Muon Beam Dynamics in HFOFO Cooling Channel

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Introduction

- Goal: HFOFO optimization using stochastic reference particle.
  - Implement a stochastic reference particle within G4beamline and validate against the nominal reference particle (stochastic disabled by default)
  - Run multiple simulations and summarize the ensemble to find the representative trajectory.
  - Use representative stochastic particle directly for HFOFO channel tuning and optimization eliminating the need for manual tuning.

Implementation & Validation

- Stochastic reference particle is implemented in G4beamline with eventID -3 Slides
- Below table compares the average energy loss of beam and reference particles (without stochastic effects) through carbon and LiH absorbers.

	Reference particle	Pencil beam, no stochastics	Bethe-bloch
2.5 mm carbon absorber "tube" 200 MeV muon	3.24	3.24	3.29
25 mm C	3.26	3.26	3.29
LiH wedge absorber HFOFO 200 MeV muon	1.72	1.72	1.73

Implementation & Validation

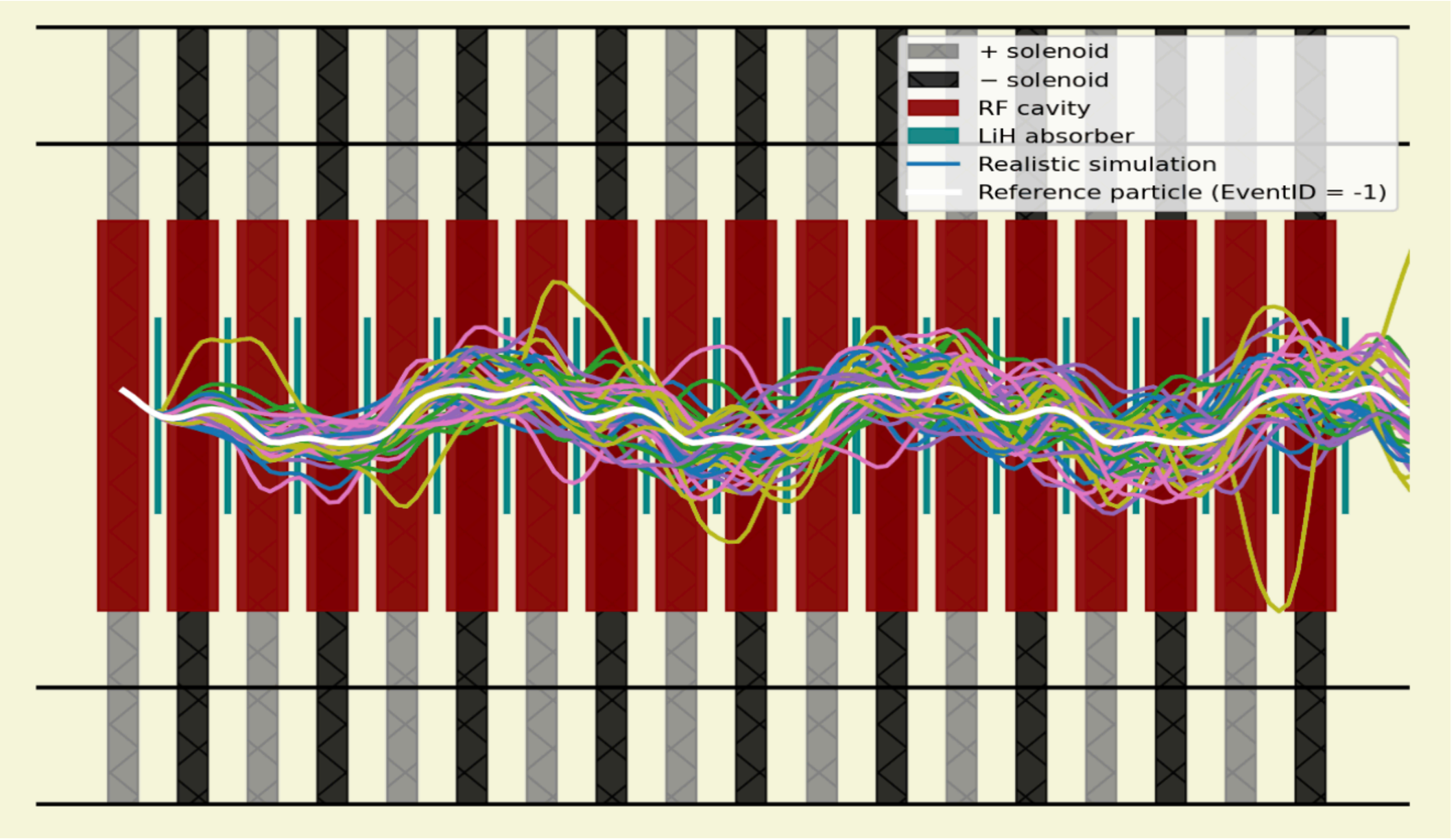
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	Pencil beam, with stochastics			**Stochastic reference particle**			**Bethe-bloch**
	Mean dE/dx (MeV/cm)	σ (standard deviation)	σ/√N N is number of survived particles	Mean dE/dx (MeV/cm)	σ (standard deviation)	σ/√N	
2.5 mm carbon absorber "tube" 200 MeV muon	3.24	1.20	0.0038 N=**99982**	3.19	0.85	0.121 N=50	3.29
25 mm C 200 MeV muon	3.26	0.37	0.0012	3.26	0.39	0.027 N=200	3.29
LiH wedge absorber HFOFO	1.72	0.24	0.0008 N=**99904**	1.72	0.24	0.011 N=500	1.73

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Stochastic Reference Particle: Multiple Simulations



Stochastic Reference Particle: Ensemble Summary

- Mean: point-by-point (at each z) average of the ensemble

$$\langle \Delta \rangle(z) = \frac{1}{N} \sum_{i=1}^{N} \Delta_i(z)$$

$$\Delta_i(z) = q_i(z) - q_{\text{nom}}(z), \qquad i = 1, \ldots, N$$

Deviation from reference trajectory

- **Median: point-by-point (at each z) middle value of the ensemble.**

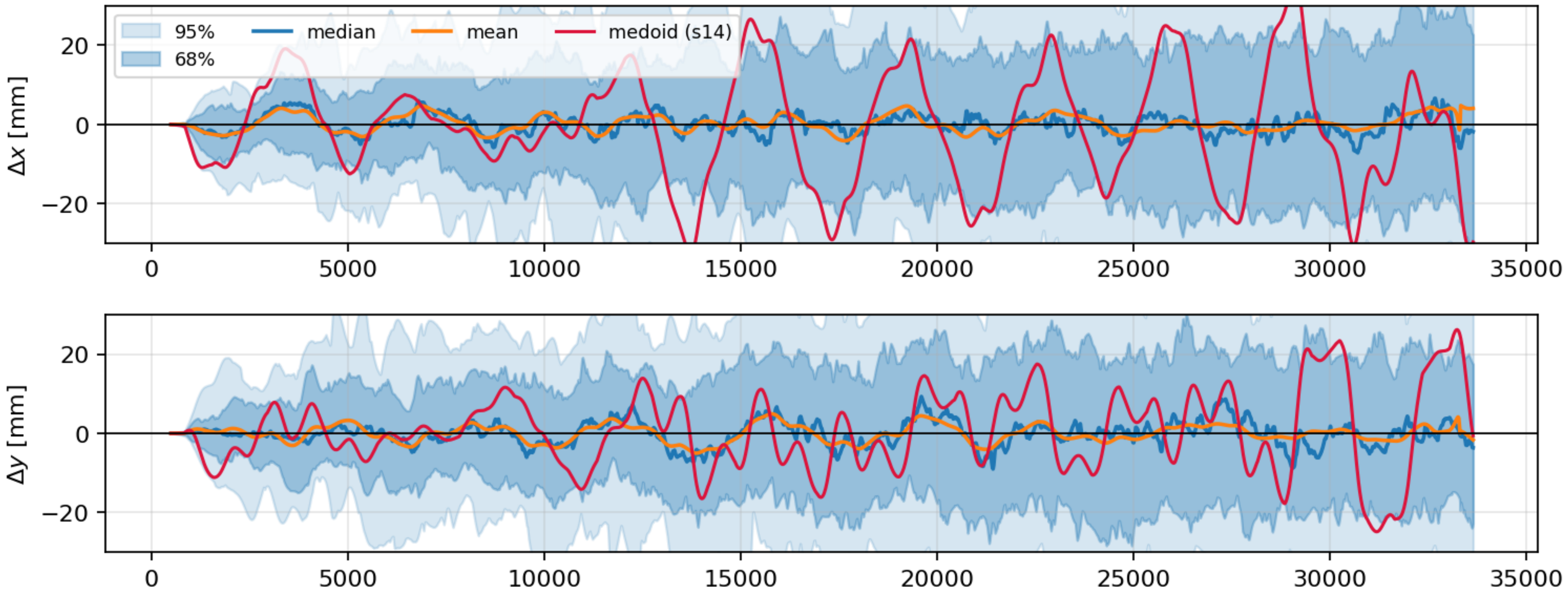
$$\operatorname{med} \Delta(z) = \begin{cases} \Delta_{\left(\frac{N+1}{2}\right)}(z), & N \text{ odd}, \\ \dfrac{\Delta_{\left(\frac{N}{2}\right)}(z) + \Delta_{\left(\frac{N}{2}+1\right)}(z)}{2}, & N \text{ even}. \end{cases}$$

