of muons in absorber materials.
2. To investigate how the distributions evolve with absorber thickness.
3. To compare deterministic and stochastic particle tracking.
4. To study the transition from Landau-like energy-loss distributions in thin absorbers to approximately Gaussian behavior in thicker materials.
5. To qualitatively compare the observed behavior with expectations from the Bethe-Bloch formalism.

Particular attention is given to the statistical properties of the energy-loss distributions, including comparisons between the Mean, Median, and Most Probable Value (MPV).

2 Physical Background

2.1 Energy Loss of Charged Particles in Matter

The average energy loss of charged particles traversing matter is described by the Bethe-Bloch equation [1, 2, 3]:

$$\left\langle -\frac{dE}{dx} \right\rangle \propto \frac{1}{\beta^2} \ln(\gamma^2 \dots) \quad (1)$$

where:

- $\beta = v/c$ is the particle velocity normalized to the speed of light,
- γ is the Lorentz factor.

Although the complete Bethe-Bloch equation contains several correction terms, the dominant qualitative behavior relevant to this work is the velocity dependence. As the muon slows down while traversing the absorber, the stopping power gradually increases because of the $1/\beta^2$ dependence.

Consequently, the local energy deposition per unit length increases as the particle penetrates deeper into the material.

2.2 Energy Straggling and Landau Fluctuations

While the Bethe-Bloch equation predicts the mean energy loss, the actual energy deposited by individual particles fluctuates event-by-event because ionization interactions occur discretely and probabilistically [10, 11].

For sufficiently thin absorbers, the energy-loss distribution becomes highly asymmetric and develops a characteristic Landau-like shape. These distributions exhibit:

- A sharp peak near the most probable energy loss,
- A long high-energy tail,
- Significant event-by-event fluctuations.

The long tail originates from rare collisions in which a large amount of energy is transferred to an atomic electron, producing energetic secondary electrons known as delta rays.

As a consequence:

$$\text{Mean} > \text{Median} > \text{MPV} \quad (2)$$

The arithmetic mean is therefore strongly influenced by the high-energy tail, whereas the MPV more accurately represents the typical energy loss in thin absorbers.

2.3 Transition Toward Gaussian Behavior

As absorber thickness increases, the particle undergoes many independent ionization interactions. According to the Central Limit Theorem, the cumulative effect of these interactions causes the overall energy-loss distribution to become increasingly Gaussian.

This transition produces several observable effects:

- The relative importance of the Landau tail decreases,
- Statistical fluctuations become averaged,
- Mean, Median, and MPV gradually converge.

Studying this transition is one of the primary objectives of the present work.

3 Simulation Framework

3.1 G4beamline and Geant4 Environment

All simulations were performed using `G4beamline`, a beam simulation framework based on the Geant4 particle transport toolkit [8, 4, 5]. Geant4 provides detailed microscopic modeling of electromagnetic and hadronic particle interactions in matter.

The simulations employed the `QGSP_BIC` physics list, which combines:

- Quark-Gluon String models for high-energy hadronic interactions,
- Binary Cascade models for intermediate-energy interactions,
- Standard Geant4 electromagnetic physics processes.

Although this study primarily focuses on electromagnetic energy loss, the complete physics list ensures self-consistent treatment of all secondary processes generated during particle transport.

3.2 Beam Configuration

A Gaussian-distributed positive muon beam was generated with a reference momentum of:

$$p = 200 \text{ MeV}/c \tag{3}$$

This momentum region is particularly relevant for muon ionization cooling studies because it lies near the minimum-ionizing region while still exhibiting substantial ionization energy loss and multiple scattering effects [3].

The primary beam configuration is summarized in Table 1.

Table 1: Primary beam configuration used in the simulations.

Parameter	Value
Particle Type	μ^+
Reference Momentum	200 MeV/c
Beam Distribution	Gaussian
Events per Run	10,000
Physics List	<code>QGSP_BIC</code>
Stochastic Processes	Enabled

3.3 Stochastic Energy Loss Modeling

A central component of this work is the inclusion of stochastic fluctuations through the Geant4 stochastic tracking option:

```
doStochastics=1
```

When enabled, Geant4 treats ionization processes as discrete interactions rather than continuous deterministic energy loss [6, 7]. This allows realistic modeling of:

- Energy straggling,
- Delta-ray production,
- Event-by-event fluctuations,
- Non-Gaussian energy-loss distributions.