- **Medoid: simulation closest to the ensemble center.**

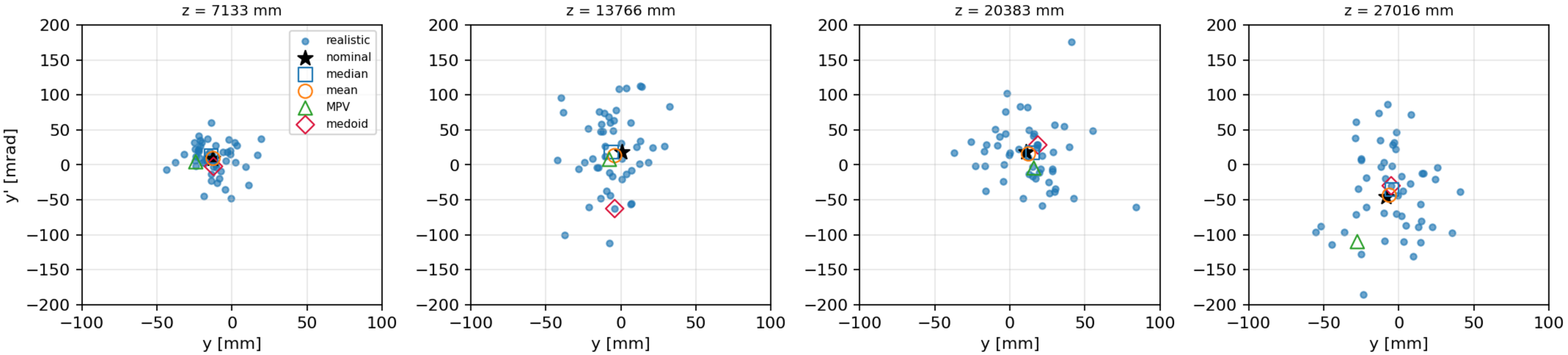
$$i_{\text{medoid}} = \underset{i=1,\ldots,N}{\arg\min}\, D_i$$

$$D_i = \sum_{k \in \mathcal{C}} w_k \sqrt{\left\langle \left(\frac{\Delta_i^k(z) - \operatorname{med} \Delta^k(z)}{\sigma_k} \right)^2 \right\rangle_z}$$