Without stochastic tracking, particles follow an effectively averaged trajectory based solely on mean energy loss. Although this approximation is useful for simplified beam-dynamics studies, it cannot reproduce the asymmetric energy-loss distributions observed in thin absorbers.

4 Absorber Configurations

4.1 Carbon Absorbers

Carbon absorbers were implemented using cylindrical tube geometries in G4beamline using the `tubs` element. The absorber configuration used in this work is defined by:

```
tubs Target
  outerRadius=200
  material=C
  length=195
  color=1,0,0
```

The material density was set to:

$$\rho_C = 1.69 \text{ g/cm}^3 \tag{4}$$

Carbon provides a useful benchmark material because its ionization properties are well understood and it produces relatively low multiple scattering [3]. Its relatively simple material properties also make it well suited for validating energy-loss and stochastic transport models in particle tracking simulations.

4.2 Lithium Hydride (LiH) Absorbers

Lithium Hydride (LiH) absorbers were implemented using wedge geometries constructed with the `trap` element in G4beamline. The absorber geometry used in this study is defined by:

```
trap wedge0
  height=350
  length=700
  upperWidth=0.005
  lowerWidth=59.5
  material=LITHIUM_HYDRIDE
  color=0,0,1
```

The wedge geometry provides a continuously varying material thickness across the transverse beam profile, which is essential for emittance exchange and six-dimensional ionization cooling in modern muon cooling lattices.

LiH is widely used in muon cooling studies because it combines:

- Low atomic number,
- Favorable ionization cooling performance,
- Reduced multiple scattering compared to higher- Z materials.

Modern cooling lattice concepts such as the HFOFO cooling channel employ LiH wedge absorbers extensively for six-dimensional muon beam cooling.

5 Source Code Enhancements

In addition to the standard G4beamline functionality, custom source-code modifications were implemented to improve the treatment of stochastic reference tracking.

New reference and tune particle definitions were introduced that preserve stochastic effects during propagation. These modifications allow direct comparison between:

1. Deterministic reference tracking,
2. Fully stochastic particle evolution.

The modified framework provides:

- Standard deterministic reference particles,
- Stochastic-aware reference particles,
- Stochastic-aware tune particles.

Traditional reference-particle tracking suppresses fluctuations and therefore cannot fully reproduce realistic beam evolution in absorber materials. The enhanced implementation enables more detailed studies of beam centroid evolution, momentum spread growth, and local energy deposition under realistic stochastic conditions.

6 Results and Discussion

6.1 Comparison of Energy Loss in Carbon and LiH Absorbers

The average energy loss behavior was compared for Carbon and Lithium Hydride (LiH) absorbers using several tracking configurations in order to isolate the role of stochastic ionization processes and material properties.

The following cases were studied:

1. Reference particle tracking without stochastic processes,
2. Reference particle tracking with stochastic processes enabled,
3. Beam particle tracking without stochastic processes,
4. Beam particle tracking with stochastic processes enabled,
5. Bethe-Bloch expectation.

Table 2 shows the mean stopping power $\langle dE/dx \rangle$ obtained for both absorber materials. In all configurations, the simulated energy loss follows the general trend predicted by the Bethe-Bloch formalism. Small deviations arise from finite detector thickness, secondary production, and stochastic fluctuations included in the full transport simulations. The comparison further demonstrates clear material dependence of the stopping power. Due to their different atomic composition, density, and effective ionization properties, Carbon and LiH exhibit distinct average energy-loss behavior over the studied momentum range. The lower- Z LiH absorber generally produces smaller stopping power and reduced fluctuation magnitude compared to Carbon. The effect of stochastic processes is also visible in the average energy-loss measurements. When stochastic ionization is enabled, additional event-by-event fluctuations broaden the distributions and slightly modify the extracted mean values. The deterministic configurations provide smoother behavior and remain closer to the idealized Bethe-Bloch expectation.

Table 2: Comparison of average energy loss $\langle dE/dx \rangle$ for different absorber materials and tracking configurations for 200 MeV muons. Values are given in MeV/cm.

Configuration	Reference particle	Beam, no stochastics	Beam, with stochastics	Reference, with stochastics	Bethe-Bloch
Carbon	3.24	3.24	3.27	3.33	3.29
LiH	1.72	1.72	1.73	1.43	1.73

To further investigate the fluctuation structure, the full dE/dx distributions for Carbon and LiH were compared for beam configuration, as shown in Figure 1. Both materials exhibit asymmetric energy-loss distributions with long high-energy tails characteristic of ionization straggling. However, the degree of asymmetry and distribution width differs between the absorbers due to their differing material properties and interaction probabilities. For thin absorber regions, the distributions display several characteristic features:

- A pronounced difference between the Mean and the Most Probable Value (MPV),

- Long non-Gaussian high-energy tails,
- Significant event-by-event fluctuations.

These observations are consistent with the expected Landau-type energy-loss behavior for charged particles traversing thin layers of matter. The comparison between Carbon and LiH therefore provides a useful validation of both the implemented stochastic transport model and the expected material dependence of muon ionization energy loss.

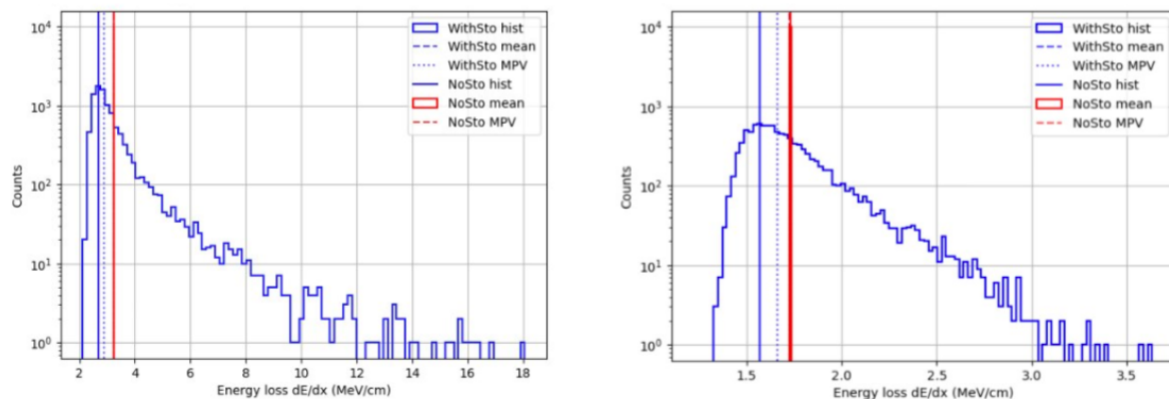


Figure 1: Comparison of dE/dx distributions for 200 MeV muons in Carbon (left) and LiH (right) absorbers. The blue histograms correspond to simulations with stochastic ionization processes enabled, while the red histograms represent simulations without stochastic processes. The inclusion of stochastic effects produces broader distributions with pronounced non-Gaussian high-energy tails, consistent with Landau energy straggling. The LiH absorber exhibits reduced fluctuation magnitude compared to Carbon due to its lower effective atomic number and stopping power.

6.2 Local Energy Loss Along the Absorber

The local stopping power was studied by measuring the energy deposited within small longitudinal slices along the absorber.

The simulations show a clear increase in local energy loss with increasing longitudinal position Z as shown in figure 2. This behavior is consistent with the expected Bethe-Bloch dependence as the muons gradually slow down while traversing the material [1, 3].

For thin slices, the energy-loss distributions exhibit pronounced asymmetry:

- The MPV lies significantly below the Mean,
- The distributions contain long high-energy tails,
- Large event-by-event fluctuations are observed.

These features are characteristic signatures of Landau energy straggling.

6.3 Cumulative Energy Loss

The cumulative energy-loss analysis integrates the total energy deposited from the entrance of the absorber to increasing material thicknesses and is shown in figure 3.