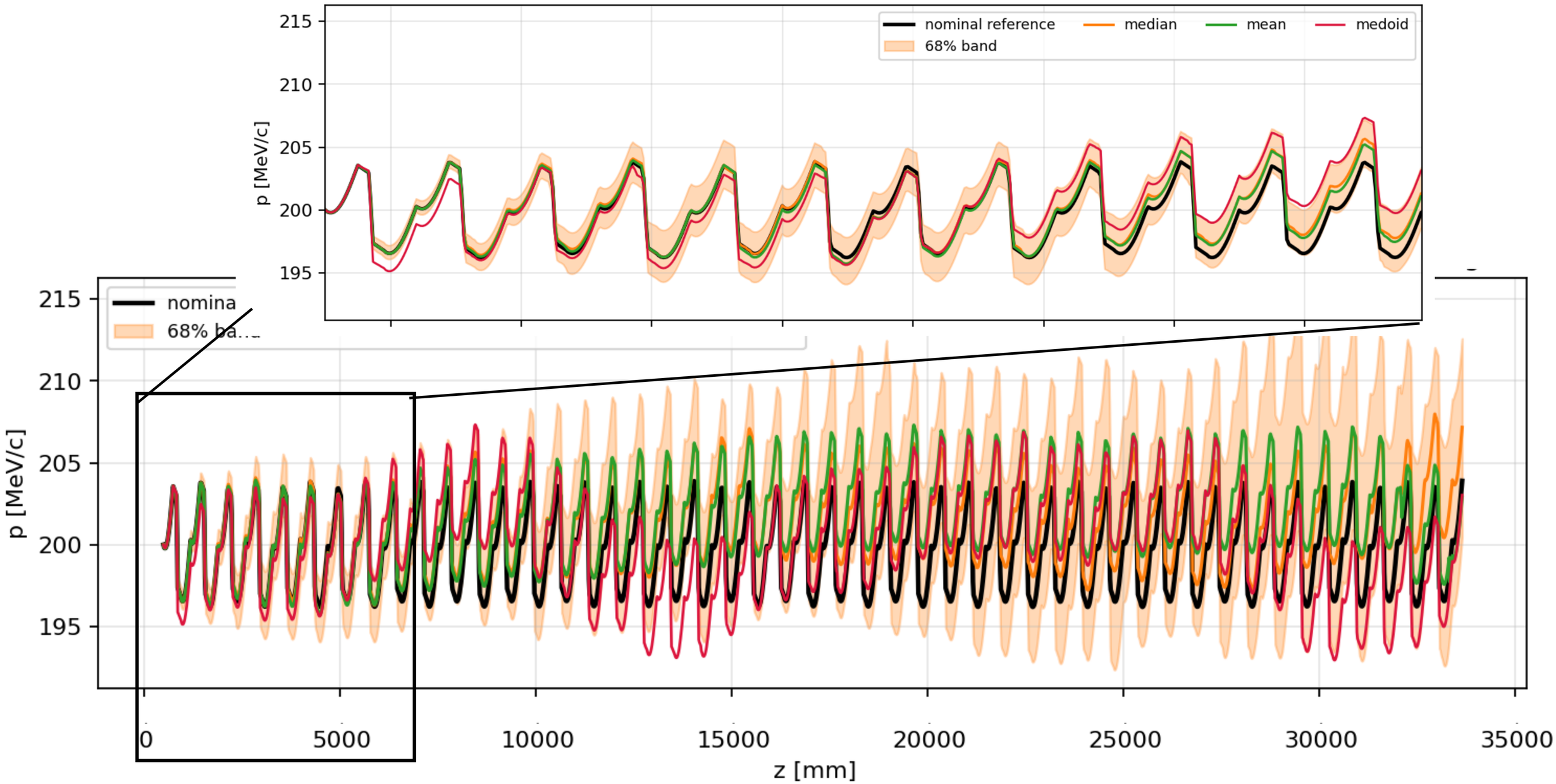
Transverse Deviation



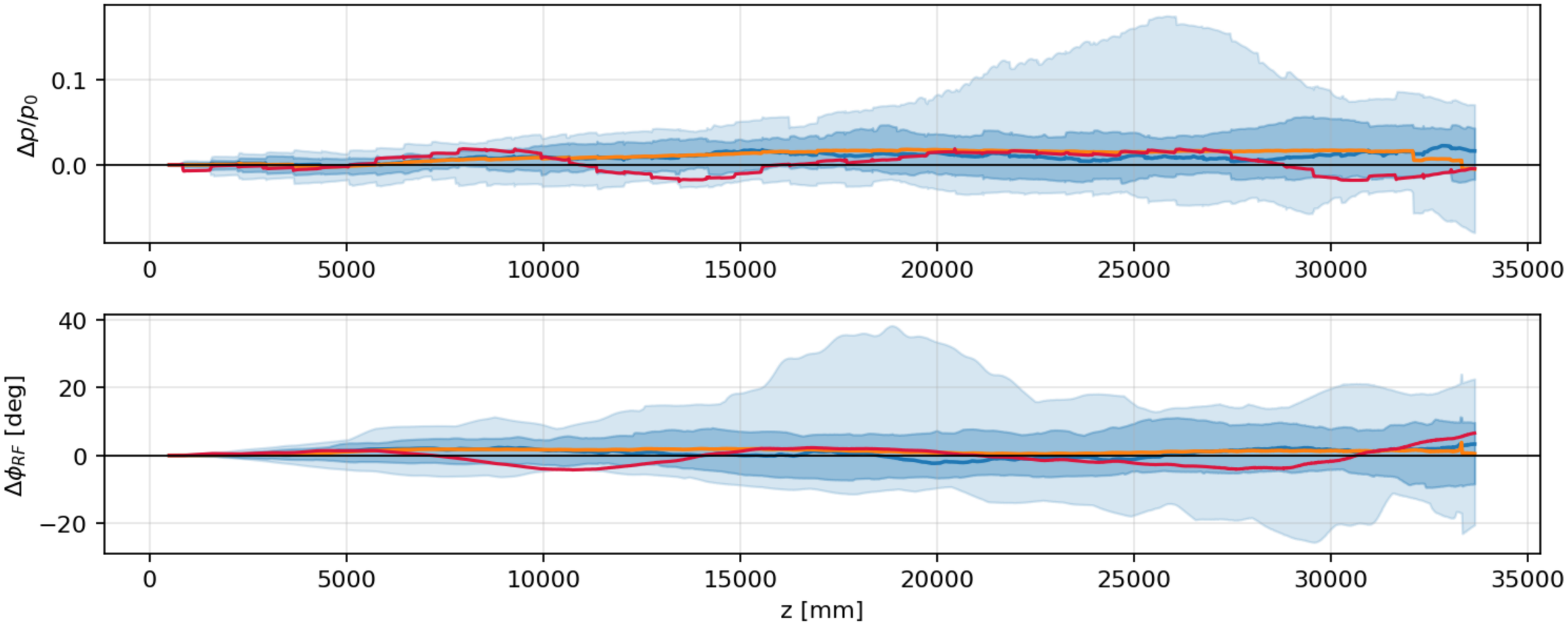
Transverse Phase Space Snapshots



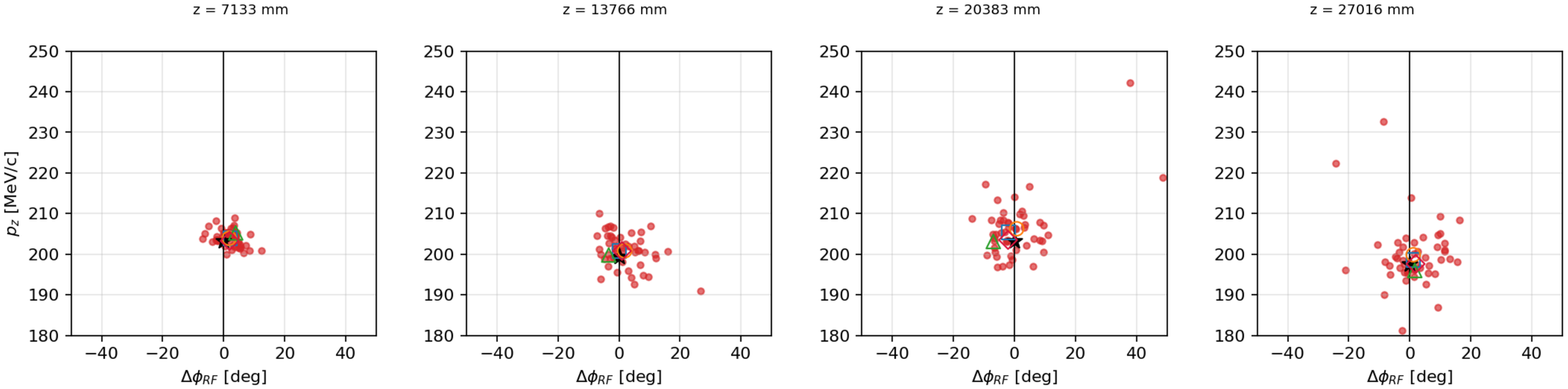
Momentum Distribution



Longitudinal Deviation



Phase Space Snapshots

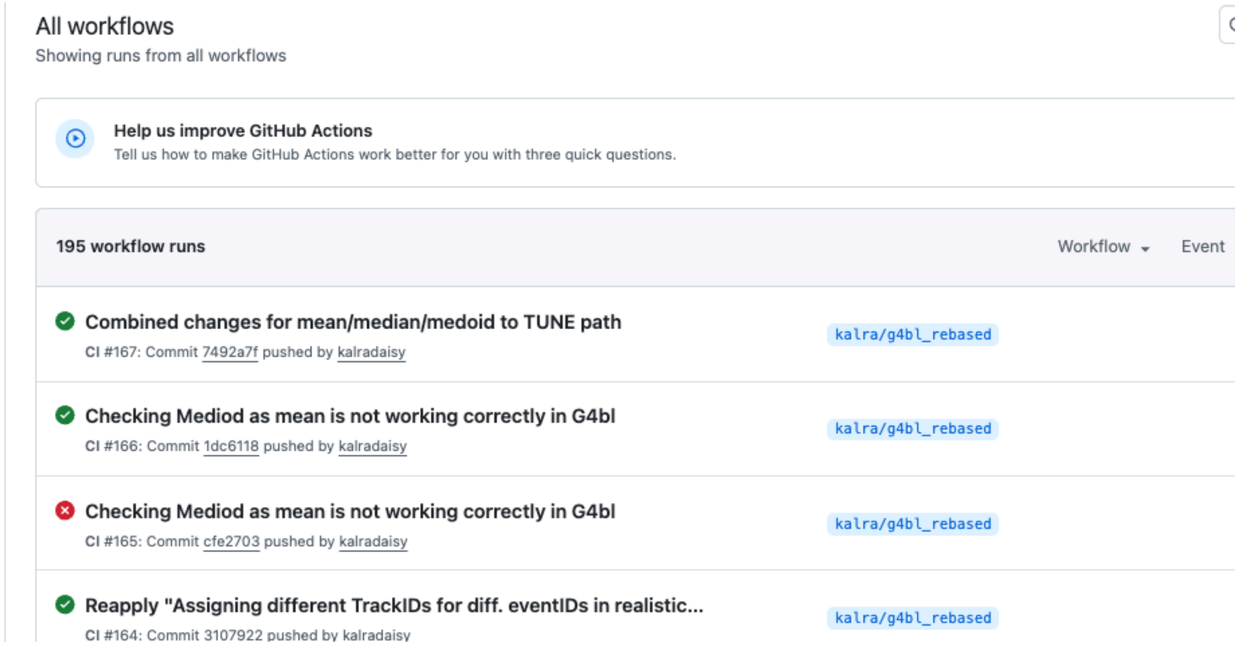


Summary

- The ensemble mean could be used as a representative stochastic particle directly for HFOFO channel tuning and optimization eliminating the need for manual tuning
- The next step is implementation within G4beamline.

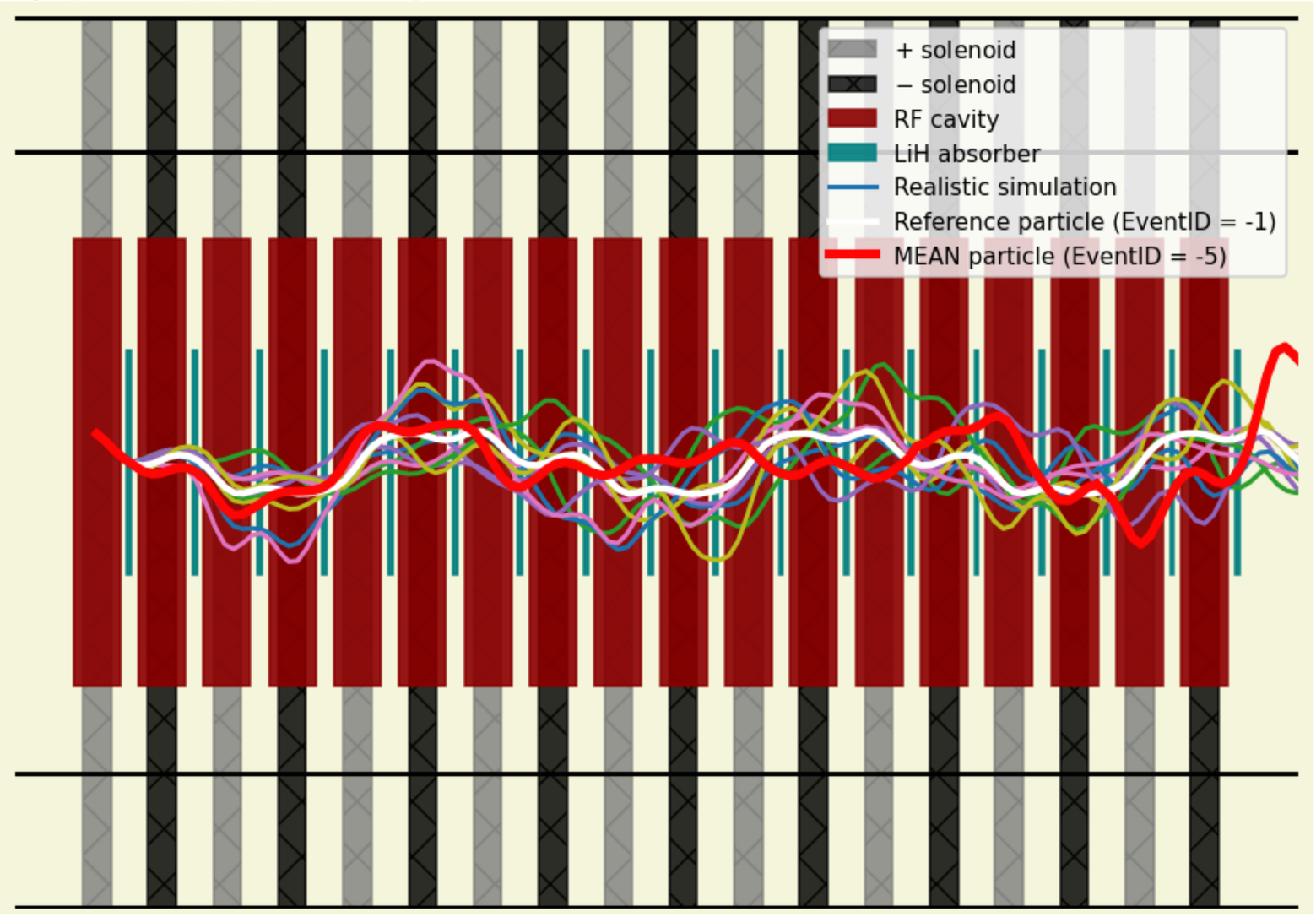
Implementation in G4beamline

- https://github.com/lawrenceleejr/g4beamline/tree/kalra/g4bl_rebased
- Instructions to run within docker container : https://indico.phys.utk.edu/event/409/contributions/1359/attachments/301/482/AccUpdate_May2026.pdf