Several important trends are observed:

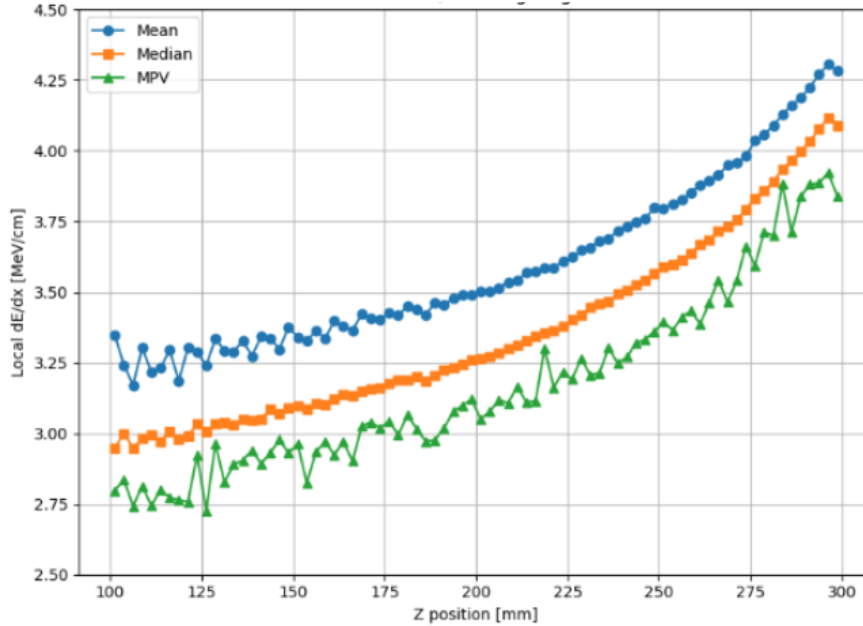


Figure 2: Local energy-loss estimators as a function of longitudinal position Z within the absorber for μ^+ particles simulated in G4beamline. The Mean energy loss (blue), Median energy loss (orange), and Most Probable Value (MPV) (green) are shown for small longitudinal slices along the absorber. All three estimators increase with increasing depth due to the rise in stopping power as the muons gradually slow down, consistent with Bethe-Bloch expectations. The persistent separation between the Mean and MPV reflects the asymmetric Landau-like energy-loss distributions and the influence of rare large-energy-transfer events associated with stochastic ionization processes.

1. The Mean cumulative energy loss increases steadily with thickness.
2. The Median and MPV gradually approach the Mean.
3. The distributions become increasingly symmetric.

As the number of ionization interactions increases, the contribution of individual large-energy-transfer events becomes less significant relative to the total accumulated energy loss.

This behavior clearly illustrates the transition from Landau-like distributions toward approximately Gaussian behavior at larger thicknesses.

6.4 Effect of Stochastic Tracking

To quantify the impact of discrete ionization processes, we compare the evolution of two statistical estimators of energy loss: the Mean and the Most Probable Value (MPV) as shown in figure. These are evaluated under four tracking configurations:

1. Reference particle tracking without stochastic processes,
2. Reference particle tracking with stochastic processes enabled,
3. Beam particle tracking without stochastic processes,
4. Beam particle tracking with stochastic processes enabled.