Ensemble Mean Trajectory Overlaid

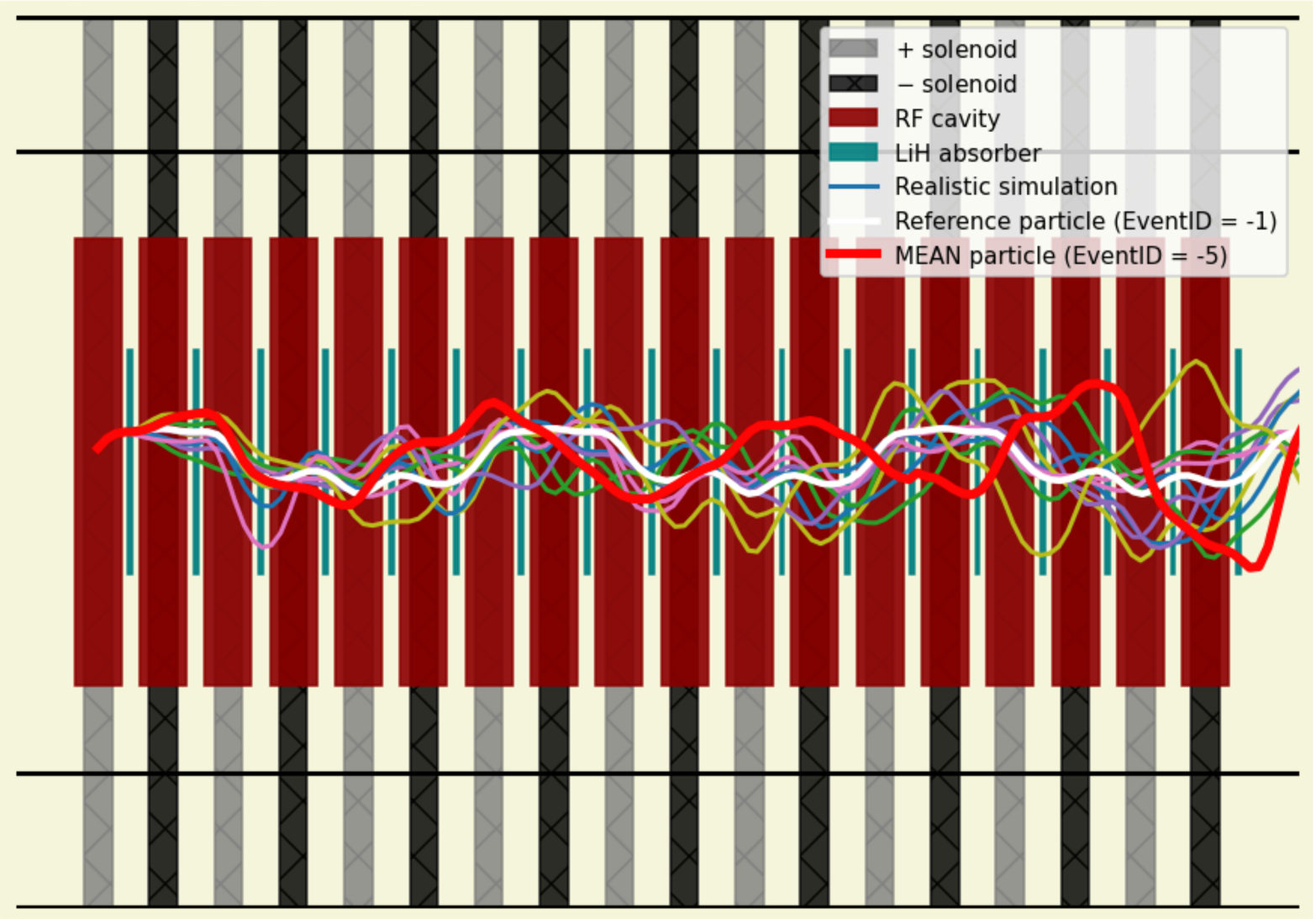
y-z space



Ensemble mean is implemented with eventID -5 in G4bl.

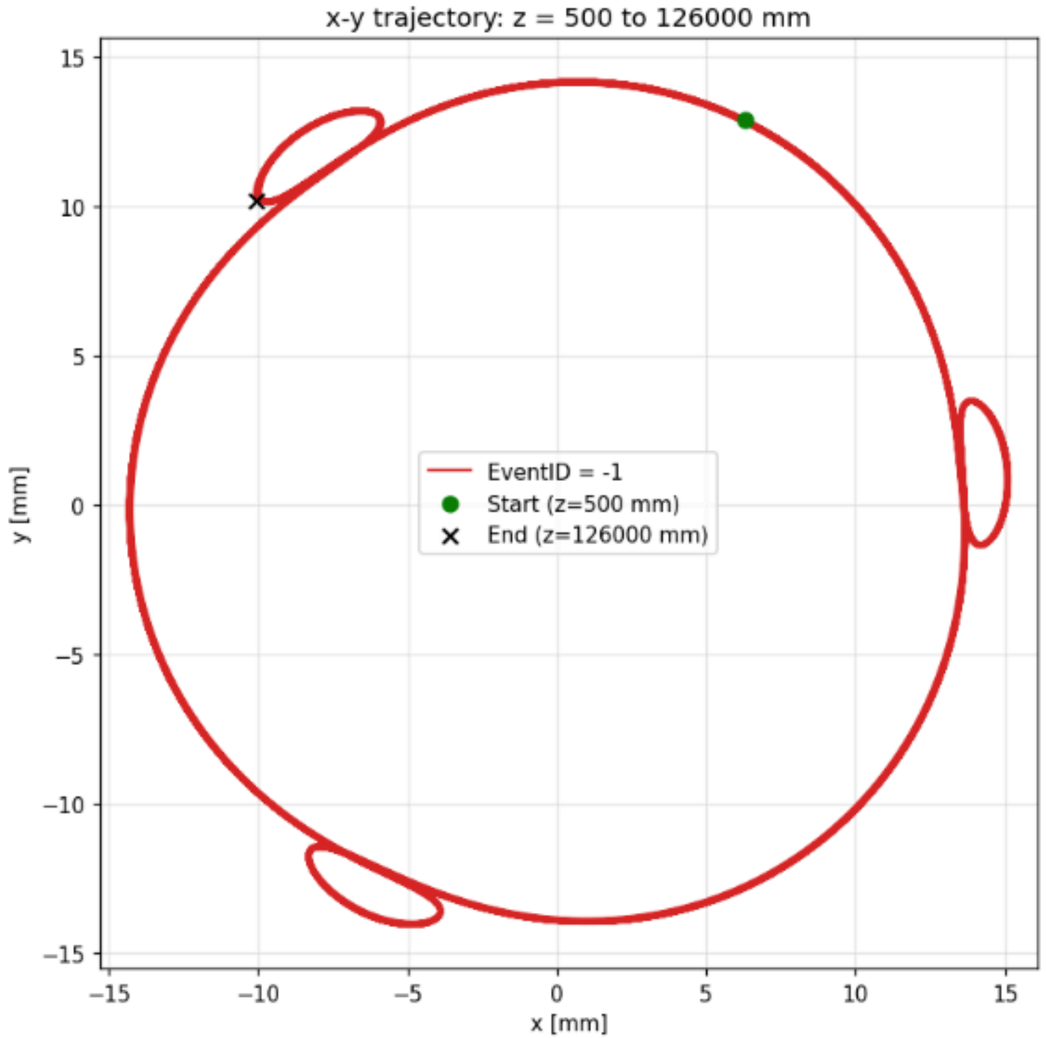
Ensemble Mean Trajectory Overlaid

x-z space

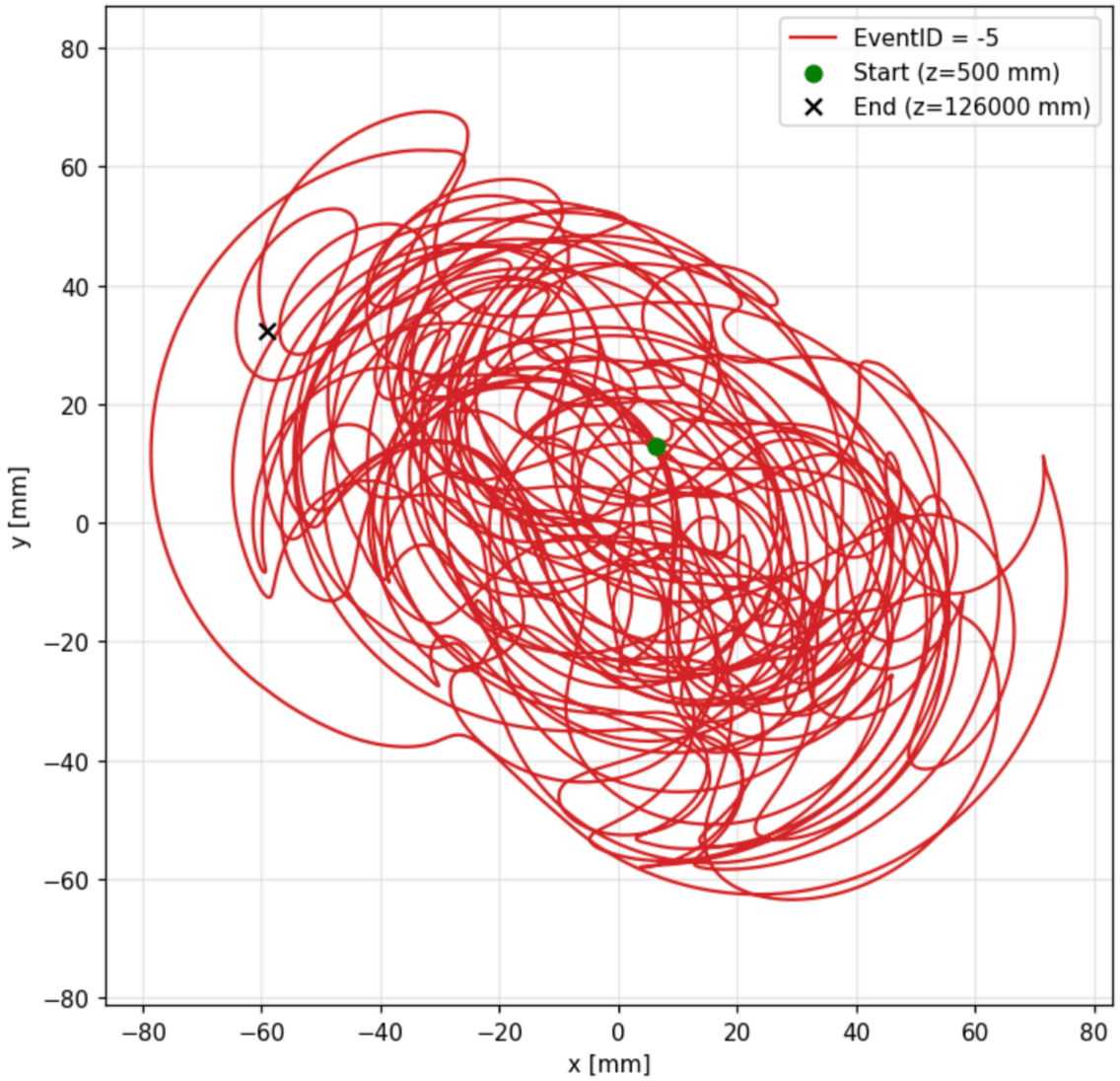


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Transverse Trajectory of Reference Particle



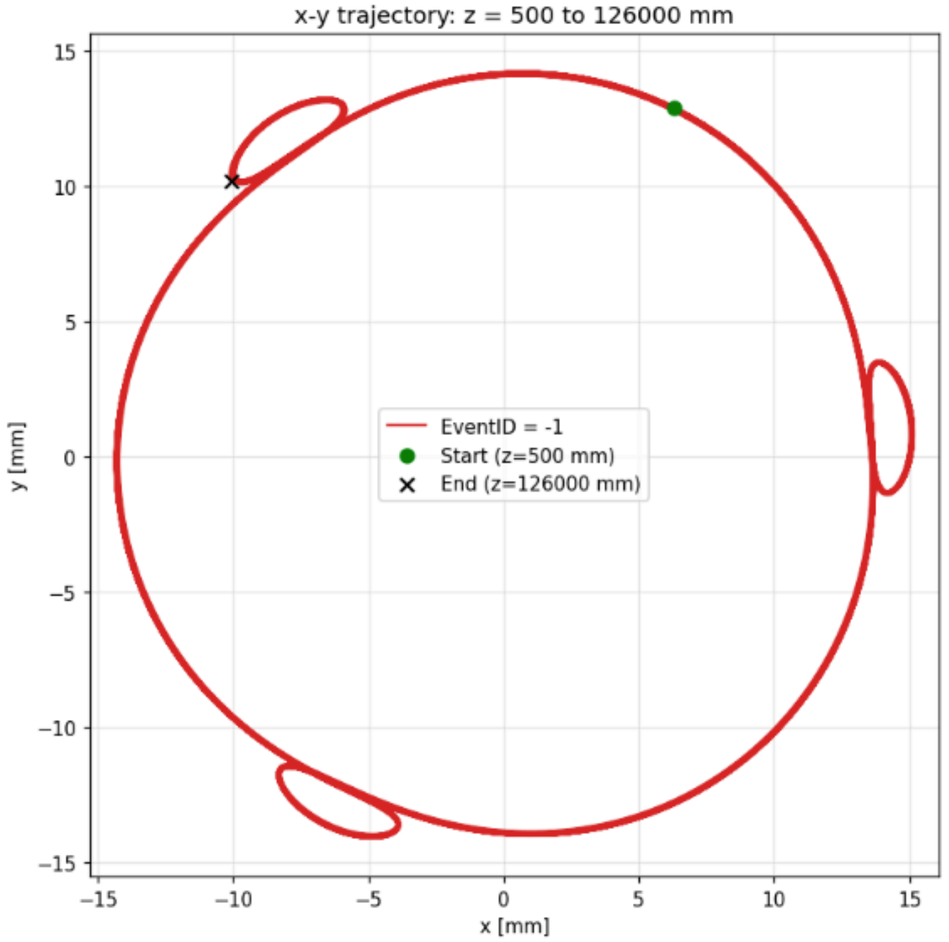
Transverse Trajectory of Stochastic Representative Particle