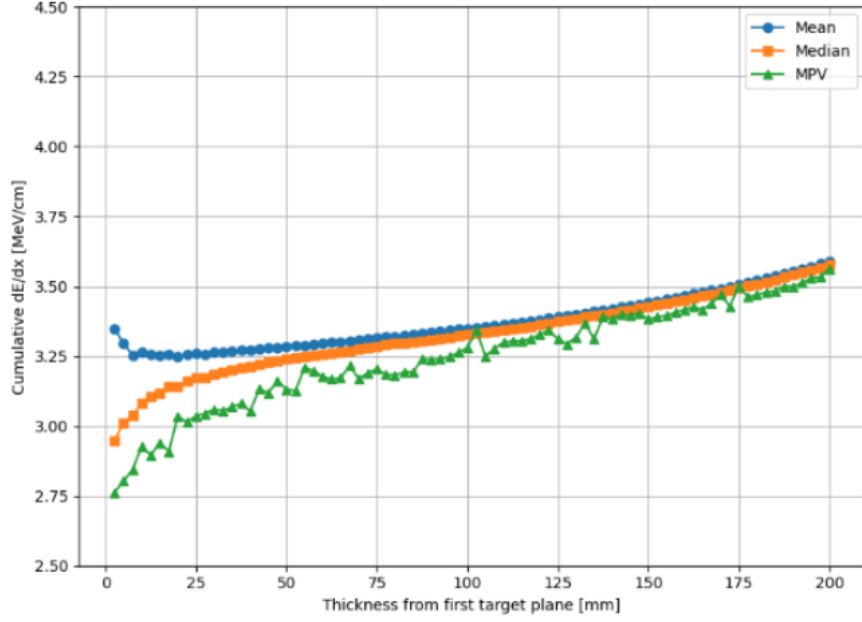


Figure 3: Cumulative energy-loss estimators as a function of absorber thickness for μ^+ particles simulated in G4beamline. The Mean (blue), Median (orange), and Most Probable Value (MPV) (green) represent the total accumulated energy loss from the entrance of the absorber up to a given longitudinal position. The Mean cumulative energy loss increases steadily with thickness, while the Median and MPV gradually converge toward the Mean as the number of ionization interactions increases. This convergence indicates a reduction in the relative importance of rare large-energy-transfer events and demonstrates the transition from asymmetric Landau-like behavior toward increasingly Gaussian energy-loss distributions in thicker absorbers.

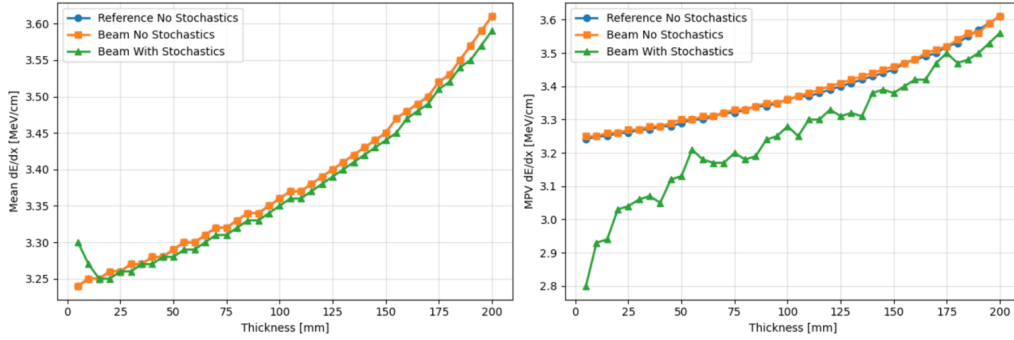


Figure 4: Comparison of energy-loss evolution using Mean (left) and MPV (right) under four tracking configurations: reference particle without stochastic processes, reference particle with stochastic processes enabled, beam particle without stochastic processes, and beam particle with stochastic processes enabled.

In the deterministic limit (stochastic processes disabled), energy loss is computed using an averaged continuous stopping power. As a result, both the Mean and MPV evolve smoothly along the absorber and remain relatively close to each other. This reflects the absence of event-by-event fluctuations and effectively suppresses energy straggling.

When stochastic processes are enabled using:

```
doStochastics=1
```

energy loss is computed through discrete ionization interactions. This introduces event-by-

event fluctuations in the energy deposition, leading to an increase in the separation between the Mean and MPV.

The effect is weak for reference particle tracking but becomes significantly more pronounced in full beam tracking, where each particle independently samples stochastic energy-loss processes. In this case:

- The Mean remains sensitive to rare large-energy-transfer events,
- The MPV reflects the most probable (typical) energy loss,
- The separation between Mean and MPV increases with absorber thickness,
- Stochastic fluctuations dominate the beam-to-beam variation.

Overall, the comparison demonstrates that deterministic reference tracking captures only the average stopping behavior, while stochastic beam tracking is required to reproduce realistic fluctuations in energy loss. The Mean–MPV separation therefore serves as a robust proxy for quantifying the impact of stochastic ionization effects in the absence of full distribution information.

7 Conclusion

This study demonstrates that G4beamline successfully reproduces the expected physical behavior of muon ionization energy loss in absorber materials.

The simulations clearly capture:

- The increase in stopping power as muons slow down,
- Landau-like energy-loss distributions in thin absorbers,
- The transition toward Gaussian behavior in thicker materials,
- The strong role of stochastic fluctuations in beam evolution.

The comparison between Mean, Median, and Most Probable Value provides a useful framework for quantifying the statistical evolution of energy-loss distributions under stochastic processes.

In addition, the custom stochastic reference-tracking implementation developed in this work extends the capabilities of G4beamline for realistic studies of ionization cooling and muon beam dynamics relevant to future Muon Collider research.

References

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