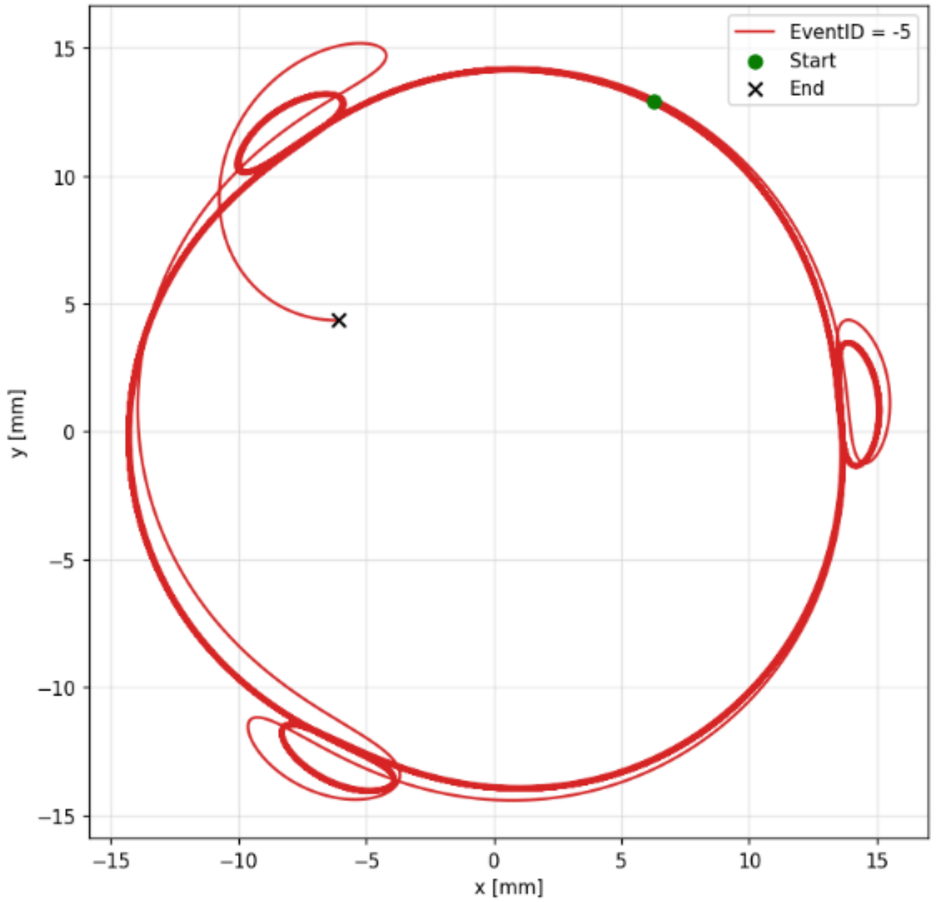


Ensemble mean is the stochastic representative particle

Back-up

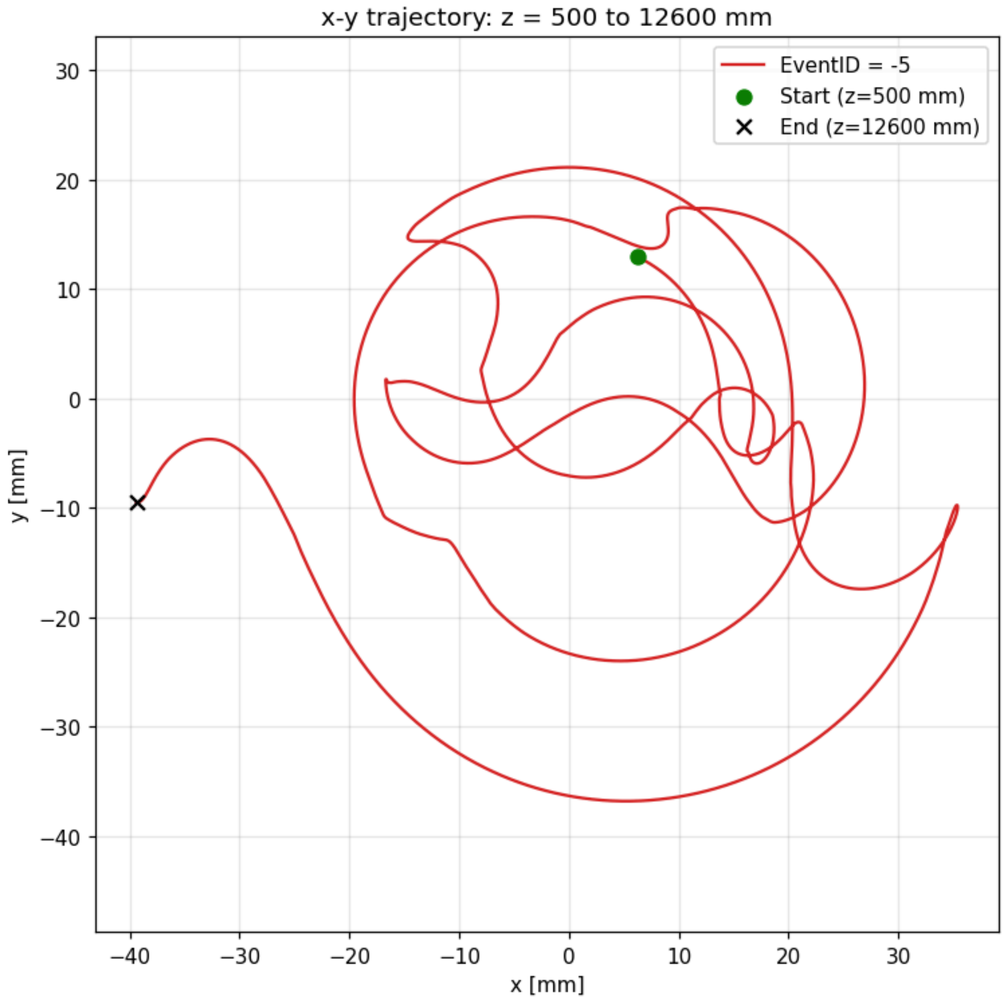
Transverse Trajectory of Reference Particle





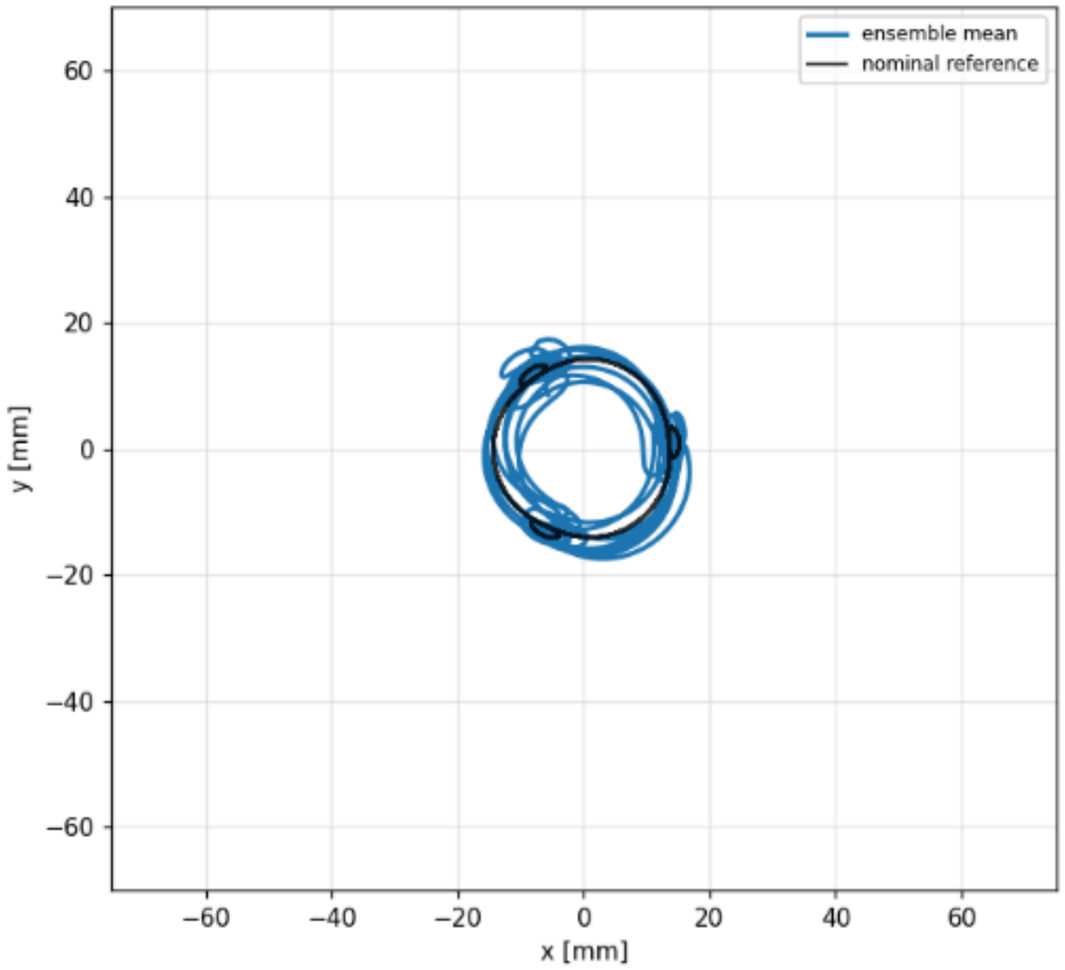
Transverse Trajectory of Stochastic Representative Particle

First three periods



Ensemble mean is the stochastic representative particle

Transverse Trajectory



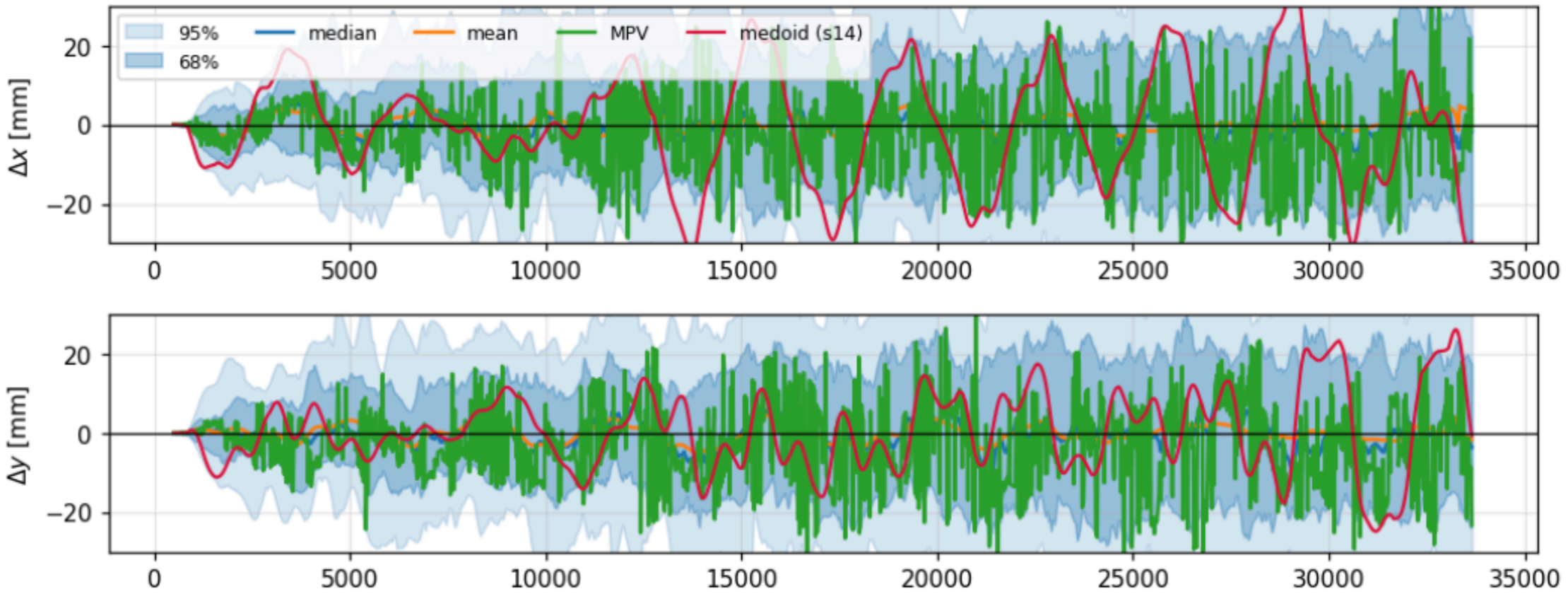
Point-by-point mean for 10 stochastic simulations

Implementation & Validation

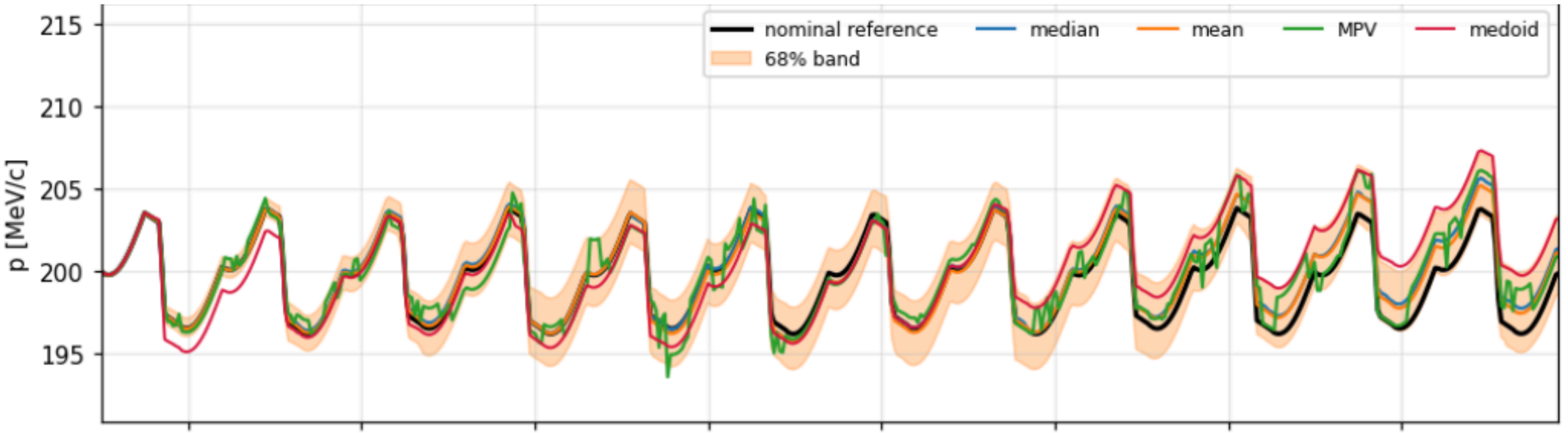
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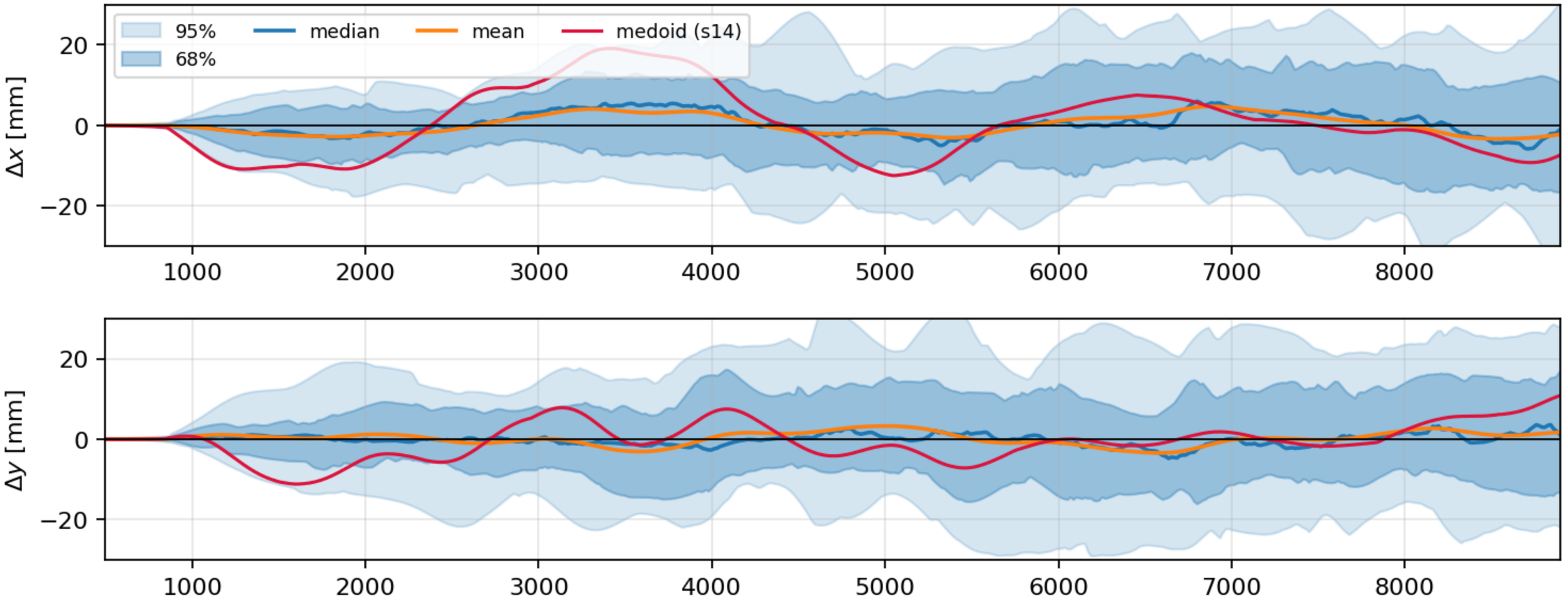
Transverse Phase Space



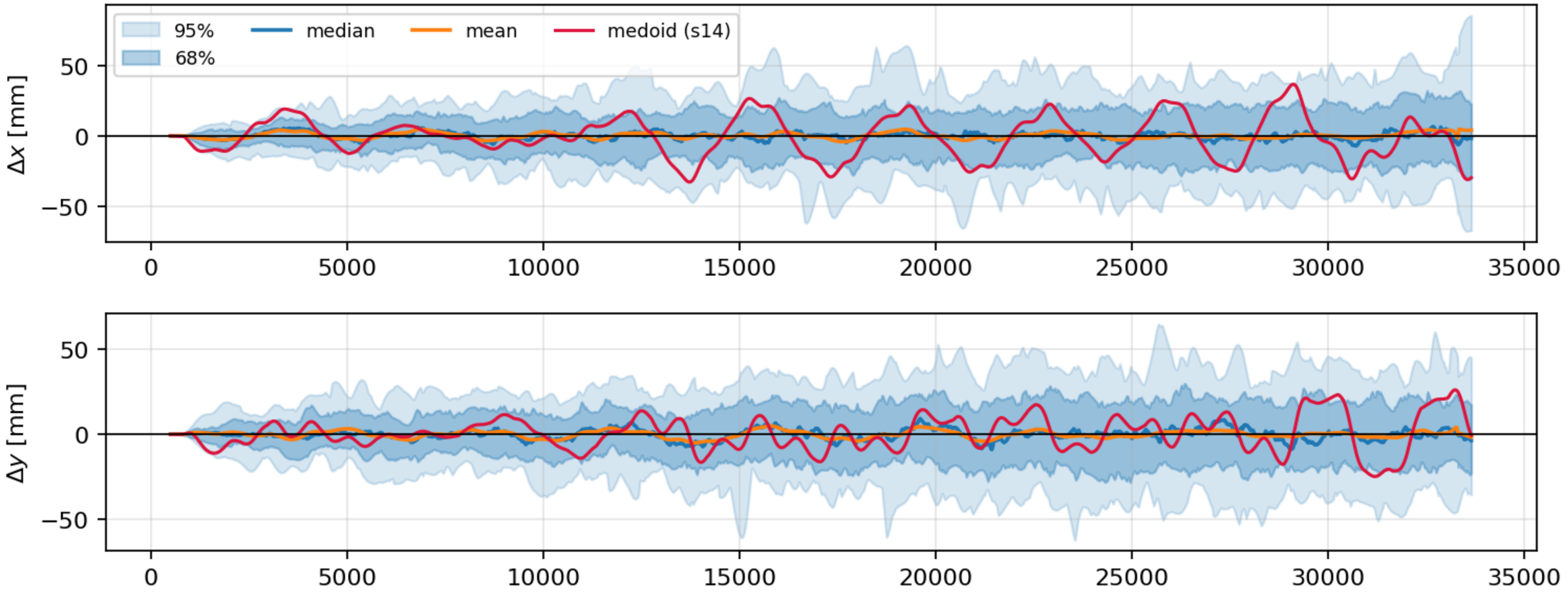
Longitudinal Momentum



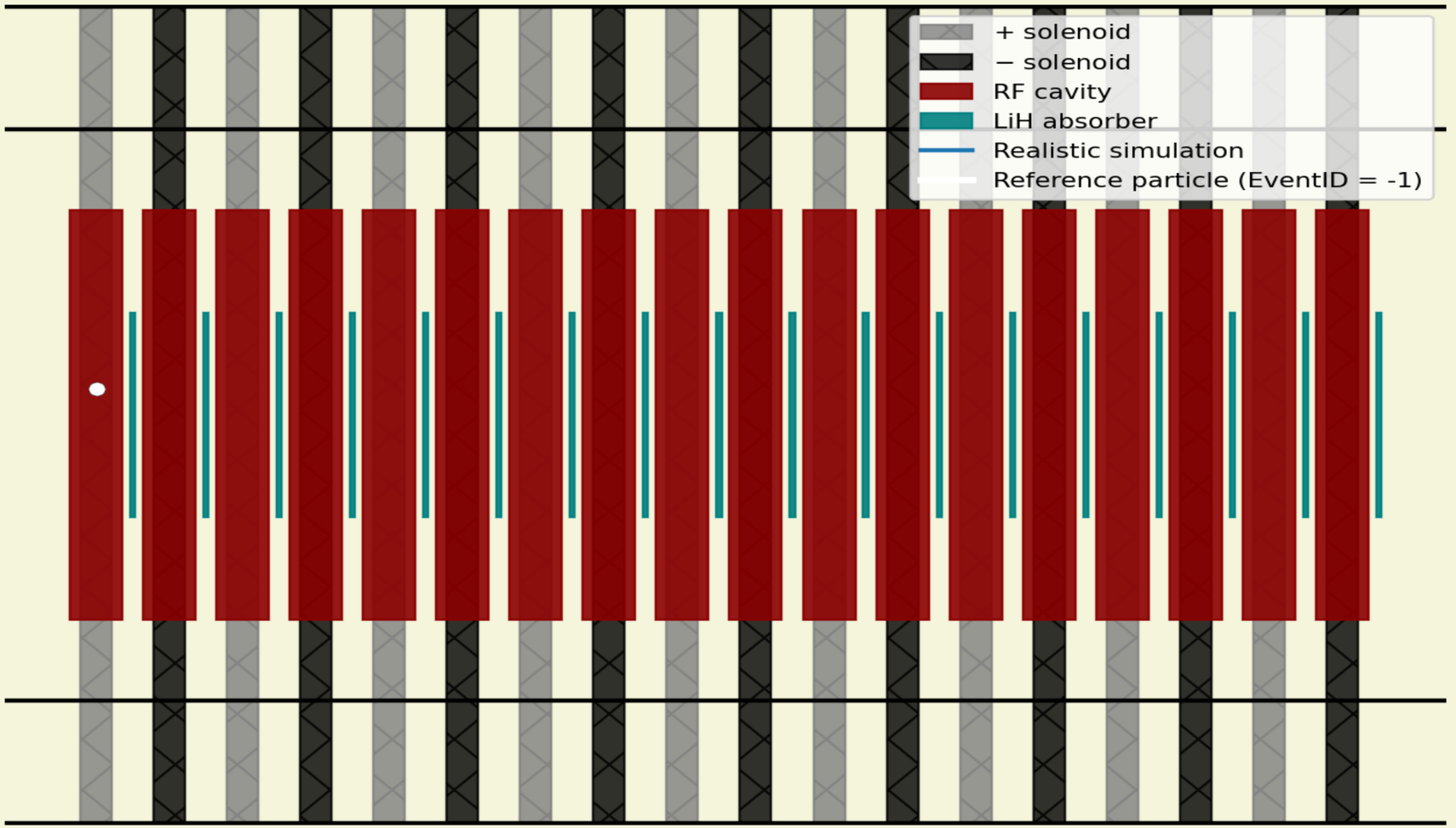
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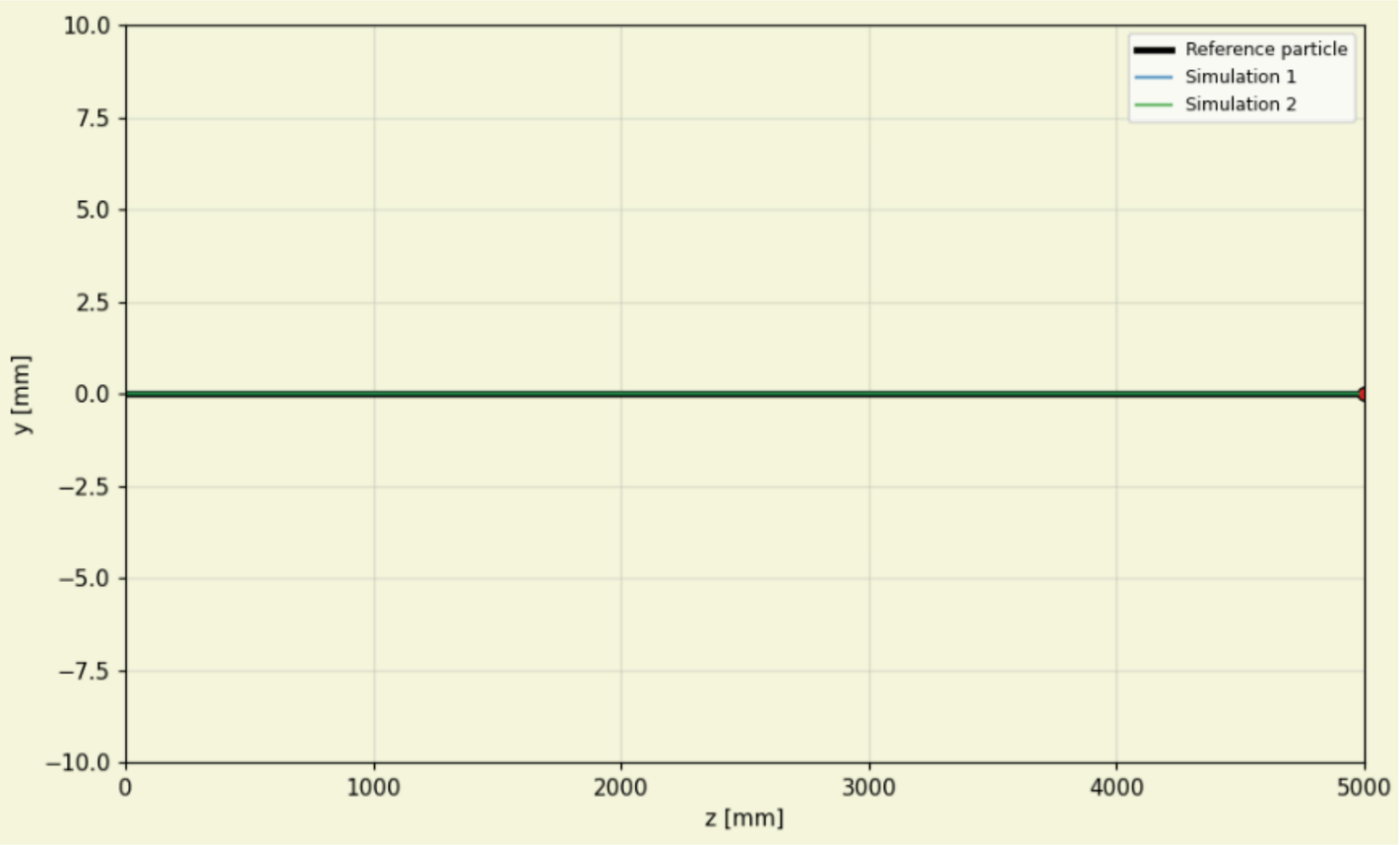
Transverse Deviation



Stochastic Reference Particle: Multiple Simulations



Closure test



- Reference and realistic reference particles through vacuum: Every realistic sample must be identical to the nominal reference.